THE PERFORMANCE AND DESIGN OF
A.C. COMMUTATOR MOTORS

THE PERFORMANCE AND DESIGN OF A.C. COMMUTATOR MOTORS

INCLUDING THE SINGLE-PHASE INDUCTION MOTOR

BY

E. OPENSHAW TAYLOR

B.Sc., A.C.G.I., D.I.C., M.I.E.E., M.Amer.I.E.E., F.R.S.E.

Assistant Professor of Electrical Engineering
Heriot-Watt College, Edinburgh

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PREFACE

THIS book has been written as a companion volume to Dr. A. E. Clayton's *The Performance and Design of D.C. Machines* and Professor M. G. Say's *The Performance and Design of A.C. Machines*. By including in the present volume the single-phase induction motor the three books have been made to cover the whole range of electrical machinery.

The a.c. commutator motor was widely investigated in the early years of the century, but from then until comparatively recently industrialists preferred the three-phase induction motor on account of its simplicity and low cost and in spite of its constant-speed characteristic and poor power factor. The gradual elimination of d.c. distribution has tended to preclude the use of the d.c. shunt motor for variable-speed drives so that modern conditions, which frequently necessitate control of speed in order to achieve maximum production and which require operation at unity power factor to obtain the best economy, have therefore again focused attention on the a.c. commutator motor. Although commutation problems still impose certain limitations, a.c. commutator motors of one type or another can be built to satisfy the requirements of practically any industrial drive; recent developments in single-phase traction also include the commutator motor as one of the most promising driving units.

The first part of the book covers items common to all types of a.c. commutator motor after which the commercially successful types are considered in turn. In each case the general principle and construction is first outlined followed by a discussion of the complexor (vector) diagram and current locus; this gives a physical picture of the behaviour of the machine but is not usually sufficiently accurate or convenient for performance calculation, and it is therefore followed by a treatment of the equivalent circuit. Since the design of a.c. commutator motors is a highly specialized field trodden by only a very few engineers, its discussion is limited to those special features showing a marked difference from the general design problems outlined in the two companion volumes.

As existing books of examples contain very few problems relating to a.c. commutator motors a number of relevant examples are included at the end of each chapter; answers are given on p. 342.

The symbols used throughout the book conform as far as possible with those listed by the British Standards Institution and with those used in the companion volumes; they are defined throughout the text as well as being summarized on pages xi–xvi. The increasing use of vector analysis for engineering problems sometimes results in ambiguities when the term "vector" is used to denote a directed line representing a quantity, such as current or voltage, that varies

sinusoidally with time; terms such as complexor, phasor and sinor have been suggested to denote the latter type of quantity, and throughout this book the term "complexor" has been used. The term "vector" is restricted to the representation of m.m.f.'s in the air gap.

I wish to thank Mr. A. C. Lane of the British Thomson-Houston Co. Ltd., and Dr. B. Schwarz of Laurence Scott and Co. Ltd., for generous assistance, Professor M. G. Say of the Heriot-Watt College for continued encouragement and help and Mr. J. Lawrence, also of the Heriot-Watt College, for valuable help in proof reading.

E. OPENSHAW TAYLOR

HERIOT-WATT COLLEGE
EDINBURGH

CONTENTS

LIST OF SYMBOLS

Instantaneous values	e,	i
Peak values	$\hat{e}$,	$\hat{\imath}$
Root-mean-square values	E,	I
Complex values	$\mathbf{E}$,	$\mathbf{I}$

Suffixes—

1, 2 and 3 indicate primary, secondary and tertiary windings
d and q indicate direct- and quadrature-axis values
t and r applied to e.m.f.'s indicate transformer and rotational e.m.f.'s

L as a suffix indicates a leakage flux or reactance

Primes (′) when applied to impedances, currents or e.m.f.'s indicate equivalent values referred to a common winding, usually the primary.

Primes when applied to turns indicate effective values ($T' = Tk_w$).

A	parameter relating to capacitor motor.
a	pairs of parallel circuits in commutator winding.
	scalar value of $\mathbf{a}$.
$\mathbf{a}$	proportionality constant relating secondary current and injected e.m.f. in "series" auxiliary machine ($\mathbf{a} = \mathbf{E}_j/\mathbf{I}_2$).
ac	specific electric loading (ampere-conductors per metre).
B	flux density (webers per square metre).
	parameter relating to capacitor motor.
$\bar{B}$	specific magnetic loading.
B_c	flux density under compole.
B_α	flux density at angle α along periphery.
b	scalar value of $\mathbf{b}$.
$\mathbf{b}$	proportionality constant relating secondary e.m.f. and injected e.m.f. in "shunt" auxiliary machine ($\mathbf{b} = \mathbf{E}_j/\mathbf{E}_2$).
C	coils in commutator winding = no. of commutator segments.
	parameter relating to capacitor motor.
c	scalar value of $\mathbf{c}$.
$\mathbf{c}$	$(1 - \mathbf{z}_1/\mathbf{z}_0)$.
D	core diameter (metres).
	parameter relating to capacitor motor.
D_c	commutator diameter (metres).
E	e.m.f.

E_b	e.m.f. across brush.
	e.m.f. due to backward-field component of flux in single-phase induction motor.
E_c	e.m.f. per coil.
E_f	e.m.f. due to forward-field component of flux in single-phase induction motor.
E_j	e.m.f. injected into secondary circuit of induction machine for regulating speed and/or power factor.
E_{jq}	quadrature component of E_j.
E_L	e.m.f. due to leakage flux.
E_{ra}	rotational e.m.f. in armature.
E_{rb}	brush e.m.f. due to rotation.
E_{ta}	transformer e.m.f. in armature.
E_{tb}	brush e.m.f. due to transformer action.
E_{ts}	transformer e.m.f. in stator.
E_{xb}	brush e.m.f. due to reactance effect.
e	base of natural logarithms.
F	force on conductor (newtons).
	m.m.f,; maximum value of fundamental of wave (ampere-turns).
F_x	m.m.f. at angle α along the periphery of gap.
F_m	total m.m.f. per pole $= I_t T_p/2$.
F_R	resultant m.m.f.
f	frequency (c/s).
f_b	backward frequency (single-phase induction motor).
	frequency of e.m.f. at brushes.
f_f	forward frequency (single-phase induction motor).
	rotational frequency of field $= pn_f$.
f_r	rotational frequency of armature $= pn_r$.
G	output coefficient.
I	current (amperes).
I_a	current in auxiliary winding of single-phase induction motor.
	total current entering armature.
I_b	current entering armature at individual brush arm $= I_a/a$.
	current due to backward-component of e.m.f. in single-phase induction motor.
I_c	circulating current in coil short-circuited by brush.
I_{1d}, I_{2d}, I_{3d}	currents in windings on direct axis.
I_f	current due to forward-component of e.m.f. in single-phase induction motor.
I_g	current in main winding of single-phase induction motor.
I_i	iron-loss component of current.
I_{2k}, I_{2n}	branch currents in equivalent circuit of motor with shunt regulating machine.
I_m	magnetizing current.

I_q	quadrature (reactive) component of current.
I_{1q}, I_{2q}, I_{3q}	currents in windings on quadrature axis.
I_t	current per turn or per conductor.
	regulator current (3-ph. doubly-fed motor).
I_w	power component of current.
J	current component (Schrage motor).
j	90° operator.
K	constant.
k	constant.
k_e	coil-span factor $= \cos(\varepsilon/2)$.
k_m	distribution factor $= \sin(\sigma/2)/(\sigma/2)$.
k_w	winding factor $= k_m \,.\, k_e$.
L	core length (metres).
	inductance (henrys).
M	mutual inductance (henrys).
m	in-phase component of **p**.
	no. of coils in series in winding.
N	no. of phases.
	rotational speed, r.p.m.
n	order of harmonic.
	quadrature component of **p**.
	rotational speed (rev/sec).
n_1	synchronous speed.
n_f	speed of rotating field.
n_r	rotational speed of rotor.
P	power.
P_x, P_y	co-ordinates of centre of circle diagram.
p	no. of pole pairs.
	scalar value of **p**.
p	ratio of effective tertiary to secondary turns of Schrage or Osnos motor $= (T_3/T_2)e^{j\beta} = m + \mathrm{j}n$.
q	any integer.
R, r	resistance (ohms).
r_a	resistance of auxiliary stator winding of single-phase induction motor.
r_b	resistance to backward currents of single-phase induction motor.
r_c	resistance per conductor.
	resistance representing iron loss in series motor.
r_{1d}, r_{2d}, r_{3d}	resistances of windings in direct axis.
r_e	resistance of auxiliary machine.
r_f	resistance to forward current components of single-phase induction motor.
r_g	resistance of main stator winding of single-phase induction motor.
r_m	magnetizing-branch resistance (parallel circuit).
r_0	magnetizing-branch resistance (series circuit).
r_{1q}, r_{2q}, r_{3q}	resistances of windings in quadrature axis.

r_s	shading-ring resistance of shaded-pole motor.
S	ratio of running speed to synchronous speed $= n_r/n_1 = f_r/f_1$.
s	fractional slip $= (n_1 - n_r)/n_1 = 1 - S$.
T	turns in series on winding.
T'	effective turns $= Tk_w$.
T_a	turns on commutator winding $= CT_c$.
T_c	turns per coil.
T_e	equivalent-star turns per phase of 3-ph. commutator winding.
T_p	transformer primary turns.
	turns per pole-pair.
T_{pph}	turns per pole-pair per phase.
T_s	transformer secondary turns.
TM	torque.
t	time.
t_c	time of commutation.
U	parameter relating to single-phase induction motor.
V	applied voltage.
v	peripheral speed of rotor.
v_c	peripheral speed of commutator.
W	parameter relating to single-phase induction motor.
w_c	width of commutator segment.
X, x	reactance (ohms).
x_1, x_2, x_3	total reactance $(x_m + x_L)$ of primary, secondary and tertiary windings.
x_{L2s}	secondary leakage reactance that is a function of slip.
x_{L1}, x_{L2}, x_{L3}	leakage reactance of primary, secondary and tertiary windings.
x_{La}	leakage reactance of auxiliary winding of single-phase induction motor.
x_{Lc}	leakage reactance of auxiliary machine.
x_{Lg}	leakage reactance of main winding of single-phase induction motor.
x_{Ls}	leakage reactance of shading ring of shaded-pole motor.
x_{Lt}	leakage reactance of auxiliary transformer.
x_m	mutual or magnetizing reactance (parallel circuit).
x_o	magnetizing reactance (series circuit).
Y, y	admittance (mhos).
Z, z	impedance (ohms).
z_1, z_2, z_3	leakage impedance of primary, secondary and tertiary windings $= \sqrt{(r^2 + x_L{}^2)}$.
z_{2s}	secondary leakage impedance that is a function of slip.
z_a	leakage impedance of auxiliary winding of single-phase induction motor.
z_g	leakage impedance of main winding of single-phase induction motor.
z_m	magnetizing-branch impedance (parallel circuit).

z_n magnetizing-branch impedance (series circuit).

α angle between brush axis and flux axis.
angular distance along periphery of air gap.
brush displacement from low-impedance position (repulsion motor).
phase angle between main- and auxiliary-winding currents in single-phase induction motor.

β angle between centre-line of brush separation and flux axis.
angle between injected e.m.f. and secondary e.m.f.
shift of movable brushes in three-phase series motor.

γ angle between the plane of the coil and flux axis of stator winding.
angle of separation for N phases $= 2\pi/N$.
half phase spread of single-phase winding (Fig. 1.3).
$1 + j(x_{L1}/z_o)\{p/(1 + p)\}$. (Schrage motor.)

δ angle between phase-winding axis and flux axis.
half interpolar angle of salient-pole machine (Fig. 1.1).
lag of secondary m.m.f. due to transformer magnetizing current in three-phase series motor.

ε chording angle of short-pitched winding.

θ angular position of coil on armature.
brush separation.
brush shift from short-circuit position (three-phase series motor).

Λ permeance.

λ 120° operator.

ρ brush shift from neutral position in three-phase motors.
turns ratio in single-phase shunt motor.

σ phase spread.

Φ flux (webers).

Φ_a flux component in auxiliary axis at starting of single-phase induction motor.

Φ_b backward component of flux in single-phase induction motor.

Φ_c compole flux.

Φ_d direct-axis component of flux.

Φ_f forward component of flux in single-phase induction motor.

Φ_g flux component in main axis at starting of single-phase induction motor.

Φ_L leakage flux.

Φ_m main or mutual flux.

Φ_q quadrature-axis component of flux.

Φ_s shading-ring flux in shaded-pole motor.

ϕ phase angle between voltage and current.

ϕ_2 phase angle between secondary current and secondary e.m.f.

ϕ_a	phase angle between voltage and current in auxiliary winding of single-phase induction motor.
ϕ_g	phase angle between voltage and current in main winding of single-phase induction motor.
ϕ_j	phase angle between secondary current and injected e.m.f.
ψ	phase angle between back e.m.f. and current.
	phase angle in three-phase series motor (Fig. 11.3).
	slot-pitch angle.
ω	angular frequency $= 2\pi f$.

UNITS

M.K.S. units are used throughout: the following conversion factors are useful.

Flux:	1 weber (Wb) = 10^8 maxwells (lines)
Flux density:	1 weber per square metre (Wb/m^2) = 10^4 gauss (lines/cm^2)
Force:	1 newton (N) = 10^5 dynes = 0·225 lb-force (Lb)
Torque:	1 newton-metre (N-m) = 10^7 dyne-cm = 0·737 Lb-ft

INTRODUCTION

The first mention of the fact that a d.c. motor would, if its field system were laminated, operate on alternating current was made by Alexander Siemens in 1884 at a meeting of the Society of Telegraph Engineers (now the Institution of Electrical Engineers). A few years later, in 1888, Prof. Ernest Wilson patented the first three-phase commutator motor. Considerable ingenuity was then shown, both in this country and elsewhere, in devising a number of different types of motor, many of which are described in an important paper by Ll. B. Atkinson read before the Institution of Civil Engineers in 1898.

The invention of the induction motor by Tesla in 1885 and its subsequent commercial development gave the world a simple and robust type of a.c. motor which adequately fulfilled most of the requirements of the industry of that time, and interest in commutator motors therefore flagged.

An exception to this arose from the development of single-phase traction for railways about 1904 for which motors having series speed-torque characteristics were required, and several types of series and repulsion motors were satisfactorily produced for the purpose. This sphere still forms a widespread field of application for the a.c. series motor and one which, with the modern development of 50-c/s traction systems, may well increase in importance.

In recent years the more exacting requirements of industrial drives have shown the limitations of the induction motor, particularly where a variable speed or good power factor is desirable. Attention has therefore again been focused on the commutator motor and improvements in design have led to the commercial use of a large number of three-phase motors in sizes up to several hundred horse-power.

The single-phase induction motor was developed coincidentally with the three-phase motor, but the complications necessary to overcome its inherent lack of starting torque hindered its development for many years. After the 1914–1918 war, however, a demand arose for motors in sizes up to about 5 h.p. for domestic and for small industrial and agricultural appliances where only a single-phase supply was available. This led to intensive work on improved methods of starting the single-phase induction motor and resulted in the modern capacitor and repulsion-start motors.

The rapidly expanding domestic field has resulted in a considerable demand for single-phase fractional-horse-power induction motors and series motors and some millions of such motors have been manufactured by mass-production methods. The single-phase series motor has the special feature of being the only type of motor which will operate satisfactorily on both alternating and direct current.

It is interesting to note that, although the constant-speed characteristic of the single-phase induction motor is suitable for the majority of drives, there is a number of applications where a variable-speed single-phase motor would be desirable—no commercially practicable motor of this type has, however, yet been developed.

PART I: GENERAL PRINCIPLES

CHAPTER 1

WINDINGS AND MAGNETOMOTIVE FORCES

THE current in the windings gives rise to the m.m.f.'s upon which the action of a motor depends, so that a study of windings and their m.m.f.'s forms a natural starting point for the study of motor performance and design.

Types of Winding

The stator windings of a commutator motor are usually single- or three-phase, depending on the type of motor, and are generally similar in arrangement to the distributed single- and double-layer windings used for induction and synchronous machines. For small fractional-horse-power single-phase motors, however, and for Scherbius machines, a salient-pole winding is often adopted.

The commutator windings, mounted on the rotor, are lap- or wave-connected as in the ordinary d.c. machine. Lap connexion is more common, as commutator voltages must be kept low.

In some machines there may also be a phase winding on the rotor, fed through slip rings, and, in the case of the single-phase induction motor, the rotor winding is usually of the cage type.

The m.m.f. distribution of any of these windings can be drawn in the usual way as a stepped wave, and an expression for the mean line through the steps given as a Fourier series. In discussing the behaviour of the machines it is generally only the fundamental that need be considered although the harmonics may have important secondary effects in certain cases; it is, however, the usual aim of designers to keep the harmonics as small as practicable.

Stator Windings

SALIENT-POLE ARRANGEMENT. The arrangement of such a winding is shown, for two poles, in Fig. 1.1(*a*). The m.m.f. wave, assumed rectangular, is shown in Fig. 1.1(*b*), and the Fourier expression* for this wave is—

$$F_\alpha = \hat{F}_m \sin \omega t \times \frac{4}{\pi}\left(\cos \delta \sin \alpha + \frac{\cos 3\delta}{3} \sin 3\alpha + \cdots + \frac{\cos n\delta}{n} \sin n\alpha\right) \qquad (1.1a)$$

* See Appendix 1.

where δ is as shown in Fig. 1.1,

$\hat{F}_m$ is the peak ampere-turns per pole $= \hat{I}_t T_p/2$,

$\hat{I}_t$ is the peak current per turn,

T_p is the turns per pole-pair,

α is angular distance measured along the armature periphery.

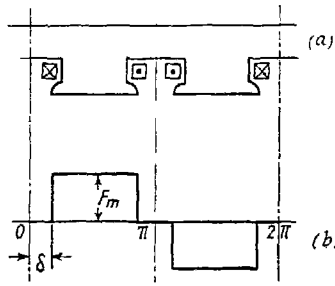


FIG. 1.1. M.M.F. OF SALIENT-POLE WINDING

(a) Winding arrangement.
(b) M.m.f. wave.

If I_t is the r.m.s. current in each turn, the peak value of the fundamental of the m.m.f. wave is

$$F = \{2(\sqrt{2})/\pi\} I_t T_p \cos \delta = 0{\cdot}9 I_t T_p \cos \delta \text{ ampere-turns} \quad . \quad (1.1b)$$

The magnitudes of the 3rd, 5th and 7th harmonics expressed as a percentage of the fundamental are shown in Fig. 1.2 for various values of the ratio pole-arc/pole-pitch, a ratio of about 0·67 being typical for small single-phase motors.

DISTRIBUTED SINGLE-PHASE WINDING. Non-salient-pole single-phase windings are generally of the single-layer type as shown in Fig. 1.3(a). The slots here shown empty may, in an actual motor, contain an auxiliary winding, e.g. the starting winding of a single-phase induction motor.

The expression for the m.m.f. of the trapezoidal wave (Fig. 1.3(c)) resulting from this winding is—

$$F_\alpha = \hat{F}_m \sin \omega t \times \frac{4}{\pi\gamma} \left(\sin\gamma \sin\alpha + \frac{\sin 3\gamma}{9} \sin 3\alpha + \cdots + \frac{\sin n\gamma}{n^2} \sin n\alpha \right) \quad . \quad (1.2a)$$

where γ is as shown in Fig. 1.3 and $\hat{F}_m = \hat{I}_t T_p/2$ ampere-turns.

If I_t is the r.m.s. current per turn the peak value of the fundamental of the wave is—

$$F = \frac{2\sqrt{2}}{\pi} \times \frac{\sin\gamma}{\gamma} \times I_t T_p = 0{\cdot}9 \frac{\sin\gamma}{\gamma} I_t T_p$$
$$= 0{\cdot}9 k_{m1} I_t T_p \text{ ampere-turns} \quad . \quad . \quad . \quad (1.2b)$$

where k_{m1} is the distribution factor (fundamental) for a winding having a phase spread of $\sigma = 2\gamma$, the values being shown in Fig. 1.4.

The magnitudes of the 3rd, 5th and 7th harmonics for various ratios of wound to total slots are shown in Fig. 1.5, from which it can be seen that an arrangement with about 70 per cent of the slots wound gives the minimum low-order harmonics.

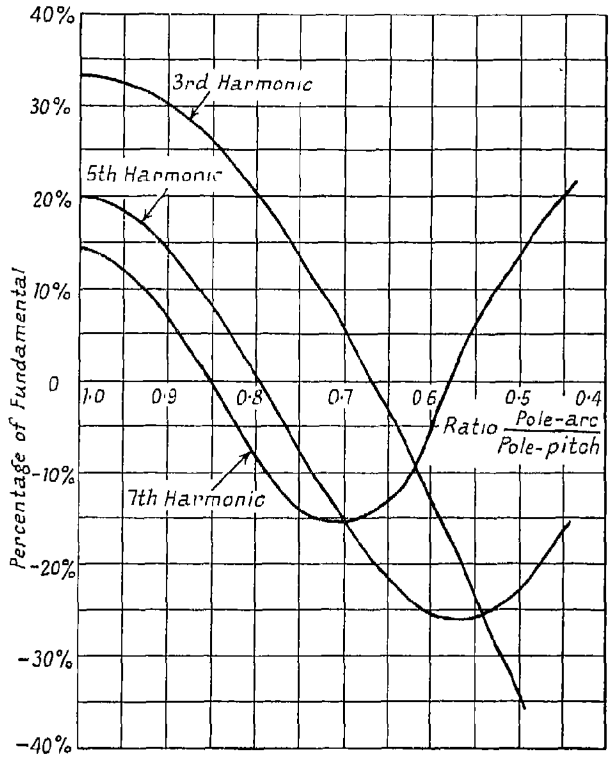


FIG. 1.2. HARMONICS AS PERCENTAGE OF FUNDAMENTAL FOR RECTANGULAR M.M.F. WAVE

In some small single-phase motors it may be desirable to reduce the harmonics still further by grading the winding, i.e. having different numbers of conductors in each slot thereby giving an m.m.f. wave which more nearly approaches a sine wave.

Suppose, for instance, that a motor with 9 slots per pole is to be wound as shown in Fig. 1.6, the desired sine wave of m.m.f. being as shown in the lower part of the diagram. A stepped wave can be superimposed on the sine wave as shown and it can be seen that the m.m.f. is given by $F \sin 10°$ at tooth A, $F \sin 30°$ at tooth B, etc.,

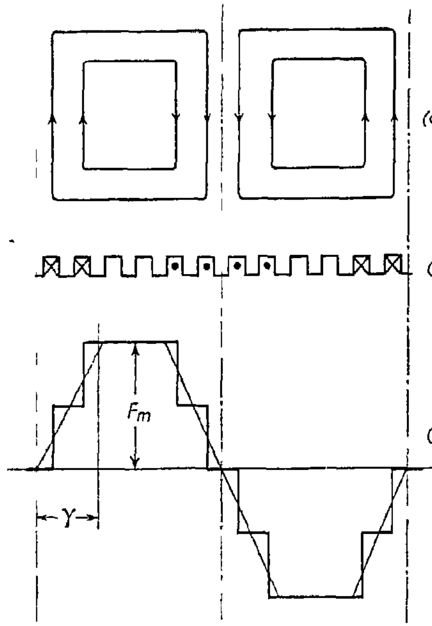


Fig. 1.3. Single-phase Distributed Winding

(a) Winding diagram.
(b) Current distribution.
(c) M.m.f. wave.

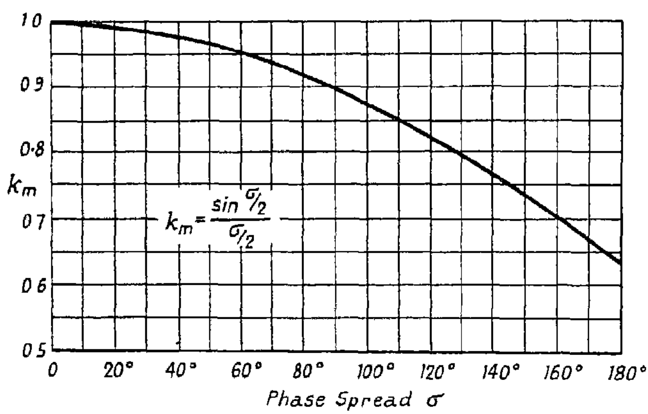


Fig. 1.4. Values of Distribution Factor for Fundamental with Various Values of Phase Spread

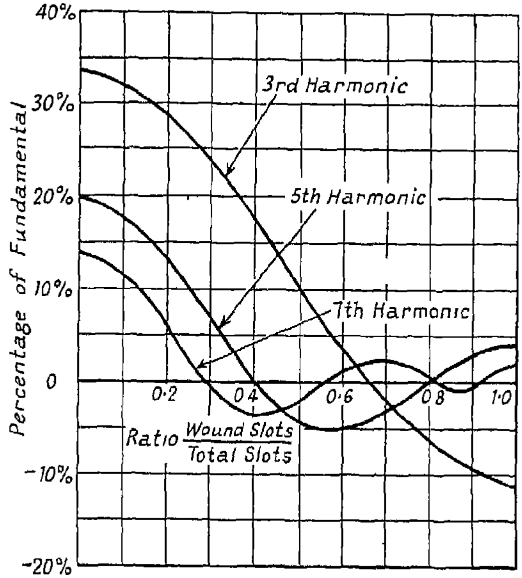


FIG. 1.5 HARMONICS AS PERCENTAGE OF FUNDAMENTAL FOR TRAPEZOIDAL WAVE

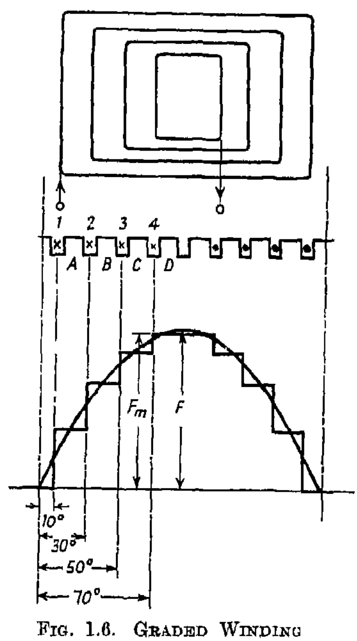


FIG. 1.6. GRADED WINDING

or, in general terms, by $F \sin (\psi/2)$, $F \sin (\psi + \psi/2)$, etc., ψ being the angular slot pitch.

In the above case, with $\psi = 20°$, the values are—

Tooth A: $F \sin 10° = 0{\cdot}174\ F$
Tooth B: $F \sin 30° = 0{\cdot}50\ F$
Tooth C: $F \sin 50° = 0{\cdot}766\ F$
Tooth D: $F \sin 70° = 0{\cdot}940\ F$

The total m.m.f. of all the turns is F_m occurring at tooth D so that $F_m = 0{\cdot}940\ F$. The above values in terms of the total m.m.f. thus become—

Tooth A: $(0{\cdot}174/0{\cdot}94)F_m = 0{\cdot}185\ F_m$
Tooth B: $(0{\cdot}500/0{\cdot}94)F_m = 0{\cdot}532\ F_m$
Tooth C: $(0{\cdot}766/0{\cdot}94)F_m = 0{\cdot}818\ F_m$
Tooth D: $= 1{\cdot}0\ F_m$

Hence the m.m.f. due to the conductors in the different slots is—

Slot 1: $0{\cdot}185\ F_m$
Slot 2: $(0{\cdot}532 - 0{\cdot}185)F_m = 0{\cdot}347\ F_m$
Slot 3: $(0{\cdot}818 - 0{\cdot}532)F_m = 0{\cdot}286\ F_m$
Slot 4: $(1{\cdot}0 - 0{\cdot}818)F_m = 0{\cdot}182\ F_m$

The total number of conductors in the group can thus be split up in the ratio of these m.m.f.'s and the nearest possible approach to a sinusoidal m.m.f. wave will be obtained.

THREE-PHASE STATOR WINDINGS. The three-phase single- and double-layer winding is treated in a companion volume* where it is shown that if the windings and the currents in them are symmetrical there will be—

(i) A fundamental rotating m.m.f. of constant magnitude moving at synchronous speed and having a peak value

$$F = \{3(\sqrt{2})/\pi\}k_w I_t T_{pph} \text{ ampere-turns} \qquad (1.3)$$

where T_{pph} is the turns per pole-pair per phase,

$k_w = k_m \cdot k_e$, k_m and k_e being the distribution and coil-span factors for the fundamental.

(ii) Harmonic m.m.f.'s of the order $6q - 1$ (q being any integer) moving in the *opposite* direction to the fundamental at $1/n$ of synchronous speed (n being the order of the harmonic); this group includes the 5th, 11th, 17th, etc., harmonics, and the magnitude of each is $1/n^2$ of that of the fundamental, e.g. 1/25 or 4 per cent for the 5th harmonic.

(iii) Harmonic m.m.f.'s of the order $6q + 1$ moving in the *same* direction as the fundamental at $1/n$ of synchronous speed. This

* Say, *The Performance and Design of A.C. Machines*, Ch. X (Pitman).

group includes the 7th, 13th, 19th, etc., harmonics, and the magnitude is also $1/n^2$ of that of the fundamental, e.g. 2·04 per cent for the 7th harmonic.

Rotor Phase Windings

Some commutator motors have a three-phase winding on the rotor fed through slip rings. If the rotor is stationary the conditions are similar to those obtaining in the stator winding, i.e. a fundamental rotating m.m.f. is set up moving at synchronous speed, together with the various harmonics. If the rotor is moving, the speed of the fundamental m.m.f. will still be synchronous speed relative to the rotor conductors but, relative to the fixed stator, the speed will be (synchronous speed $\pm$ rotor speed), the positive sign referring to the case where the rotor is moving in the same direction as the m.m.f. If the rotor is moving at synchronous speed in the opposite direction to the m.m.f., the speed of the m.m.f. relative to the fixed stator will be zero, i.e. it will be stationary as in a d.c. machine.

Commutator Windings

SINGLE-PHASE. With two brushes per pole-pair spaced 180° (electrical) apart the arrangement is similar to that of an ordinary d.c. machine. With full-pitch coils a triangular m.m.f. wave results which is stationary relative to the brushes as shown in Fig. 1.7. Analytically this is a special case of the trapezoidal wave (eq. (1.2*a*), p. 2) with $\gamma = \pi/2$. If the coils are short pitched the coil-span factor for each harmonic can be added to each term giving the expression for the m.m.f. as—

$$F_\alpha = \hat{F}_m \sin \omega t \times \frac{8}{\pi^2}\left(k_{e1} \sin \alpha + k_{e3} \times \frac{1}{9} \sin 3\alpha + \cdots + k_{en} \times \frac{1}{n^2} \sin n\alpha\right) \quad . \quad (1.4a)$$

where $\hat{F}_m = \hat{I}_t(T_a/2p)$,

T_a is the total number of rotor turns,

$\hat{I}_t$ is the peak current per turn $= (\sqrt{2})I_a/2a$,

I_a is the total r.m.s. current entering the armature.

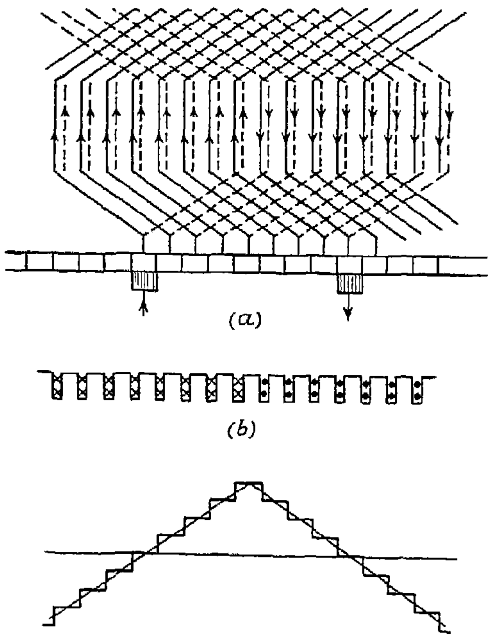
The peak value of the fundamental of the wave is thus—

$$F = (8/\pi^2) \times k_e \hat{I}_t(T_a/2p)$$
$$= \{2(\sqrt{2})/\pi^2\}k_e(I_a T_a/ap) \text{ ampere-turns} \quad . \quad . \quad (1.4b)$$

It should be noted that the peaks of the m.m.f. wave occupy the same position as the conductors undergoing commutation, i.e. the axis of the m.m.f. is at 90° to the axis of the coils short-circuited by the brushes. Under these conditions the m.m.f. is said to be acting

along the brush axis and it pulsates at the frequency of the current supplied to the brushes.

If the armature is rotating, the actual conductor occupying any particular position is continually changing but the current distribution around the armature is not affected. For a given current, therefore, the magnitude and the frequency of pulsation of the m.m.f. are independent of the speed of rotation.



(a) (b) (c)

FIG. 1.7. M.M.F. OF SINGLE-PHASE COMMUTATOR WINDING

(a) Winding diagram.
(b) Current distribution.
(c) M.m.f. wave.

THREE-PHASE: THREE BRUSHES PER POLE-PAIR. The winding may be represented diagrammatically as in Fig. 1.8(a) or by a winding diagram as in Fig. 1.9. It is thus similar to a three-phase mesh-connected winding with 120° phase spread. If symmetrical three-phase currents of r.m.s. value I_b are entering at each of the brushes the currents in each turn of the winding will be $I_t = I_b/\sqrt{3} = I_a/a\sqrt{3}$ and will be displaced by 30° from the brush currents as shown in Fig. 1.8(b). The m.m.f. wave can be drawn by considering a particular moment in the cycle, e.g. when the current entering brush A is at its peak value of, say, 100 A; the currents in

the winding are then as shown in Fig. 1.8(*a*), and the current distribution and m.m.f. wave can be drawn as in Fig. 1.9(*b*) and (*c*). It can be seen that at this particular moment in the cycle the m.m.f. wave is acting along an axis at 90° to the axis of the coil being commutated by brush *A*. As in the single-phase case, the rotation of the armature does not affect the current distribution around the periphery so that the m.m.f. wave is the same as would be set up in a stationary armature, i.e. it has a fundamental and harmonics

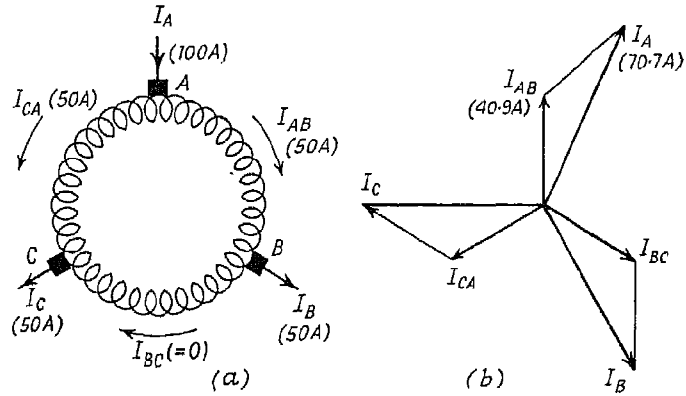


FIG. 1.8. CURRENTS IN THREE-PHASE COMMUTATOR WINDING

(*a*) Diagram of winding (instantaneous current values when I_A is at its peak value).
(*b*) Complexor diagram (r.m.s. currents).

rotating relative to the fixed brushes at the appropriate fractions of synchronous speed.

The magnitude of the fundamental is, from eq. (1.3),

$$F = \{3(\sqrt{2})/\pi\}k_{m120}k_e I_t T_{pph} = \{(\sqrt{6})/\pi\}k_{m120}k_e(I_a/a)T_{pph}$$
$$= \{3(\sqrt{2})/2\pi^2\}k_e(I_a/a)(T_a/p) \text{ ampere-turns} \quad . \quad . \quad (1.5)$$

where T_{pph} is the rotor turns per pole-pair per phase and T_a is the total rotor turns.

An alternative, and often more convenient, expression is given, on p. 14, in terms of the equivalent-star turns.

THREE-PHASE: DIAMETRIC BRUSHES. An alternative arrangement with six brushes per pole-pair is shown in Fig. 1.10, each phase being led to diametrically opposite brushes. The fundamental m.m.f. can be found by extending eq. (1.4*b*) for the three phases—

$$F = F_{\mathrm{I}} + F_{\mathrm{II}} + F_{\mathrm{III}}$$
$$= \{2(\sqrt{2})/\pi^2\}k_e(I_a/a)(T_a/p)\{\sin \omega t \sin \alpha$$
$$+ \sin(\omega t - 2\pi/3)\sin(\alpha - 2\pi/3)$$
$$+ \sin(\omega t - 4\pi/3)\sin(\alpha - 4\pi/3)\}$$
$$= (3/2)\{2(\sqrt{2})/\pi^2\}k_e(I_a/a)(T_a/p)\cos(\omega t - \alpha)$$

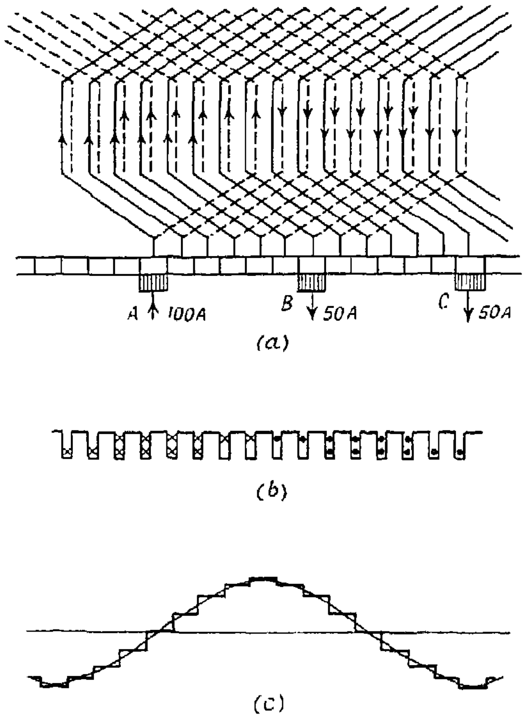


FIG. 1.9. M.M.F. OF THREE-PHASE COMMUTATOR WINDING

(a) Winding diagram (instantaneous currents when I_A is at its peak value).
(b) Current distribution diagram
(c) M.m.f. wave.

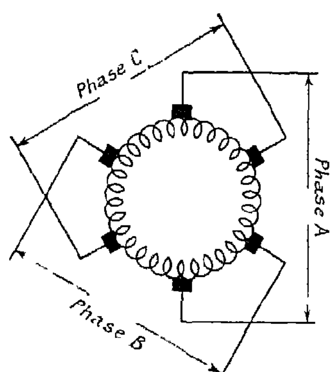


FIG. 1.10. THREE-PHASE DIAMETRIC CONNEXION

The peak of the fundamental is therefore—

$$F = \{3(\sqrt{2})/\pi^2\}k_e(I_a/a)(T_a/p) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.6)$$

It is thus seen that for a given armature the m.m.f. produced by a given brush current is twice that of the three-brush arrangement. It is shown later, however, that the voltage required is also twice that of the three-brush arrangement so that the volt-amperes for a given m.m.f. is the same.

Summary of Three-phase M.M.F.'s

The different circumstances in which rotating m.m.f.'s are produced by three-phase currents are summarized in Table 1.1.

TABLE 1.1
PRODUCTION OF ROTATING M.M.F.'S

	Arrangement	Nature of M.M.F. (fundamental)
1	3-phase currents fed to stationary 3-phase winding.	M.m.f. rotates at synchronous speed n_1.
2	3-phase currents fed to 3-phase winding rotating at n_r.	M.m.f. rotates at $n_1 \pm n_r$.
3	3-phase currents fed to 3 brushes on rotating commutator winding.	M.m.f. rotates at synchronous speed corresponding to frequency of currents and independent of conductor speed.

Resultant M.M.F.

The stator and rotor m.m.f.'s normally act simultaneously giving a resultant m.m.f. which actually sets up the flux in the air gap. The value of this resultant m.m.f. can be obtained by drawing the two m.m.f. waves in their correct relative positions and adding the ordinates at the various points around the gap periphery. A quicker and more convenient method, however, is to represent the m.m.f. waves by their fundamentals and draw the space vectors corresponding to these.

PULSATING M.M.F.'s. In the case of a pulsating m.m.f. the space vector will represent the peak value of the fundamental of the m.m.f. wave and will point along the axis in which the m.m.f. is acting.

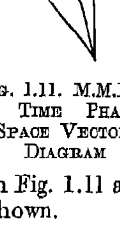


FIG. 1.11. M.M.F.'s IN TIME PHASE; SPACE VECTOR DIAGRAM

If the stator and rotor m.m.f.'s pulsate at the same frequency and are in time phase but acting in different directions, their vectors will be as shown in Fig. 1.11 and the resultant m.m.f. F_R will be their vector sum as shown.

If the two m.m.f.'s are not in time phase, but are displaced as in Fig. 1.12(*a*), the resultant m.m.f. at various instants of time throughout the cycle can be found as in Fig. 1.12(*b*). It can be seen that this resultant rotates, traversing one revolution in one cycle, and also varies in magnitude throughout the revolution. The field produced by such an m.m.f. is known as an *elliptic rotating field.* If the two m.m.f.'s are equal in magnitude and displaced by 90° in space the rotating field becomes constant in magnitude and rotates at a constant speed as in a two-phase motor.

ROTATING M.M.F.'s. In any practical machine in which polyphase currents set up rotating m.m.f.'s, the fundamental m.m.f.'s set up by

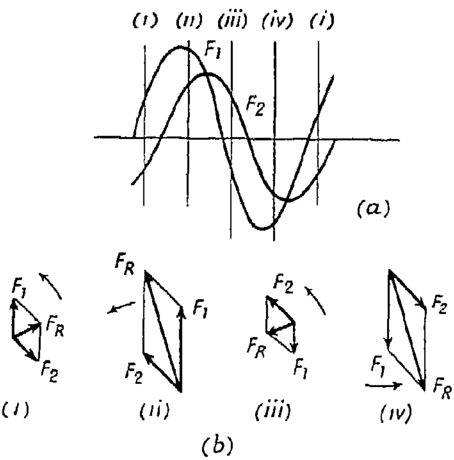


FIG. 1.12. M.M.F.'s NOT IN TIME PHASE—ROTATING RESULTANT
(*a*) Time diagram
(*b*) Space diagram

the different windings always rotate at the same speed and in the same direction around the air gap. The resultant m.m.f. will therefore also rotate at this speed and can be found by adding the waves in their correct relative positions or by representing their fundamental components by vectors as in the pulsating field case.

Diagrammatic Representation of Windings

It is convenient to have standard conventions for representing the windings diagrammatically so that the relative positions of the m.m.f.'s can be seen at a glance on a simple diagram.

PHASE WINDINGS. The usual method of representing a phase winding such as is normally wound on the stator is shown in Fig. 1.13(*a*). The m.m.f. produced is coaxial with the coils of the winding. The sense of the m.m.f. depends on the direction of the current, and

a convenient convention is to assume it to be the same as the general direction of current through the winding as shown.

The three-phase winding is as shown in Fig. 1.13(*b*). M.m.f. diagrams show that the direction of the rotating m.m.f. at any

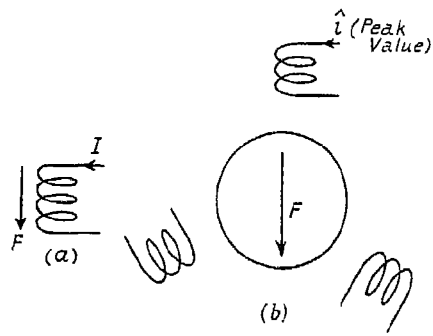


FIG. 1.13. DIAGRAMMATIC REPRESENTATION OF STATOR WINDINGS
(*a*) Single-phase winding
(*b*) Three-phase winding.

instant is along the axis of the phase carrying the peak current at that instant and, if the above convention is adhered to, it will be in a sense similar to that of the current along that phase.

COMMUTATOR WINDINGS. The usual way of representing a commutator winding is shown in Fig. 1.14, the brushes being represented as standing on the armature periphery and in contact with the top coil sides to which they are actually connected, i.e. the conductors which are undergoing commutation; due to the inclination of the overhang this is about half a pole-pitch away from their actual position on the commutator. Reference to Fig. 1.7 shows that with this convention for drawing the winding, the direction of the m.m.f. is along the line of the brushes, i.e. along the *brush axis*. The sense of the m.m.f. is specified by the positions of the dots and crosses representing the current directions in the conductors. Whether the dots are placed to the left-hand or the right-hand side of the axis is purely arbitrary, and a convenient convention, adopted here, is to put the dots to the left-hand side of the axis when looking in the direction of current flow in the circuit leading to and from the brushes as shown in Fig. 1.14.

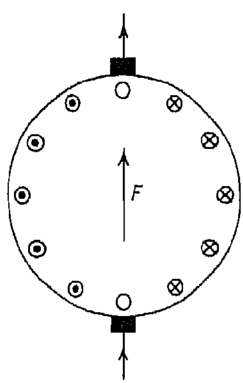


FIG. 1.14. DIAGRAMMATIC REPRESENTATION OF SINGLE-PHASE COMMUTATOR WINDING

COMMUTATOR WINDING CARRYING THREE-PHASE CURRENTS. A commutator winding with three brushes per pole-pair is shown in Fig. 1.8 and is seen to be a delta-connected circuit. It is convenient when studying the behaviour of three-phase commutator motors to represent this by its equivalent-star circuit as shown in Fig. 1.15.

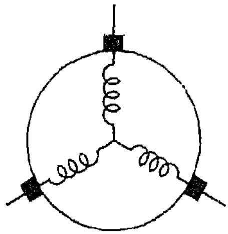


FIG. 1.15. EQUIVALENT-STAR REPRESENTATION OF THREE-PHASE COMMUTATOR WINDING

Reference to Fig. 1.9 shows that, when the current entering brush A is at its peak value, the m.m.f. is acting along an axis through that brush and in a sense similar to the direction of current in the circuit connected to the brush.

The current in the equivalent-star winding is equal to the brush current, i.e. equal to $\sqrt{3}$ times the actual winding current between brushes, and the equivalent impedance per phase is 1/3 of the actual impedance of the section of winding between brushes; the number of turns per phase is $1/\sqrt{3}$ times the actual turns between brushes. The expression (1.5) for the m.m.f. thus becomes, in terms of the total currents and the equivalent-star turns in series per phase T_e—

$$F = \{3(\sqrt{2})/\pi\}k_{m120°}k_e(I_aT_e/a) \text{ ampere-turns} \qquad (1.7)$$

Fluxes

The m.m.f.'s due to the currents in the windings give rise to the fluxes upon which the action of the machine depends, and these may be divided into two main components as shown in Fig. 1.16.

MAIN OR MUTUAL FLUXES. These cross the air gap and link both stator and rotor windings; they are produced by the resultant

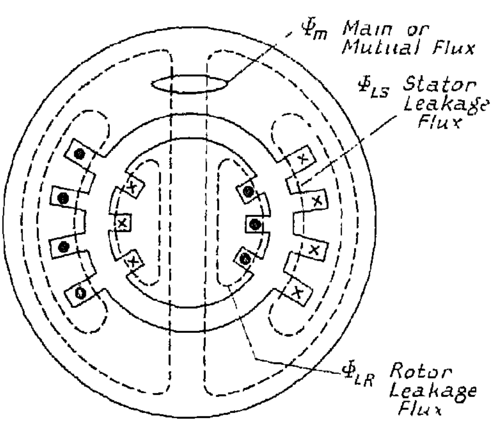


FIG. 1.16. FLUXES IN MACHINE

m.m.f. of the various windings acting on the main flux paths, and are shown as Φ_m in Fig. 1.16.

LEAKAGE FLUXES. These link with only the stator or rotor windings and do not cross the air gap. They are produced by the m.m.f. of one winding only (except in special cases) and are shown as Φ_{LS} and Φ_{LR} in Fig. 1.16.

The action of the machine depends primarily on the main flux although the leakage fluxes have important secondary effects.

It is often convenient, particularly with single-phase machines, to assume that each m.m.f., e.g. the stator and rotor m.m.f.'s, each produce a separate flux and to assume, in discussing the behaviour of the machine, that each flux has a separate existence. Such a procedure simplifies the analysis but is only accurate if saturation of the magnetic circuit may be neglected.

Torque

The force on the conductors, and therefore the torque, results from the interaction of the current in the conductors and the flux in accordance with the fundamental equation

$$F = BiL \text{ newton-metres*} \qquad (1.8)$$

The general effect may be seen by assuming the m.m.f.'s on the stator and rotor to give rise to separate fluxes. In Fig. 1.17(*a*) the

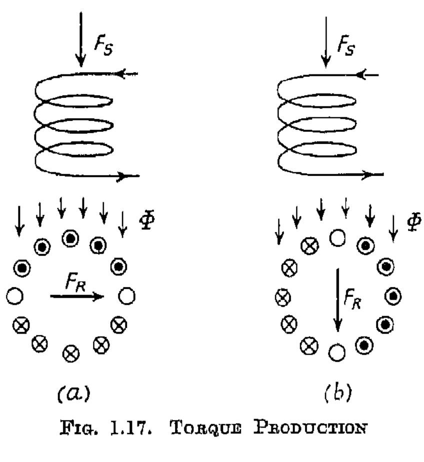


FIG. 1.17. TORQUE PRODUCTION

current in the stator winding produces an m.m.f., and therefore a flux, acting vertically as shown. The rotor conductors are lying in this flux and have a force exerted on them which, according to Fleming's left-hand rule, tends to turn each conductor clockwise. In Fig. 1.17(*b*) the stator and rotor m.m.f.'s are both acting vertically,

* 1 newton-metre = 0·737 Lb-ft.

and it can be seen, by considering the direction of the force on each conductor, that there is no resultant torque.

It may thus be stated that a torque will only arise as a result of currents in two windings when their m.m.f.'s are at 90° or have components at 90°. The precise value of the torque can be calculated from eq. (1.8) when the dimensions and winding arrangements for the machine in question are known.

EXERCISES 1

1. A motor has its winding arranged in a single slot per pole. Each slot contains 30 conductors each carrying 10 A (r.m.s.). Determine the magnitude of the fundamental, 3rd and 5th harmonics of the m.m.f. wave produced by the winding at the instant when the current is at its peak value. Express the harmonics as fractions of the fundamental.

2. The brushes of a 4-pole commutator winding having 120 full-pitch turns are connected to terminals. There are (*a*) 2 brushes, (*b*) 3 brushes and (*c*) 6 brushes per pole-pair and in each case 100 A (r.m.s.) is fed to the terminals. Find the fundamental of the m.m.f. wave in each case. Determine for case (*c*) the equivalent-star turns in series for the winding and use this value to find the fundamental of the m.m.f. wave.

3. A 2-pole stator has 24 slots and carries a 1-phase, 1-layer winding arranged in 14 slots with 10 conductors per slot, each conductor carrying 7·0 A. Draw the m.m.f. wave when the current is at its peak value. Sketch the fundamental of the wave and compare with the calculated value.

It is desired to devise a winding for the above machine which will give a closer approximation to a sine wave by having different numbers of conductors in the slots. Work out details of such a winding and draw its m.m.f. wave.

4. A 4-pole, lap-connected commutator winding is arranged in 60 slots and has 6 conductors per slot. The winding is short-pitched by two slot-pitches. Draw to scale the peak m.m.f. wave for one pole-pair when 50 A (r.m.s.) is fed to the winding. Calculate and sketch the fundamental.

5. A 4-pole, 3-phase commutator motor has a 1-layer stator winding arranged in 84 slots with 8 conductors per slot connected in two parallel paths. The rotor carries a full-pitch lap-connected winding arranged in 108 slots with 6 conductors per slot. Three-phase currents of 100 A pass through the stator winding and are fed to the commutator winding by three brushes per pole-pair spaced at 120° (elec.). Draw, when the current in one stator phase is a maximum, the m.m.f. wave due to (*a*) the stator winding, (*b*) the rotor winding and (*c*) both windings when the brushes are located so that the m.m.f.'s act along axes displaced from each other by 45° (elec.). Sketch the fundamental of the resultant and compare with the calculated value.

6. A 2-pole machine has two windings arranged with their axes in quadrature and carrying currents displaced from each other by 90° in time phase. Both windings give sinusoidally-distributed m.m.f. waves, one having a maximum value of 1,000 AT and the other of 700 AT. Draw the resultant m.m.f. vector at various moments in the cycle and sketch its locus.

CHAPTER 2

ELECTROMOTIVE FORCES

Generation of E.M.F.

The fluxes in the machine give rise to e.m.f.'s in the windings as a result either of relative motion between the flux and conductors (*rotational e.m.f.'s*) or of pulsations of the flux linked with the windings (*transformer e.m.f.'s*); rotational e.m.f.'s are denoted by the suffix r and transformer e.m.f.'s by the suffix t. In a.c. commutator motors both actions may occur separately or simultaneously in the same winding. The e.m.f.'s produced may also be grouped according as to whether they are set up by main or mutual fluxes or by leakage fluxes.

Transformer E.M.F.'s in Phase Windings

IN TERMS OF FLUX. The fundamental expression for the e.m.f. induced in a single coil linked with an alternating flux, $\hat{\Phi}$ sin ωt webers,* is—

$$e_c = -\omega T_c \hat{\Phi} \cos \omega t \text{ volts}$$

The r.m.s. value is

$$E_c = (1/\sqrt{2})\omega T_c \hat{\Phi} = (\sqrt{2})\pi f T_c \hat{\Phi} \text{ volts}$$

If the winding has several coils distributed over the armature surface as in Fig. 2.1 it can be seen that although coil aa is linked with the whole of the flux, the coil bb is linked with only a portion of it, i.e. with $\Phi \cos \gamma$ if the flux is sinusoidally distributed, γ being the angle between the plane of the coil and the flux axis.

If there are m coils connected in series arranged symmetrically with respect to the flux, the total e.m.f. is—

$$E_t = (\sqrt{2})\pi f m T_c \sum_{-\sigma/2}^{\sigma/2} \hat{\Phi} \cos \gamma \text{ volts}$$

If m is large

$$E_t = (\sqrt{2})\pi f m T_c (1/\sigma) \int_{-\sigma/2}^{\sigma/2} \hat{\Phi} \cos \gamma \, d\gamma$$

$$= (\sqrt{2})\pi f T \hat{\Phi} \frac{\sin (\sigma/2)}{\sigma/2}$$

$$= (\sqrt{2})\pi f T \hat{\Phi} k_m \text{ volts}$$

where $k_m = \sin (\sigma/2)/(\sigma/2)$ is the distribution factor allowing for the spread of the winding, the values for various phase spreads being

* Since the *peak* value of an alternating flux is invariably used in calculations the circumflex is normally omitted.

shown in Fig. 1.4, p. 4. The spread for a three-phase winding is normally 60°, but in commutator-motor practice any value up to 180° may occur.

If the coils are short-pitched the coil-span factor* (given by $k_e = \cos(\varepsilon/2)$, ε being the electrical angle by which the coils are

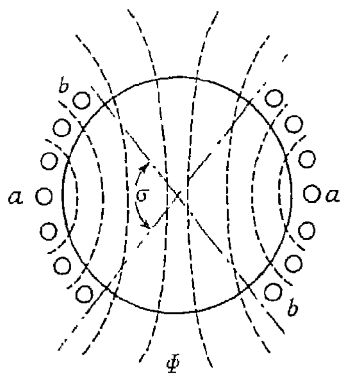


FIG. 2.1. WINDING LINKED WITH ALTERNATING FLUX

short-pitched) must also be introduced so that the complete expression for the transformer e.m.f. is—

$$E_t = (\sqrt{2})\pi f T\hat{\Phi} k_m \,.\, k_e \text{ volts} \qquad (2.1)$$

If the axis of the winding is inclined at an angle δ to that of the flux, only that component of flux which is coaxial with the winding, i.e. $\Phi \cos\delta$, will be instrumental in generating transformer e.m.f., so that the expression then becomes—

$$E_t = (\sqrt{2})\pi f T\Phi \cos\delta \,.\, k_m \,.\, k_e \qquad (2.2)$$

The expression Tk_mk_e is referred to as the *effective turns* and written T' so that

$$E_t = (\sqrt{2})\pi f T'\Phi \cos\delta \qquad (2.3)$$

IN TERMS OF MUTUAL REACTANCE. Instead of expressing the e.m.f. in terms of the flux it is often convenient, particularly when studying the behaviour of machines by their equivalent circuits, to express the e.m.f. in terms of the mutual reactance between the various windings.

Before expressing it in this way it is convenient to assume all circuits to be reduced to the same number of turns, e.g. the number of effective turns on the primary winding, so that the mutual inductance M between all windings linked with the same flux is the

* Say, *The Performance and Design of A.C. Machines*, Ch. X (Pitman).

same. To do this the currents, e.m.f.'s, impedances and admittances of any other circuits must be expressed in terms of the primary as follows.

Currents referred to primary—

$$I_2' = I_2(T_2'/T_1'); \; I_3' = I_3(T_3'/T_1'); \text{ etc.} \quad . \quad (2.4a)$$

E.m.f.'s referred to primary—

$$E_2' = E_2(T_1'/T_2'); \; E_3' = E_3(T_1'/T_3'); \text{ etc.} \quad . \quad (2.4b)$$

Impedances referred to primary—

$$z_2' = z_2(T_1'/T_2')^2; \; z_3' = z_3(T_1'/T_3')^2; \text{ etc.} \quad . \quad (2.4c)$$

Admittances referred to primary—

$$y_2' = y_2(T_2'/T_1')^2; \; y_3' = y_3(T_3'/T_1')^2; \text{ etc.} \quad . \quad (2.4d)$$

If there are, say, three windings combining to produce one particular flux, the total flux may be written—

$$\mathbf{\Phi} = \mathbf{\Phi}_1 + \mathbf{\Phi}_2 + \mathbf{\Phi}_3$$

where $\mathbf{\Phi}_1$, $\mathbf{\Phi}_2$ and $\mathbf{\Phi}_3$ are the fluxes due to the m.m.f. of each winding by itself.

Since all the windings now have the same number of effective turns the mutual inductance may be written—

$$M = \Phi_1 T_1'/\hat{\imath}_1 = \Phi_2 T_1'/\hat{\imath}_2' = \Phi_3 T_1'/\hat{\imath}_3'$$

so that, using complex quantities,

$$\mathbf{\Phi}_1 = M\mathbf{I}_1(\sqrt{2})/T_1'; \; \mathbf{\Phi}_2 = M\mathbf{I}_2'(\sqrt{2})/T_1'; \; \mathbf{\Phi}_3 = M\mathbf{I}_3'(\sqrt{2})/T_1'$$

Substituting for the flux in the expression (2.3) and omitting $\cos \delta$ thus gives for the primary winding—

$$\begin{aligned} E_{t1} &= (\sqrt{2})\pi f T_1'\{M(\sqrt{2})/T_1'\}(\mathbf{I}_1 + \mathbf{I}_2' + \mathbf{I}_3') \\ &= \omega M(\mathbf{I}_1 + \mathbf{I}_2' + \mathbf{I}_3') \text{ volts} \end{aligned}$$

Written symbolically, since the e.m.f. lags the flux by 90°, this becomes—

$$\mathbf{E}_{t1} = -\,\mathrm{j}\omega M(\mathbf{I}_1 + \mathbf{I}_2' + \mathbf{I}_3') = -\,\mathrm{j}x_m(\mathbf{I}_1 + \mathbf{I}_2' + \mathbf{I}_3') \text{ volts} \quad . \quad . \quad . \quad (2.5)$$

where x_m is the mutual reactance.

In drawing complexor diagrams and writing down analytical expressions relating to this voltage it is usual to regard it as a voltage drop, similar to an IR drop, so that—

$$\text{voltage drop} = \mathrm{j}x_m(\mathbf{I}_1 + \mathbf{I}_2' + \mathbf{I}_3')$$

Since the windings all are assumed to have the same number of turns, $\mathbf{E}_{t2}'$ and $\mathbf{E}_{t3}'$ will be the same as $\mathbf{E}_{t1}$ and the actual values of

the e.m.f.'s $\mathbf{E}_{t2}$ and $\mathbf{E}_{t3}$ can be found from (2.4*b*). In using the above expression it must be remembered that the currents are complexor quantities variable in time phase and proper attention must be paid to their direction and sign. The application of these two methods of dealing with the e.m.f. is illustrated in the following example.

Example 2.1. A single-phase, 50-c/s motor has a mutual reactance between its stator and rotor windings of 15·7 Ω (referred to primary). There are 250 effective turns on the primary (stator) each carrying 5 A and 200 on the rotor each carrying 4 A, the two m.m.f.'s being coaxial and the currents in time phase with each other. Find the e.m.f. induced in each winding.

(*a*) From total flux—

Secondary current referred to primary $= 4 \times 200/250 = 3{\cdot}2$ A

Mutual inductance $= 15{\cdot}7/(2\pi \times 50) = 0{\cdot}05$ H

$$\text{Flux due to primary current} = (0{\cdot}05 \times 5\sqrt{2})/250 = 0{\cdot}00141 \text{ Wb}$$

$$\text{Flux due to secondary current} = (0{\cdot}05 \times 3{\cdot}2\sqrt{2})/250 = 0{\cdot}00091 \text{ Wb}$$

Total mutual flux $= 0{\cdot}00232$ Wb

Hence from e.m.f. eq. (2.3)

$$E_{t1} = (\sqrt{2})\pi \times 50 \times 250 \times 0{\cdot}00232 = 129 \text{ V}$$
$$E_{t2} = (\sqrt{2})\pi \times 50 \times 200 \times 0{\cdot}00232 = 103 \text{ V}$$

(*b*) From mutual reactance—

Secondary current referred to primary $= 3{\cdot}2$ A

Hence from eq. (2.5)

$$E_{t1} = 15{\cdot}7(5 + 3{\cdot}2) \text{ V} = 129 \text{ V}$$
$$E_{t2} = 129 \times 200/250 = 103 \text{ V}$$

LEAKAGE E.M.F.'s. Leakage fluxes also set up transformer e.m.f.'s and, as in other types of machine, these are conveniently treated as reactance e.m.f.'s. The leakage flux Φ_L is, except in certain special cases, due to the current in one winding only so that the leakage inductance is $L = \Phi_L T/\hat{\imath}$ and

$$E_L = (\sqrt{2})\pi f T \Phi_L \text{ volts, lagging the flux by } 90°$$

$$\therefore \quad \mathbf{E}_L = -\mathrm{j}\omega L\mathbf{I} = -\mathrm{j}\mathbf{I}x_L \text{ volts}$$

In drawing complexor diagrams to include this e.m.f. it is again usual to represent the voltage necessary to overcome it, this voltage

being known as the reactance drop and leading by 90° the current setting up the flux, i.e.

$$\text{voltage drop} = j\mathbf{I}x_L \quad . \quad . \quad . \quad (2.6)$$

Rotational E.M.F.'s in Phase Winding

The e.m.f. induced as a result of a relative rotation between the conductors of a phase winding and a steady flux Φ, one of which is rotating at $n = f/p$ r.p.s., is given elsewhere* as

$$E_r = (\sqrt{2})\pi f T \Phi k_m k_e \text{ volts}$$
$$= (\sqrt{2})\pi f T' \Phi \text{ volts} \quad . \quad . \quad . \quad (2.7a)$$

It may be noted that this is similar in form to eq. (2.1) for the transformer e.m.f. in a phase winding.

If both flux and the conductors are rotating at speeds corresponding to frequencies f_f and f_r respectively, the expression becomes—

$$E_r = (\sqrt{2})\pi(f_f \pm f_r)T'\Phi \text{ volts} \quad . \quad . \quad (2.7b)$$

the plus or minus sign depending on whether the two rotations are in the opposite or in the same directions.

Transformer E.M.F.'s in Commutator Winding

The total number of turns on a commutator winding are divided among $2a$ parallel paths where $a = 1$ for a simple wave winding and $a = p$ for a simple lap winding. For duplex windings, which are sometimes used, $a = 2$ or $2p$ respectively. It is, however, generally convenient to express e.m.f.'s in terms of the total turns T_a on the winding rather than the turns per phase or the turns in series between brushes.

The e.m.f. appearing between two diametrically-spaced brushes located with their axes coincident with that of the flux can be found from the expression (2.1) given for a phase winding if $T_a/2a$ is substituted for the turns and if the distribution factor is taken as $2/\pi = 0{\cdot}636$, corresponding to a phase spread of 180°.

Thus

$$E_t = (\sqrt{2})\pi f(T_a/2a)\Phi(2/\pi)k_e$$
$$= (\sqrt{2})f(T_a/a)\Phi k_e \text{ volts} \quad . \quad . \quad . \quad (2.8a)$$

If the brush axis is displaced by an angle α from the flux axis, then

$$E_t = (\sqrt{2})f(T_a/a)\Phi k_e \cos\alpha \text{ volts} \quad . \quad . \quad (2.8b)$$

The e.m.f. may also be expressed in terms of the mutual reactance between the windings associated with the flux Φ as in the case of the phase windings (page 18), remembering that the effective turns on the commutator winding are $T_2' = (T_a/2a)(2/\pi) = T_a/a\pi$ assuming $\alpha = 0$ and $k_e = 1$.

* Say, *The Performance and Design of A.C. Machines*, Ch. X (Pitman).

Substituting for T_a/a in terms of T_2' and for Φ as before gives

$$E_t = (\sqrt{2})fT_2'\pi(M/T_1')\sqrt{2}(\mathbf{I}_1 + \mathbf{I}_2' + \mathbf{I}_3')$$

$$\therefore \quad \mathbf{E}_t = -\mathrm{j}(T_2'/T_1')x_m(\mathbf{I}_1 + \mathbf{I}_2' + \mathbf{I}_3') \text{ volts}$$

The transformer e.m.f. referred to the primary is

$$\mathbf{E}_t' = \mathbf{E}_t \times T_1'/T_2' = -\mathrm{j}x_m(\mathbf{I}_1 + \mathbf{I}_2' + \mathbf{I}_3') \text{ volts} \qquad (2.8c)$$

Rotational E.M.F.'s in Commutator Winding

STATIONARY FIELD. Suppose that the commutator winding is rotating in a steady flux Φ and two brushes are placed on the commutator as shown in Fig. 2.2. The e.m.f. indicated by the

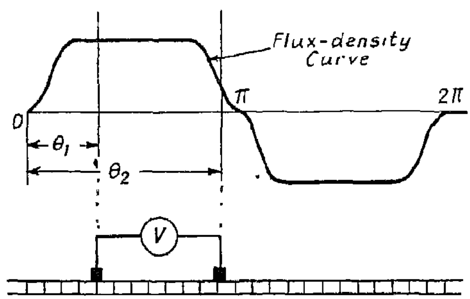


FIG. 2.2. ARRANGEMENT OF BRUSHES ON COMMUTATOR

voltmeter connected between the brushes will be the sum of the e.m.f.'s in all the individual coils connected to the segments lying between the brushes, i.e.

$$e = \Sigma e_c$$

The e.m.f. of a coil occupying any particular position will always be the same, although the coil will change due to the rotation. The e.m.f. between the brushes will thus always be the same, i.e. a steady e.m.f. of zero frequency.

The e.m.f. of any coil is proportional to the ordinate, B, of the flux-density curve at that particular position so that

$$e = 2LvT_c\Sigma B$$

If there are m coils between the brushes there will be m ordinates and

$$e = 2LvT_cm \times (\text{average ht. of ordinates } B \text{ between } \theta_1 \text{ and } \theta_2)$$

If the flux-density curve is assumed sinusoidal, i.e. if only its fundamental is considered, and if there are a large number of coils,

the average height of the ordinates of the flux-density curve between θ_1 and θ_2 may be written

$$\frac{1}{\theta_2 - \theta_1}\int_{\theta_1}^{\theta_2} \hat{B} \sin \theta \, d\theta$$

so that

$$E = 2LvT_c \frac{m}{\theta_2 - \theta_1}\int_{\theta_1}^{\theta_2} \hat{B} \sin \theta \, d\theta \text{ volts}$$

If there are C coils in the winding arranged in a pairs of parallel circuits, the number of coils in series between the brushes will be

$$m = (C/a)(\theta_2 - \theta_1)/2\pi$$

so that

$$E = 2LvT_c \frac{C}{a} \times \frac{\theta_2 - \theta_1}{2\pi} \times \frac{1}{\theta_2 - \theta_1}\int_{\theta_1}^{\theta_2} \hat{B} \sin \theta \, d\theta$$

Integrating and substituting

$$\hat{B} = \bar{B}(\pi/2) = (\Phi/YL)(\pi/2) = \Phi p/DL$$
$$v = \pi Dn$$
$$\text{total turns } T_a = CT_c$$

gives—

$$E = (T_a/a)f_r\Phi(\cos \theta_1 - \cos \theta_2) \text{ volts}$$

It is evident from Fig. 2.3 that this will be a maximum when the line bisecting the angle between θ_1 and θ_2 is coincident with the

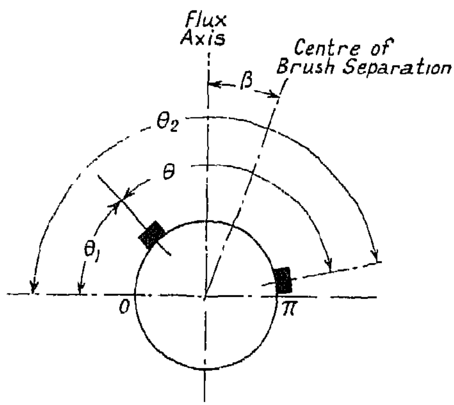


FIG. 2.3. DIAGRAMMATIC REPRESENTATION OF BRUSHES ON COMMUTATOR WINDING

position of maximum flux density and will be zero if the brushes are moved to a position 90° from this.

If the brushes are drawn in the conventional way as shown in Fig. 2.3, where θ is the angle of brush separation and β is the angle

of displacement between the centre-line of the brush separation and the flux axis, the expression for the e.m.f. may be written as follows.

Since

$$\theta_1 = 90 + \beta - \theta/2$$

and

$$\theta_2 = 90 + \beta + \theta/2$$

$$\begin{aligned} E &= (T_a/a)f_r\Phi\{\cos(90 + \beta - \theta/2) - \cos(90 + \beta + \theta/2)\} \\ &= 2(T_a/a)f_r\Phi \sin(\theta/2)\cos\beta \text{ volts} \qquad (2.9) \end{aligned}$$

If f_r is written as pn, the case of an ordinary d.c. machine, with $\theta/2 = 90°$ and $\beta = 0$ becomes the well-known expression—

$$E = 2(T_a/a)pn\Phi \text{ volts} \qquad (2.10)$$

It may be noted that, if the brushes are diametrically spaced and at 90° to the flux axis, the average height of the flux-density ordinates is $\bar{B}$, the mean flux density, and $m = C/2a$ so that eq. (2.10) can be obtained directly without any assumption with regard to the shape of the flux wave—the e.m.f. is therefore dependent only on the total flux and is not affected by its distribution around the gap.

ROTATING FIELD. If the field considered in the last paragraph rotates at a speed corresponding to a frequency f_f instead of being stationary, the magnitude of the e.m.f. depends on the rate at which the conductors cut the flux, i.e. on $(f_f \pm f_r)$. When the field axis is midway between the brushes the e.m.f. is at a peak value and 90° later, when $\beta = 90°$, it is zero. The e.m.f. induced thus alternates at a frequency depending on the speed at which the field moves past the brushes, i.e. f_f, this frequency being quite independent of the speed of rotation.

The r.m.s. value of the e.m.f. is, assuming it to vary sinusoidally, $1/\sqrt{2}$ of its peak value so that—

$$E_r = (\sqrt{2})(T_a/a)(f_f \pm f_r)\Phi \sin(\theta/2) \text{ volts} \qquad (2.11)$$

and its frequency is f_f c/s.

ALTERNATING FIELD. If the commutator winding is rotating in an alternating field, as is common in single-phase machines, the rotational e.m.f. alternates between peak values when the field is at its peak and zero values when it is zero, i.e. it is an alternating e.m.f. and has the same frequency as that of the field in which it is rotating. Furthermore it is in time phase with the field. The magnitude of the e.m.f. is, of course, proportional to the speed of rotation, i.e. to f_r, so that assuming diametrically-spaced brushes ($\theta = 180°$) as is usual under these circumstances, the e.m.f. is—

$$E_r = (\sqrt{2})(T_a/a)f_r\Phi \sin\alpha \text{ volts} \qquad (2.12)$$

It may be noted that this has the same form as the transformer e.m.f. of eq. (2.8*b*) except that $\cos\alpha$ is replaced by $\sin\alpha$, i.e. the

rotational e.m.f. is a maximum when the brushes are at 90° to the flux axis and the transformer e.m.f. is a maximum when they are in line with the flux axis.

It is often convenient to express this e.m.f. in terms of the mutual reactance as in the case of the transformer e.m.f. discussed on p. 18. It is the effective rotor turns ($T_2' = T_a/2a \times 2/\pi$) and not the actual turns which are effective in setting up the flux contributing to the mutual reactance. Substituting for T_a/a in terms of T_2' and for Φ as in the case of the transformer e.m.f. gives—

$$E_r = (\sqrt{2})T_2'\pi f_r(M/T_1')\sqrt{2}(\mathbf{I}_1 + \mathbf{I}_2' + \mathbf{I}_3' + \cdot\cdot\cdot)$$

Since E_r is in phase with Φ

$$\mathbf{E}_r = (T_2'/T_1')S\omega M(\mathbf{I}_1 + \mathbf{I}_2' + \mathbf{I}_3' + \cdot\cdot\cdot) \text{ volts} \qquad (2.13a)$$

where M is referred to the primary and $f_r/f_1 = S$ (not to be confused with s = slip; $S = 1 - s$).

The rotational e.m.f. referred to the primary is

$$\mathbf{E}_r' = \mathbf{E}_r(T_1'/T_2') = S\omega M(\mathbf{I}_1 + \mathbf{I}_2' + \mathbf{I}_3' \qquad \text{volts} \qquad (2.13b)$$

The e.m.f. in a single turn has a peculiar wave shape as shown in Fig. 2.4 which is drawn for a coil rotating at an angular speed

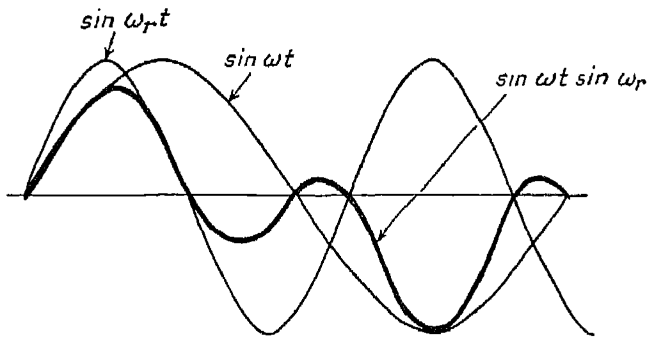


FIG. 2.4. WAVE SHAPE OF E.M.F. IN SINGLE TURN ROTATING IN PULSATING FLUX

$\omega_r = 2\pi f_r$ radians (electrical) per second in a flux alternating at a frequency $f = \omega/2\pi$ c/s where $\omega_r = 0{\cdot}6\,\omega$, i.e. it is running at 60 per cent of synchronous speed. It can be seen that the rotational e.m.f. becomes zero due to the flux passing through the zero value and also due to the coil passing the neutral axis. The curve is clearly a function of the product $\sin \omega t \,.\, \sin \omega_r t$.

Action of Commutator as Frequency Converter

Consider a commutator winding fitted with both slip rings and a commutator. If polyphase currents at a frequency f_1 are fed to the

slip rings with the winding stationary, a rotating field will be set up moving at $n_1 = f_1/p$ rev/sec. This field will cut the conductors and set up in them e.m.f.'s of frequency f_1. Also the field will be moving at a speed n_1 relative to the fixed brushes and, as already explained, the e.m.f.'s at the brushes will be at a corresponding frequency, i.e. f_1. Slip-ring currents, conductor currents and brush currents and the corresponding e.m.f.'s are therefore all at frequency f_1.

Suppose now that the winding is rotating at a speed n_r. The speed of the field relative to the conductors will still be n_1 since currents at frequency f_1 are being fed to fixed points in the winding. The frequency of conductor e.m.f.'s and currents is thus still f_1. The field is, however, moving at $n_1 \pm n_r$ relative to the fixed brushes, and the frequency of the e.m.f.'s appearing at the brushes, and therefore of the currents flowing from them, will be $f_1 \pm f_r$.

The commutator thus changes the frequency f_1 of the currents in the conductors or at the slip rings to a frequency $f_1 \pm f_r$ at the brushes, i.e. from a value in the conductors corresponding to the speed of the field past the conductors to a value at the brushes corresponding to the speed of the field past the brushes. This feature of the commutator is of fundamental importance in the behaviour of commutator machines.

EXERCISES 2

1. A 2-pole armature carrying a full-pitch commutator winding having 240 turns rotates at 1,200 r.p.m. in a flux of peak value 0·01 Wb and which alternates at 50 c/s. Determine the magnitude and frequency of the e.m.f. between diametrically-spaced brushes located (*a*) in line with the flux axis, (*b*) at 90° to the flux axis and (*c*) at 30° to the flux axis. Draw to scale the e.m.f. wave in a single turn.

2. A 2-pole rotating armature has a closed double-layer winding having full-pitched coils. There are 48 coils having 5 turns each. The front end of the winding is connected to a commutator and the back to 6 equally-spaced slip-rings. Calculate the magnitude and frequency of the e.m.f. appearing between the brushes on the commutator, between adjacent slip-rings and between diametrically-opposite slip-rings if—

(*a*) The armature is rotating at 3,000 r.p.m. in a stationary flux of 0·01 Wb per pole, two brushes being placed on the commutator in an axis at 90° to that of the flux.

(*b*) As in (*a*) but with the flux rotating at 200 r.p.m. in the same direction as the armature.

(*c*) As in (*a*) but with the flux rotating at 200 r.p.m. in the opposite direction to the armature.

(*d*) As in (*a*) but with three brushes spaced at 120°, one brush being on the flux axis.

(*e*) As in (*b*) but with three brushes at 120°.

3. A 4-pole armature has 104 lap-connected coils, each having 4 turns, and is running at 1,500 r.p.m. in a stationary field of 0·02 Wb per pole. The e.m.f. between two brushes symmetrically spaced relative to the flux axis is 200 V. Find the angle of brush separation, assuming the flux to be sinusoidally distributed over the pole-pitch. What would be the magnitude and frequency of

the e.m.f. if the armature were running at 3,000 r.p.m. and the field were moving at 1,500 r.p.m. in the same direction, the brush separation remaining the same?

4. The data refer to the lap-connected commutator winding of a 6-pole, 3-ph. motor running at 550 r.p.m. in a rotating field moving in the same direction at 1,000 r.p.m. Three brushes per pole-pair are placed on the commutator 120° (elec.) apart. Symmetrical 3-ph. currents of 30 A enter the terminals and pass through the winding, the winding currents being in phase opposition to the e.m.f.'s generated between the brushes. The total number of armature turns (full-pitch) is 304, the flux per pole is 0·01 Wb and the equivalent-star leakage reactance per phase for the whole winding at the above speed is 0·3 Ω. Calculate the current in the conductors, the e.m.f. generated between brushes and the equivalent-star terminal voltage.

5. A 1-ph., 50-c/s motor has a mutual reactance between stator and rotor windings (referred to primary) of 20 Ω. There are 300 turns (effective) on the stator each carrying 10 A, and 150 on the rotor each carrying 12 A. The m.m.f.'s of the two windings act along the same axis and the currents are displaced in time phase by 180°. Find the e.m.f. in each winding and the total flux linking the two.

6. The stator of a 2-pole commutator motor has 100 turns (effective) and the armature has 100 full-pitch turns, the mutual reactance between stator and armature being 8 Ω (referred to stator). A pair of brushes is placed on the commutator in the stator-winding axis and connected in series with the stator winding. A current of 15 A passes through the stator winding and these brushes, the m.m.f.'s of the two windings being in opposition. Another pair of brushes is placed on the commutator in quadrature with the stator axis. Calculate the e.m.f. across each pair of brushes and across the stator winding if the armature is running at twice synchronous speed.

CHAPTER 3

COMMUTATION

Commutation With A.C. Machines

As is well-known from a study of d.c. machines, when a coil undergoes commutation, i.e. while it is short-circuited by the brush, the current in the coil reverses; this reversal is opposed by the e.m.f. due to the coil inductance resulting from its leakage flux. Unless proper steps are taken this e.m.f. (known as the reactance e.m.f.) delays the change of current in the coil so that the reversal is not complete by the time the coil leaves the brush and sparking ensues. With the a.c. machine a precisely similar state of affairs exists but is further complicated by the fact that other e.m.f.'s may also be induced in the short-circuited coil due to its rotation in, or due to its being linked with an alternating component of, the air-gap flux—these cause circulating currents to flow in the short-circuited coils leading not only to poor commutation but also to additional losses.

Change of Current During Commutation

The process of commutation involves a change of current in the coil during the time for which it is short-circuited by the brush. For normal speeds and brush widths this time is generally between 0·2 and 0·4 milliseconds.

D.C. Machine. The case of the d.c. machine is illustrated in Fig. 3.1 which shows the current in an armature coil as it moves round

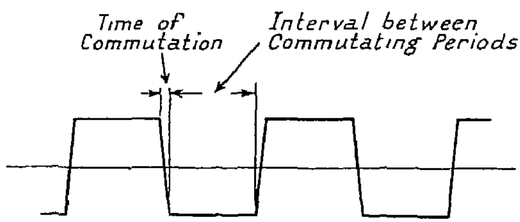


Fig. 3.1. Commutation in D.C. Machine

the armature. If I is the current in the coil, it changes during the time of commutation from $+I$ to $-I$, a change of $2I$ amperes.

Single-phase Machine. If an alternating current at, say, 50 c/s is passed through an armature with diametrically-spaced brushes and running at above synchronous speed, the conditions will be as shown in Fig. 3.2. The time of commutation is small compared to that of a cycle of the alternating current, and there may be several

current changes within one cycle. It can be seen that the magnitude of the current change varies, depending on the moment in the cycle at which it occurs, from a value of twice the maximum current to zero. The reactance e.m.f. set up thus alternates at supply frequency

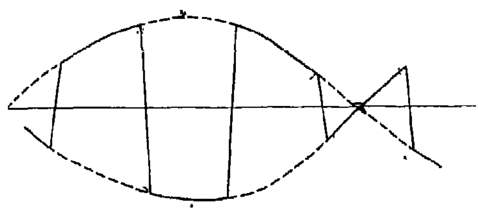


FIG. 3.2. COMMUTATION IN SINGLE-PHASE MACHINE

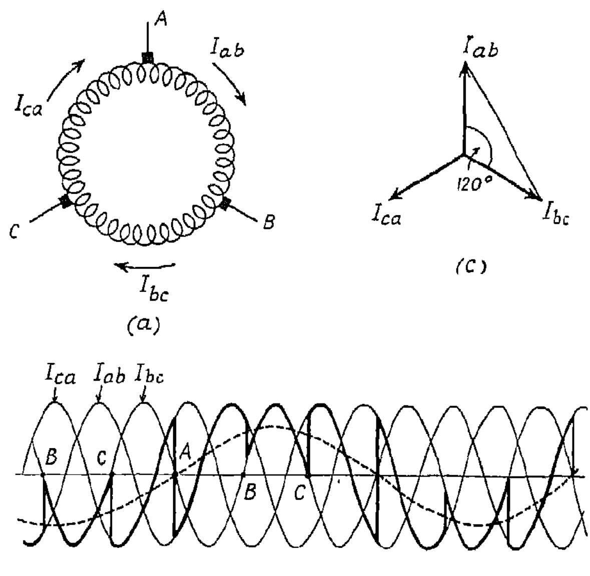


FIG. 3.3. COMMUTATION OF THREE-PHASE MACHINE

(*a*) Diagram of winding.
(*b*) Current in coil.
(*c*) Complexor diagram.

and, since it is a maximum at maximum current, it is in phase with the current entering the brush.

THREE-PHASE MACHINE. The arrangement is as shown in Fig. 3.3(*a*), currents at, say, frequency f_1 being supplied to the brushes.

The currents I_{ab}, I_{bc} and I_{ca} flowing in the conductors are also at frequency f_1 and are as shown by the waves in Fig. 3.3(*b*). When an individual coil passes from the band *ab* to the band *bc* by moving past the brush *B* the current in it changes from the value of I_{ab} to the value of I_{bc} at that instant. If the speed of the armature is such that a coil passes from brush *A* to brush *B* in 1/2 cycle the brushes can be located on Fig. 3.3(*b*) as shown. At brush *B*, therefore, the current changes from the value of I_{ab} at this point to the value of I_{bc} at this point as shown by the thick vertical line. As the coil proceeds past the various brushes the current in it changes in accordance with the heavy line in Fig. 3.3(*b*). Movement of a coil from brush *A* to brush *B* in 1/2 cycle corresponds to a speed frequency f_r of 2/3 of the frequency of the currents supplied to the brushes, and the wave shape drawn refers to this particular speed. It can be seen that the current consists of a number of sections in which it varies at frequency f_1 with a further variation at frequency $f_1 - f_r$ superimposed as shown by the dotted line.

It can also be seen that the maximum current change is 1·73 times the peak value of the conductor current, i.e. it is not so great as in the single-phase case. The magnitude of this change can also be seen from the complexor diagram of the conductor currents in Fig. 3.3(*c*) since it is the largest value of the difference between a pair of adjacent currents. This difference is also the brush current, so that the maximum current change is equal to the peak brush current $\hat{I}_b$.

General Polyphase Machine. If the diagram of Fig. 3.3(*c*) is drawn for any number of phases *N* separated by an angle $\gamma = 2\pi/N$, the largest value of the difference between currents in adjacent phases is $2 \,.\, \hat{I}_{ab} \,.\, \sin(\gamma/2)$. The value of this for different numbers of phases is shown in Table 3.1.

Table 3.1

Current Change During Commutation with Different Numbers of Phases

Number of Phases N	2	3	4	5	6	7
Angle γ	180°	120°	90°	72°	60°	51·4°
Maximum Current Change $= K\hat{I}_{ab} = \hat{I}_b$	$2{\cdot}0\hat{I}_{ab}$	$1{\cdot}73\hat{I}_{ab}$	$1{\cdot}41\hat{I}_{ab}$	$1{\cdot}17\hat{I}_{ab}$	$1{\cdot}0\hat{I}_{ab}$	$0{\cdot}84\hat{I}_{ab}$

It is thus seen that commutating conditions are likely to be less severe, from the point of view of reactance e.m.f., in a machine having a larger number of phases, and a number greater than three is sometimes employed for this reason.

Brush E.M.F.'s

As mentioned previously, the e.m.f. set up in the coils short-circuited by the brushes is made up of one or more of the following—

reactance e.m.f.
rotational e.m.f.
transformer e.m.f.

This e.m.f. appears continuously between the heel and toe of the brush although it must be remembered that, when the machine is running, it is produced from a succession of different coils.

Reactance E.M.F.

As the rate of change of current due to commutation is large compared to the rate of change due to its cyclic variation, the calculation of the reactance e.m.f. can be carried out by the methods normally employed for the d.c. machine and described in a companion volume.*

MAGNITUDE OF E.M.F. The leakage flux associated with a coil undergoing commutation is found from the permeance of the leakage paths associated with the slot and the corresponding overhang. For a single slot and its overhang this permeance may be written Λ weber/ampere-turn.

The leakage flux associated with a slot containing z_s conductors each carrying a current i amperes is thus

$$iz_s\Lambda \text{ webers}$$

If I_a is the r.m.s. current in the conductors, then during the time of commutation, t_c seconds, it will change by an amount $K(\sqrt{2})I_a$ where K is a constant depending on the number of phases (see Table 3.1).

If commutation takes place simultaneously in all the conductors of the slot, the average rate of change of flux, and therefore the average e.m.f. induced in a conductor during the time of commutation, is

$$K(\sqrt{2})I_a z_s \Lambda / t_c \text{ webers/second or volts/conductor}$$

If there are T_c turns per coil and similar conditions obtain in both sides of the coil, the average e.m.f. induced will be

$$2 \,.\, K(\sqrt{2})I_a z_s T_c \Lambda / t_c \text{ volts per coil}$$

and this appears across the brush.

This is the e.m.f. occurring at the moment of maximum current change—at other moments in the cycle it will be less and will vary sinusoidally at the frequency of the currents being commutated.

* Clayton, *The Performance and Design of D.C. Machines* (Pitman).

Since the current is alternating the reactance e.m.f. will alternate similarly and the r.m.s. value will be—

$$E_{xb} = 2KI_a z_s T_c \Lambda / t_c$$
$$= 2I_b z_s T_c \Lambda / t_c \text{ volts} \quad . \quad . \quad . \quad (3.1)$$

If the coil pitch is not the same as the brush pitch the conductors in the lower coil-side will not be undergoing commutation at the same time as those in the upper coil-side so that the variable part of the leakage flux will be halved, and therefore the reactance e.m.f. will be reduced to one quarter. Further complications are introduced when there are several coil-sides in the slot which are commutated successively, but the relevant calculations are similar to those for a d.c. machine. It can be seen that the magnitude of the reactance e.m.f. is reduced by an increase in the number of phases on account of the reduction of the constant K.

PHASE OF E.M.F. The reactance e.m.f. E_{xb} is alternating and has a peak value when the brush current is at its peak, so that the e.m.f. is in phase with the brush current and in such a direction round the short-circuited coil as to oppose the current change in it.

LIMITING VALUES. The reactance e.m.f. calculated above is the fundamental component of the e.m.f. that actually occurs, and experience shows that the value should not exceed about 0·5 V. Superimposed on it there are considerable high-frequency ripples due to the slots, segments and the non-uniform rate of change of current.

Rotational E.M.F.

The coils undergoing commutation are rotating and therefore moving relative to the main flux Φ in the air gap; a rotational e.m.f. is thus set up in the short-circuited coil. The flux Φ may be constant in magnitude but rotating as in polyphase machines or it may be alternating but fixed in space as in most single-phase machines.

If the flux is constant and rotating at a speed corresponding to a frequency f_f and the conductors are rotating at a speed corresponding to f_r, the e.m.f. induced will be, from eq. (2.7*b*),

$$E_{rb} = (\sqrt{2})\pi(f_f \pm f_r)T_c \Phi k_e \text{ volts per coil} \quad . \quad (3.2a)$$

the negative sign referring to rotation of the conductors in the same direction as that of the field. The frequency of the e.m.f. depends on the speed of the field past the fixed brushes, i.e. on f_f. The e.m.f. has its peak value in a particular coil when the flux axis is at 90° to the coil axis.

If the flux is alternating but stationary in space with its axis coincident with the brush axis (at 90° to the short-circuited coil axis), the e.m.f. in the short-circuited coil can be found as follows. Supposing the flux is steady at its peak value, the e.m.f. induced in

a coil as it rotates is, from eq. (2.7a), $E = (\sqrt{2})\pi f_r T_c \Phi k_e$ volts (r.m.s.). The short-circuited coil, however, always occupies the same position relative to the flux, i.e. the position giving the peak value of the above; but the flux is changing sinusoidally so that the e.m.f. also changes sinusoidally at the same frequency as that of the flux, and in phase with it. Hence the r.m.s. value is also

$$E_{rb} = (\sqrt{2})\pi f_r T_c \Phi k_e \text{ volts per coil} \qquad (3.2b)$$

If the brush axis is at an angle α to the flux axis the above must be multiplied by $\cos \alpha$.

Transformer E.M.F.

The third of the e.m.f.'s that may be induced in the coils undergoing commutation is due to their being linked with an alternating flux and this occurs in most single-phase machines. If the coil axis is at an angle δ to the flux axis the magnitude of the e.m.f., from eq. (2.1), is

$$E_{tb} = (\sqrt{2})\pi f T_c \Phi k_e \cos \delta \quad . \quad . \quad . \qquad (3.3)$$

and lags the flux by 90°.

Total Brush E.M.F.

For a simple lap winding the e.m.f. between adjacent segments is equal to that of a coil, but for a simple wave winding there are a coils between adjacent segments so that the voltage is correspondingly increased. In general the voltage between the segments is $E_b p/a$. Furthermore, as a brush usually bridges more than one segment the voltage between the toe and heel of the brush may be two or three times E_b; as it is essential to keep this voltage to a minimum the brushes of an a.c. commutator motor rarely have a width of more than two segments.

Limitations to Flux Per Pole

Even if some of the special devices described later are used for assisting commutation, practical experience indicates that if the voltage across the brush exceeds two or three volts bad commutation will result. To keep the values of the rotational and transformer e.m.f.'s down to this figure involves a limitation of the flux per pole and therefore of the output of the machine. Although conditions vary somewhat with different types of machine, an idea of the maximum possible values of the flux per pole for any a.c. commutator machine can be found from—

$$\Phi = E_b/\{(\sqrt{2})\pi f T_c\} \text{ webers} \qquad (3.4)$$

where E_b is either E_{rb} or E_{tb} and f is the frequency appearing in the voltage equations (3.2) or (3.3).

As mentioned later, single-turn coils and lap windings ($p/a = 1$) are generally employed so that the limiting flux per pole for machines

with full-pitch coils in the commutator winding is as shown in Table 3.2.

TABLE 3.2

LIMITING VALUES OF FLUX PER POLE (WEBERS)

Frequency \ E_b	1·5	2·0	2·5	3·0	5·0	7·0
$16\frac{2}{3}$	0·0202	0·0272	0·0346	0·0403	0·0676	0·0912
25	0·0135	0·018	0·0225	0·027	0·0451	0·0633
50	0·00675	0·009	0·0112	0·0135	0·0226	0·0315
75	0·0045	0·006	0·0075	0·009	0·0150	0·0211
100	0·00339	0·00452	0·0056	0·0068	0·0113	0·0167

If it is practicable to short-pitch the coils of the commutator winding, correspondingly larger values of flux can be used, but except in the case of the Osnos motor extensive short pitching is not desirable.

Limitations to Voltage and Output

The limitation to the voltage per coil fixes a practical limit to the voltage and power that may be handled by a commutator winding.

VOLTAGE LIMIT. Assuming E_b to be the same for all coils, the voltage, E_r, between any pair of adjacent brushes on the commutator is given by

$$E_r = E_b \,.\, k_m \text{ (no. of segments between brushes)}$$
$$= E_b \,.\, k_m \left(\frac{\text{periphery of commutator between brushes}}{\text{segment width } (w_c)}\right)$$

If the brushes are displaced by θ electrical degrees—

$$k_m = \frac{\sin(\theta/2)}{\theta/2}$$

and the periphery between brushes $= \pi D_c(\theta/2\pi p) = D_c\theta/2p$

so that

$$E_r = E_b\frac{\sin(\theta/2)}{\theta/2}\cdot\frac{D_c\theta}{2pw_c} = E_b\cdot\frac{D_c}{pw_c}\sin\frac{\theta}{2} \text{ volts}$$

For diametrically-spaced brushes this becomes $E_r = E_b \,.\, D_c/pw_c$ and for 120°-spaced brushes $E_r = E_b\{(\sqrt{3})/2\}(D_c/pw_c)$.

It is thus evident that a large-diameter commutator with small segments will be necessary to obtain high voltages. The commutator diameter is limited by the safe peripheral speed $v_c = \pi D_c n$. Taking n as synchronous speed, to which the running speed is always

related, $v_c = \pi D_c f/p$ so that $D_c/p = v_c/\pi f$. Inserting this in the e.m.f. expression gives

$$E_r = E_b(v_c/\pi f w_c) \sin (\theta/2) \tag{3.5}$$

Putting in typical permissible figures for diametrically-spaced brushes ($E_b = 4$ V, $v_c = 2{,}500$ cm/sec, $w_c = 0{\cdot}4$ cm, $f = 50$ c/s) gives—

$$E_r = 4 \times 2{,}500/(\pi \times 50 \times 0{\cdot}4) \times 1 = 160 \text{ V}$$

It is thus evident that a commutator winding cannot normally be connected to a circuit of 420 V or more without a step-down transformer.

OUTPUT. The volt-amperes handled by a single-phase commutator winding are $E_r I$, and by a three-phase commutator winding are $\sqrt{3} E_r I$, where E_r is the voltage between brushes and I the total armature current.

For the single-phase case, assuming single-turn coils,

$$VA = \sqrt{2}(T_a/a) f_r \Phi I$$

Since Φ is limited by the brush voltage to $E_b/\sqrt{2}\pi f$ [eq. (3.4)] and I is limited by the specific electric loading, $ac = (I/2a)(2T_a/\pi D)$ ampere-conductors per metre of periphery, the volt-amperes may be written

$$\begin{aligned} VA &= \sqrt{2}(T_a/a) f_r (E_b/\sqrt{2}\pi f)(ac \, . \, a \, . \, \pi D/T_a) \\ &= E_b(f_r/f) D \, . \, ac \end{aligned}$$

But $v = \pi D n_r$ so that $D = vp/\pi f_r$ and therefore

$$VA = (E_b \, . \, v \, . \, ac)/(\pi f) \text{ volt-amperes per pole-pair} \tag{3.6}$$

Inserting limiting figures ($E_b = 4$ V; $v = 30$ m/sec; $ac = 30{,}000$ A-conductors/m; $f = 50$) gives $VA = 23 \, . \, 10^3$.

Assuming unity power factor for the winding the maximum power output is thus about 30 h.p. per pole-pair.

For a three-phase winding the current per conductor is $I/\sqrt{3}a$ instead of $I/2a$ so that VA becomes $(3/2)(f_r/f)E_b D ac$, i.e. $1{\cdot}5$ times the single-phase value or about 45 h.p. per pole-pair.

Machines of large output will thus require a large number of poles and possibly a special type of winding, e.g. a duplex winding in which $a/p = 2$. The actual output of many commutator motors may, however, be greater than would appear from this figure since the commutator may not handle the whole of the motor power.

Methods of Improving Commutation

As in the d.c. machine the voltage drop between brush and commutator is an essential factor in producing good commutation; in the a.c. machine, however, its influence must be augmented by reducing the magnitude or the effect of the e.m.f.'s induced in the short-circuited coils. This can be done as follows.

(i) Appropriate design limitations—use of single-turn coils, small values of flux per pole, short core lengths and duplex windings. These are effective in reducing both reactance and transformer or rotational e.m.f.'s.

(ii) Neutralization of the e.m.f.'s by compoles—effective for reactance and transformer or rotational e.m.f.'s.

(iii) Reduction of circulating current by resistance connectors—reduces the effect of transformer and rotational e.m.f.'s.

(iv) Use of discharge windings—effective for reactance e.m.f.

USE OF SINGLE-TURN COILS. All three e.m.f.'s are proportional to the number of turns per coil so that this number must be kept to a minimum, and, on all except fractional-horse-power machines, single-turn coils are usual. This, of course, leads to a large number of commutator segments and tends to make the machine expensive but it is a necessary condition with large a.c. commutator motors.

SMALL VALUES OF FLUX PER POLE AND SHORT CORE LENGTHS. Reducing the flux per pole directly reduces the transformer and rotational e.m.f.'s and, since it results in smaller core and overhang lengths, indirectly reduces the leakage flux and hence the reactance e.m.f. As explained on page 34, this limitation of flux per pole also limits the output.

USE OF DUPLEX WINDING. An ordinary duplex winding in which there are two similar and distinct windings connected to alternate commutator segments doubles the number of segments and therefore halves the voltage per segment for a given flux per pole; alternatively the flux per pole for a given voltage per segment can be doubled and the output per pole quadrupled. Duplex windings thus offer considerable possibilities for large machines. If, however, the two windings are electrically separate there is no reason for a particular commutator segment to hold itself at a potential midway between those of the two adjacent to it and, in fact, commutation with such an arrangement would be impracticably bad. To avoid this, intermediate segments could be held at mid-potential by joining them to the back end of the coils connected to the adjacent segments by means of equalizing connexions; although this results in improved commutation the currents in the equalizing connexions, which must pass beneath the core to avoid induced e.m.f.'s, cause considerable shaft currents. By a suitable choice of coil span for the two windings it is possible to eliminate currents in the equalizing connexions and so make the arrangement practicable although somewhat complicated. The introduction of a discharge winding, described later, leads to a simpler method of coupling the two halves of the winding, and it is in this form that such windings are commonly used.

NEUTRALIZATION OF E.M.F.'S BY THE USE OF COMPOLES. In the d.c. machine where the only e.m.f. that has to be considered is the reactance e.m.f., a compole can be designed to set up in the short-circuited coil an e.m.f. which completely neutralizes the reactance

e.m.f. under all conditions of load and speed so that commutation is almost perfect.

Although it is not practicable to fit compoles to most forms of a.c. machine, in certain special cases, e.g. the single-phase series motor, they can be fitted and made to partially neutralize both the reactance and the transformer e.m.f.'s. Such an arrangement is described more fully in Chapter 14 (page 238).

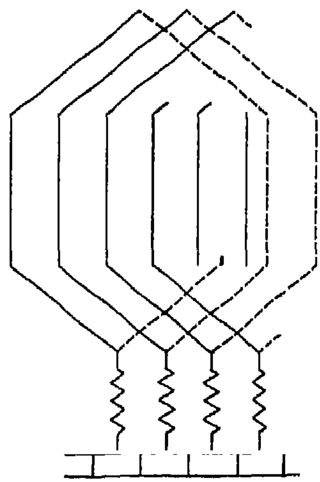


Fig. 3.4.
Resistance Connectors

Resistance Connectors. The circulating currents caused by rotational and transformer e.m.f.'s can be limited by resistors connected between the coils and the commutator segments as shown in Fig. 3.4. Any one of these carries current only during the commutating period so that the losses in them and the heating caused are not excessive. There is, however, a danger of their being burnt out if the motor stalls or if there is delay in starting. Use of resistance connectors is therefore avoided if possible, but they are necessary in some cases where it is desired to increase the flux per pole above the values in Table 3.2. A reasonable design results if each resistor is proportioned to give a voltage drop, when carrying full-load current, of about 0·25 V; they then enable the voltage per coil to be increased by about 30 per cent.

Use of Discharge Resistors. The use of a discharge resistor in connexion with the opening of an inductive circuit is well known, and the same principle can be used to absorb the energy stored in the leakage flux around the short-circuited coils during the commutation process. Resistors connected in parallel with the coils as

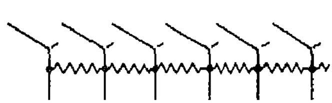


Fig. 3.5. Discharge Resistors

shown in Fig. 3.5 illustrate the principle; in practice, however, the resistor is more conveniently arranged in the form of a discharge winding placed in the same slots as the main winding. This discharge winding must have a suitably high resistance, it must not be linked inductively with the main coil with which it is in parallel, i.e. it must lie in different slots, and there should be no circulating current

due to differing e.m.f.'s being induced in the two parallel coils. Three forms of discharge winding are in common use.

ROBINSON DISCHARGE WINDING. This arrangement employs a simple double-layer winding as the discharge winding as shown in

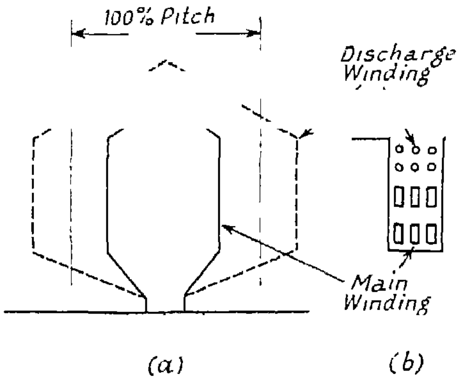


FIG. 3.6. ROBINSON DISCHARGE WINDING

(a) Winding diagram.
(b) Arrangement in slot.

Fig. 3.6. The coils which are connected in parallel are, of course, in different slots, and to ensure equality of e.m.f.'s in the two coils those of the main winding are short-pitched by a certain amount while those of the discharge winding are over-pitched by an equal amount. For constructional reasons, and also to keep the inductance low, the discharge winding is placed in the top of the slots.

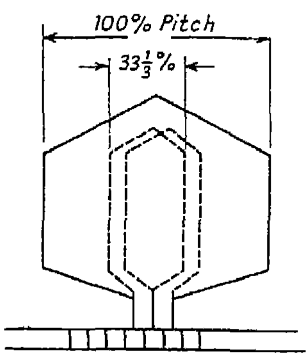


FIG. 3.7. DUPLEX DISCHARGE WINDING

DUPLEX DISCHARGE WINDING. As previously mentioned, duplex windings are commonly employed, part of such a winding being shown in Fig. 3.7. A discharge winding with two discharge coils in series, each of $33\frac{1}{3}$ per cent pitch (coil-span factor 0·5) can be connected in parallel with each full-pitch main coil thus ensuring equality of e.m.f.'s. The mid-point of the discharge coils is connected to the intermediate segment as shown so that the discharge winding acts as an equalizing winding for the two branches of the duplex winding.

INDIRECT DISCHARGE WINDING (SCHWARZ). The arrangement is shown in Fig. 3.8(*a*), the discharge winding, which is of $33\frac{1}{3}$ per cent pitch as in the duplex arrangement, being placed in the bottom of the slots and separated from the main winding by magnetic shims as shown in Fig. 3.8(*b*). It can be seen that the discharge coils for

main coil 1 are in the same slots as, and, on account of the magnetic shims, tightly coupled magnetically to, the discharge coil for main coil 2. Main coil 2 is thus coupled by transformer action to main coil 1 and acts as its discharge circuit. It is evident, since the discharge coils cannot both be coaxial with the main coils, that there will be some difference of e.m.f. between the two and circulating

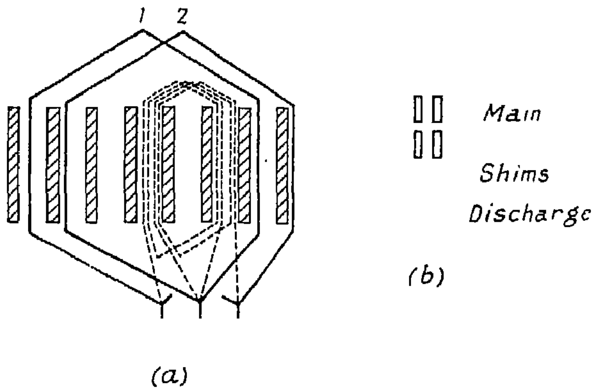


FIG. 3.8. INDIRECT DISCHARGE WINDING (SCHWARZ)

currents will result. These can, however, be avoided by special winding arrangements.

Any of the three discharge windings give satisfactory results and the choice is largely governed by the feelings of the designer, manufacturing arrangements or the patent situation.

Effect of Commutation on Rotor Reactance

It can be seen from Fig. 3.3 that the rotor current is made up of two frequencies, the frequency f_f at which the field moves past the brushes and the frequency $(f_f \pm f_r)$ at which the field moves past the conductors, i.e. the slip frequency.

While the current in the conductors is varying at field frequency the reactance e.m.f. can be expressed in the usual way as Ix volts lagging the current by 90° and being independent of speed.

The reactance e.m.f. induced by the change of current during commutation modifies this—referring to Fig. 3.9 it has been shown on page 32 that the reactance e.m.f. due to the current change is in phase with the brush current. Considering the section of winding between A and B the reactance e.m.f. at brush A is in phase with I_A and that at brush B in phase with I_B as shown in the e.m.f. complexor diagram of Fig. 3.9(c). The reactance e.m.f. at a particular brush may be assumed to act during one-half of the commutating period in the phase which the coil is leaving and during the other half in the phase which the coil is entering. In phase AB therefore E_{xbA} and E_{xbB} are acting and together they give an e.m.f.

E_{xb}' leading I_{ab} by 90°. This e.m.f. thus acts in opposition to the reactance e.m.f. due to the normal current change in the phase. It may be noted also that, if the motor has full-pitch coils, the coils being commutated by brush C will be lying in the slots of phase AB, and an e.m.f. proportional to $-E_{xbC}$ will be induced in the phase—this also leads I_{ab} by 90° and so adds to the above effect.

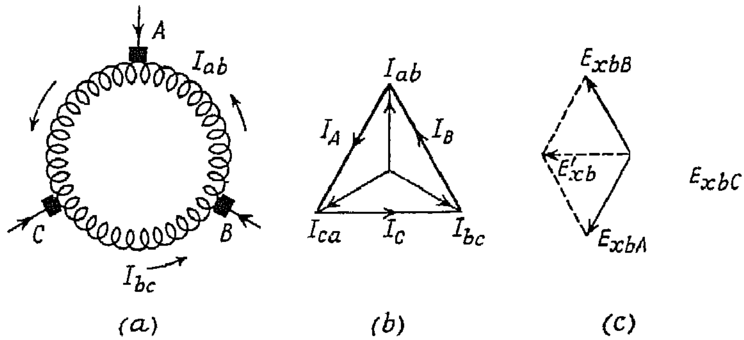


FIG. 3.9. REACTANCE E.M.F.'S DURING COMMUTATION

(a) Winding.
(b) Current complexor diagram.
(c) Reactance e.m.f. complexor diagram.

The current change at commutation thus reduces the effective rotor reactance by the introduction of an apparent negative reactance, the magnitude of which is dependent on speed.

It is shown by Arnold* and others that the effective reactance of the winding may be expressed in the form

$$x = sx_v + x_w \qquad . \quad . \quad . \quad (3.7)$$

The effective reactance is thus made up of a fixed part and a part proportional to slip, i.e. the speed of the field past the conductors. Reactance thus decreases as the speed rises reaching zero at a speed just above synchronism and then becoming negative, i.e. the rotor behaves as a group of capacitors.

The above result may also be arrived at by a consideration of leakage fluxes—these may be resolved into two parts one of which rotates around the rotor at brush frequency and the other being stationary in space but pulsating at brush frequency.

Brushes

The choice of brush† demands considerable care but the types found most suitable for a.c. commutator motors are hard carbon, high-resistance carbon and electrographitic brushes. The chief

* Arnold, *Die Wechselstromtechnik*, Vol. 2., Ch. 1.
† Hayes, *Current Collecting Brushes* (Pitman).

TABLE 3.3
BRUSH DATA

Type of Brush	Current Density (A/in.^{2})	Coeff. of Friction	Max. Periph. Speed (ft/min)
High-resistance graphite	60	0·18	12,000
Hard carbon	25–40	0·28–0·32	3,000
Low-speed electrographitic	50–65	0·17–0·22	5,000
High-speed electrographitic	65–75	0·15–0·20	10,000

properties of these are shown in Table 3.3 and typical brush drop curves are given in Fig. 3.10.

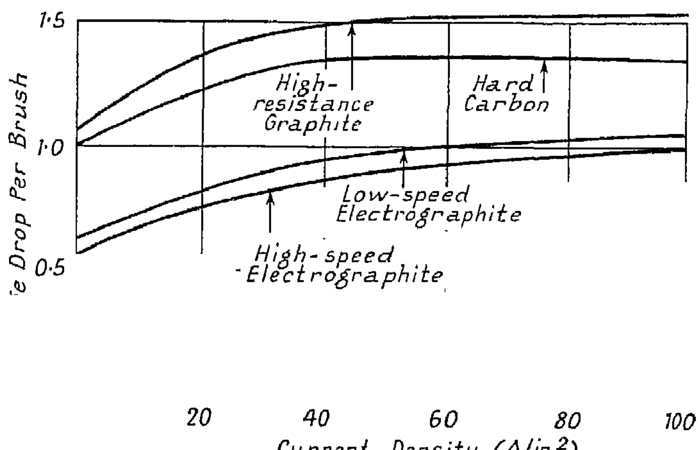


FIG. 3.10. BRUSH DROP FOR TYPICAL BRUSHES (PRESSURE = 2 LB/IN.^{2})

Radio Interference

The abrupt change of current during commutation as well as any sparking that may occur cause high-frequency voltages to appear across the armature of a commutator motor; these voltages are applied to the supply leads which may act as a transmitting aerial and cause radio interference.

The most effective remedy is to short-circuit the high-frequency voltages by a small capacitor (0·05 to 0·005 μF) connected across the brushes; best results are obtained if two capacitors in series are used with their mid-point connected to the motor frame, but if the frame is not earthed it may attain a dangerous potential unless a very small capacitor (0·005 μF) is used.

Trouble due to radio interference is generally confined to fractional-horse-power motors used for domestic apparatus since the mains

supplying such motors are commonly in close proximity to receiving aerials. All such motors should be fitted with capacitor suppressors as described above.

EXERCISES 3

1. A 3-phase commutator winding has a reactance e.m.f. of 1·5 V per segment. Estimate the value of the reactance e.m.f. if it were arranged for 7-phase operation, the current in the conductors remaining the same.

2. A 2-pole armature carrying a double-layer winding with one full-pitch turn per coil, 72 slots and 2 conductors per slot is lap-connected. The brush width is equal to 2 segments and the commutator diameter is 30 cm.

Draw, to a base of time, the wave shape of the current in a coil for the conditions given below—

(*a*) Armature running at 3,300 r.p.m. with 100 A d.c. passing between diametral brushes.

(*b*) Armature running at 3,300 r.p.m. with 100 A (r.m.s.) at $16\frac{2}{3}$ c/s passing between diametral brushes.

(*c*) Armature running at (i) 3,300 r.p.m., (ii) 2,000 r.p.m. with 25-c/s, 3-phase currents of 100 A entering three brushes spaced at 120°.

Measure the maximum current change in each case.

If the permeance associated with each slot is $25 \cdot 10^{-7}$ Wb/AT find the average reactance e.m.f. in each case. In case (*c*) find also the reactance e.m.f. if the coil span were 120°.

3. A 2-pole commutator winding with diametric brushes rotates at 1,000 r.p.m. in 50-c/s alternating fluxes of 0·01 Wb along the brush axis and 0·005 Wb at 90° to the brush axis; the latter leads the former by 90° in time phase. The winding has 3 turns per coil. Determine the transformer and rotational e.m.f.'s in the coil short-circuited by the brushes and find the total coil e.m.f. At what speed would the total e.m.f. be zero?

4. Three-phase currents of 240 A are supplied to a 4-pole armature having 180 lap-connected single-turn coils arranged in 60 slots. The armature is running at 1,800 r.p.m., the commutator diameter is 25 cm and the brush width is 0·9 cm. If the permeance associated with one slot is $20 \cdot 10^{-7}$ Wb/AT estimate the reactance voltage if (i) the coils are full-pitched and (ii) the coils have a span of 120°.

PART II: AUXILIARY COMMUTATOR MACHINES FOR REGULATING SPEED AND POWER FACTOR OF INDUCTION MOTORS

CHAPTER 4

EFFECT OF E.M.F. INJECTED INTO SECONDARY CIRCUIT

THE ordinary induction motor operates at approximately constant speed and at a lagging power factor between 0·8 and 0·9. Various means are available* for improving this power factor and for providing a means of varying the speed. The most effective methods involve the injection of an e.m.f. into the secondary circuit from some suitable external source.

This injected e.m.f. must be at the same frequency as that of the secondary currents, i.e. slip frequency, and is most conveniently obtained by means of a commutator, which, as already seen, acts as a frequency converter. This commutator may be part of a separate auxiliary machine or it may be incorporated as part of the induction motor itself. In this part of the book are considered the arrangements in which a separate auxiliary commutator machine is used. Before discussing the different types in detail, however, it is desirable to consider the general effect of the injected e.m.f. on the behaviour of the induction motor.

Injection of E.M.F. into Secondary Circuit

The effect of injecting an e.m.f. into the rotor circuit of an induction motor can be observed by studying the complexor diagrams of Fig. 4.1. In Fig. 4.1(a) is shown the diagram for an induction motor running normally with the rotor short-circuited. The e.m.f. induced in the secondary winding by the main air-gap flux is sE_2 and, since the secondary winding is short-circuited, the secondary current, I_2, is limited only by the secondary-winding impedance so that $sE_2 = I_2 z_{2s}$.† The secondary current lags sE_2 by a small angle due to the secondary-winding leakage reactance, and this lag is reflected into the primary winding and is a contributory cause of the lagging power factor of the primary current.

If an e.m.f. E_j is injected into the rotor circuit in phase opposition to E_2, i.e. at an angle of $\beta = 180°$ to E_2, it will tend to decrease the current, resulting in a decreased torque. Assuming the load torque to remain unchanged, the motor torque becomes less than the load

* Say, *The Performance and Design of A.C. Machines*, Ch. XIII (Pitman).

† The suffix s in x_{2s} and z_{2s} indicates that the secondary-circuit reactance varies with slip in a manner dependent on the particular type of machine under consideration.

torque and the speed drops; this drop results in an increase of sE_2 which increases the current so that the torques again balance with the motor running steadily at the reduced speed. Conditions are as shown in Fig. 4.1(b) (illustrating rotor quantities only) and the scalar equation $sE_2 = I_2z_{2s} + E_j$ obtains. Injecting an e.m.f. at an angle of $\beta = 180°$ thus effects a reduction of speed: except near synchronous speed I_2z_2 is relatively small so that $s \simeq E_j/E_2$. Hence

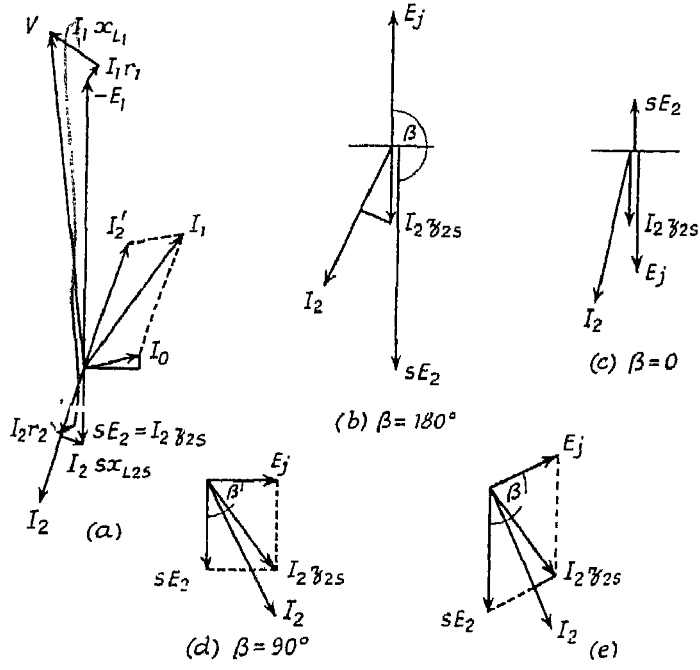


FIG. 4.1. EFFECT OF INJECTING E.M.F. E_j AT VARIOUS ANGLES β INTO SECONDARY CIRCUIT OF INDUCTION MOTOR

(a) Induction motor.
(b) $\beta = 180°$. Below synchronous speed.
(c) $\beta = 0$. Above synchronous speed.
(d) $\beta = 90°$. P.F. improvement.
(e) General case. P.F. improvement and speed reduction.

if the injected e.m.f. E_j can be varied from zero to the standstill secondary e.m.f. E_2, the speed can be varied down to zero.

If the e.m.f. is injected in phase with E_2 ($\beta = 0°$) it will tend to increase the current so that the motor torque exceeds the load torque and the speed increases, thus reducing sE_2 until stable running is obtained. If E_j exceeds I_2z_{2s}, sE_2, and therefore s, become negative and the motor runs above synchronous speed ($s \simeq -E_j/E_2$). Conditions are as shown in Fig. 4.1(c) and the equation $sE_2 = I_2z_{2s} - E_j$ obtains. An e.m.f. equal to E_2 injected at an angle $\beta = 0°$ thus gives about twice synchronous speed.

In Fig. 4.1(d) the e.m.f. is injected leading E_2 by 90°, i.e. $\beta = 90°$. The relation between the voltage drop and the e.m.f.'s can now be written as a complexor equation

$$\mathbf{I}_2\mathbf{z}_{2s} = \mathbf{sE}_2 + \mathbf{E}_j \qquad (4.1)^*$$

and it can be seen from the complexor diagram that the effect is to make I_2z_{2s}, and therefore I_2, lead the position it previously occupied. This leading secondary current is reflected into the primary and gives an improved or leading power factor. Since there is no component of E_j directly assisting or opposing sE_2, the required value of sE_2 will not change appreciably and the motor speed will not be affected. The magnitude of E_j necessary to advance the phase of the secondary current by about 45° will be of the same order as sE_2, i.e. only a few per cent of E_2 for a normally designed machine.

In the final figure, 4.1(e), the e.m.f. is injected at a general angle β to E_2 and gives rise to an improvement of power factor as well as a change of speed. It is sometimes convenient to assume that the vertical component of E_j, i.e. $E_j \cos\beta$, gives rise to a change in speed while the horizontal component $E_j \sin\beta$ gives rise to an improvement of power factor.

The complexor equation (4.1) above is a general equation covering an injected e.m.f. at any angle β; the two scalar equations given previously are special cases when $\beta = 180°$ and $\beta = 0°$.

The injection of an appropriate e.m.f. into the secondary circuit can thus give both speed and power factor control as desired. The e.m.f. must always be at slip frequency whatever the value of the slip and it can conveniently be obtained from an auxiliary commutator machine.

Sources of Injected E.M.F.

Four fundamental types of regulating machine may be used as a source of the injected e.m.f. as enumerated below; in practice various modifications and amplifications of these are employed.

LEBLANC MACHINE. This is simply an armature with a commutator, exactly similar to that normally used for a d.c. machine; it has no associated stator winding but, for a basic two-pole machine, has three brushes spaced at 120°, these being connected in the induction motor secondary circuit as shown in Fig. 4.2(a). The machine produces an e.m.f. approximately proportional to I_2, the motor secondary current, and leading or lagging it by 90° depending on the speed at which the armature is driven.

* It may be noted that some writers base their calculations on the equation $\mathbf{sE}_2 = \mathbf{I}_2\mathbf{z}_{2s} + \mathbf{E}_j$, i.e. the sign of E_j is reversed and the direction of the sE_2 complexor in the diagrams is reversed. The choice is purely arbitrary and, provided consistency is maintained throughout, makes no difference to any results obtained.

WALKER MACHINE. This is similar to the Leblanc machine except that it has a stator winding which is connected in series with the brushes as shown in Fig. 4.2(*b*). It also produces an e.m.f. which is approximately proportional to I_2 but the angle at which it is injected into the secondary circuit can be varied by suitable location of the brushes relative to the stator winding.

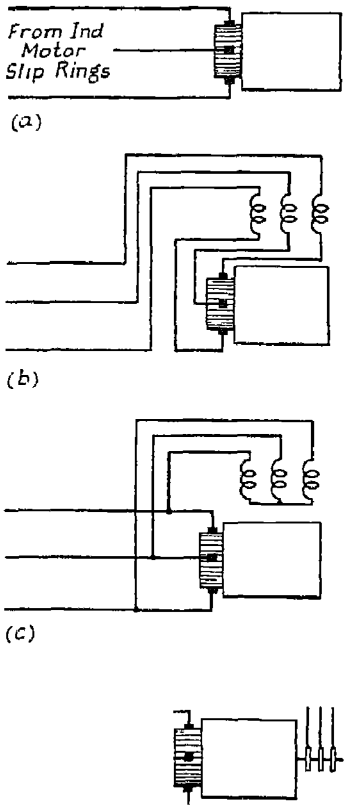


FIG. 4.2. TYPES OF REGULATING MACHINE

(*a*) Leblanc machine.
(*b*) Walker (series) machine.
(*c*) Scherbius (shunt) machine.
(*d*) Frequency converter.

SCHERBIUS MACHINE. This has a stator winding connected in parallel with the brushes as shown in Fig. 4.2(*c*). The voltage across the winding is sE_2. The reactance of the winding is proportional to the frequency of the current, i.e. proportional to slip, so that the current through the winding is approximately independent of slip. A constant flux and therefore a constant injected e.m.f. will be produced. The angle of injection can be adjusted, as with the Walker type, by appropriate design. The term "Scherbius" is sometimes used to denote a machine having series, shunt or separately-excited stator windings, and machines may have two or all three of these windings in order to give special characteristics.

FREQUENCY CONVERTER. This is an armature with commutator and slip rings as shown in Fig. 4.2(*d*), an e.m.f. at supply frequency being fed to the slip rings and converted by the commutator to slip frequency. The injected e.m.f. is thus again independent of slip as in the Scherbius arrangement.

With each of the arrangements shown the regulating machine must be driven by a separate motor or by the induction motor itself. Various additions to the four fundamental schemes shown have been devised and used, and details of those of commercial importance are given in subsequent chapters.

If the regulating machine is used only for power-factor improvement, without speed control, it is called a *phase advancer* or *a.c. exciter*, while if used for speed control it is called a *slip regulator*.

Magnetizing Volt-amperes

To produce the flux necessary for the operation of an induction motor magnetizing ampere-turns, I_mT, are required necessitating a magnetizing current in one of the windings. The flux sets up a back e.m.f. E in the winding carrying the current, this e.m.f. being proportional to frequency. The reactive volt-amperes necessary to set up the flux are thus EI_m and are proportional to the frequency of the magnetizing current.

If, in the case of a motor with a regulating machine in its secondary circuit, the injected e.m.f. has a component at 90° to the secondary e.m.f., then it can be seen from Figs. 4.1(d) and 4.1(e) that it sets up a component of current in phase with the flux, i.e. a magnetizing current, in the secondary circuit, thus magnetizing the main machine from the secondary instead of from the primary winding.

Since this magnetization is supplied at a low frequency the reactive volt-amperes required are only s times, i.e. a few per cent of, those required if magnetized from the primary in the usual way; the advantage of exciting the induction motor from its secondary is thus evident, and if a regulating machine is to be used solely as a phase advancer it will be very small in size compared to that of the main motor. Since the main motor does not now have to carry the magnetizing current its size can be slightly reduced and its efficiency increased, thus neutralizing, to some extent, the cost of the phase advancer.

The limit in reducing the frequency of the magnetizing current is, of course, reached in the synchronous machine in which it is supplied at zero frequency (direct current) and no reactive volt-amperes are required, the only input to the exciting circuit being the power to supply the resistance loss in the exciting winding.

Secondary (Slip) Power

Reference to Fig. 4.1 shows that when the e.m.f. from the regulating machine is injected at an angle suitable for giving speed control there is a component of the secondary current I_2 in phase with, or in phase opposition to, the e.m.f. Power is thus being delivered to or taken from the auxiliary machine; this power is the slip power and corresponds to that wasted in secondary resistance when speed control by rotor resistance is employed.

MAGNITUDE OF SLIP POWER. Referring to the complexor diagram of Fig. 4.3, which represents the conditions for an induction motor with an e.m.f. E_j injected into the secondary circuit at an angle β, it can be seen that the power taken from the supply to the primary

$$P = VI_1 \cos\phi$$
$$= E_1I_1 \cos\psi + I_1{}^2r_1$$

Resolving on to E_1,

$$I_1 \cos \psi = I_c + I_2' \cos \phi_2$$

$$\therefore E_1 I_1 \cos \psi = E_1 I_c + E_1 I_2' \cos \phi_2$$

$$= \text{iron loss} + \text{power transferred to secondary}$$

Hence

$$P = \text{iron loss} + \text{primary copper loss} + \text{power transferred to secondary}$$

Resolving the secondary e.m.f.'s on to I_2—

$$sE_2 \cos \phi_2 = I_2' r_2' + E_j \cos [180 - (\beta - \phi_2)]$$

Referring to the primary and multiplying by I_2'—

$$sE_1 I_2' \cos \phi_2 = (I_2')^2 r_2' + E_j' I_2' \cos [180 - (\beta - \phi_2)]$$

Adding $(1 - s)E_1 I_2' \cos \phi_2$ to both sides,

$$E_1 I_2' \cos \phi_2 = (I_2')^2 r_2' + E_j' I_2' \cos [180 - (\beta - \phi_2)] + (1 - s)E_1 I_2' \cos \phi_2 \quad . \qquad (4.2)$$

Thus

$$\begin{matrix}\text{power transferred} \\ \text{to secondary}\end{matrix} = \begin{matrix}\text{secondary} \\ \text{resistance} \\ \text{loss}\end{matrix} + \begin{matrix}\text{power } \textit{from} \text{ rotor} \\ \textit{to} \text{ regulating} \\ \text{machine} \\ \text{(slip power)}\end{matrix} + \begin{matrix}\text{mechanical} \\ \text{output}\end{matrix}$$

It can be seen that the mechanical output from the shaft is *$(1 - s)$ times the secondary input and that this will* be less or more than the secondary input according as to whether s is less or greater than 1, i.e. whether the motor is running below or above synchronous speed. Neglecting the relatively small resistance loss, the power to the regulating machine will be positive for sub-synchronous speeds and negative for super-synchronous speeds, power in the latter case being delivered from the auxiliary machine to the main motor. This distribution of secondary power is illustrated, for a constant motor input and with losses neglected, in Fig. 4.4.

It is seen that the regulating machine will have to deal with an amount of power which is proportional to slip, and, if speed control down to zero speed or up to twice synchronous speed is required, it will have to be of the same rating as the main motor. The method of disposing of this slip power gives rise to two alternative types of drive depending on whether it is returned to the supply or added to the mechanical output at the shaft of the main motor.

Constant Torque Drives. In this case the slip power is returned to the supply. The regulating machine may be coupled to a generator connected to the supply mains, and when the main motor is running at sub-synchronous speeds the regulating machine acts as a motor driving the generator which returns the power direct to the supply; at super-synchronous speeds the generator runs as a motor

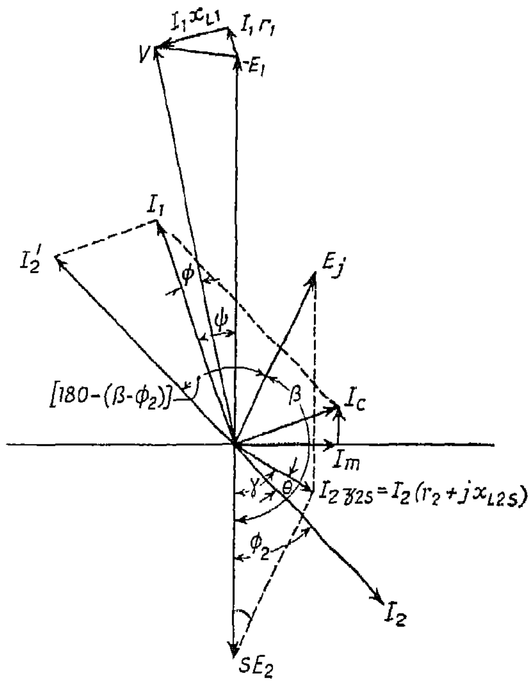


FIG. 4.3. COMPLEXOR DIAGRAM WITH E_j INJECTED AT ANGLE β

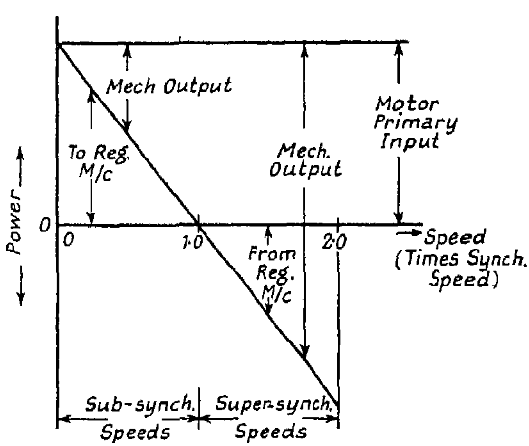


FIG. 4.4. DISTRIBUTION OF POWER AT VARIOUS SPEEDS

driving the regulating machine which supplies the slip power to the secondary circuit of the main motor. The conditions are illustrated in Fig. 4.5. For a given main-motor input it can be seen that the

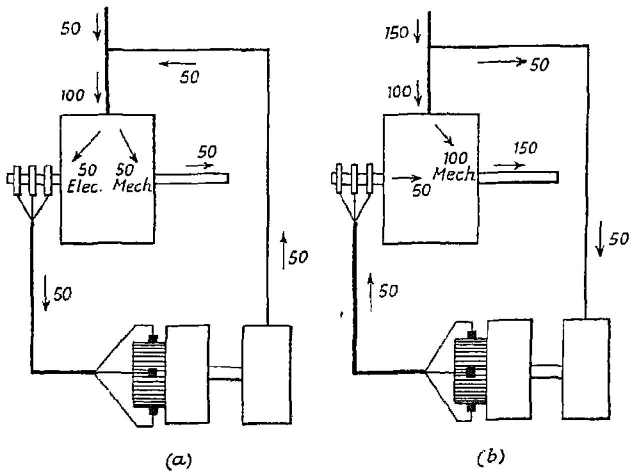


FIG. 4.5. CONSTANT TORQUE DRIVE

(a) 50 per cent of synchronous speed.
(b) 150 per cent of synchronous speed.

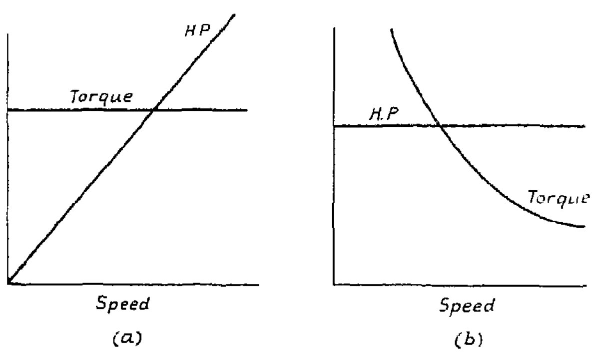


FIG. 4.6. MECHANICAL CHARACTERISTICS OF DRIVES FOR CONSTANT MAIN-MOTOR INPUT

(a) Constant torque drives.
(b) Constant h.p. drives.

mechanical output at the main-motor shaft is proportional to speed so that the torque is constant at all speeds, as shown by the curve of Fig. 4.6(*a*).

CONSTANT HORSE-POWER DRIVES. The slip power is added to or taken from the main-motor shaft, the auxiliary generator of the previous case being made to drive a third machine mounted on the main-motor shaft. The operating conditions are shown in Fig. 4.7 from which it is seen that the horse-power is constant and that the

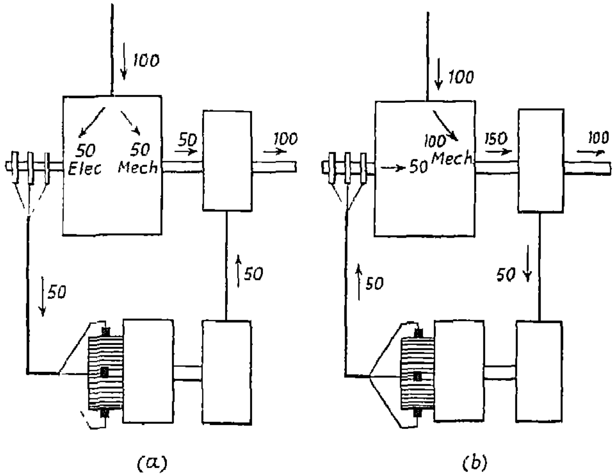


FIG. 4.7 CONSTANT H.P. DRIVE

(a) 50 per cent of synchronous speed.
(b) 150 per cent of synchronous speed.

torque varies inversely as the speed, leading to very high torques at low speeds, as shown by the curves of Fig. 4.6.

Approximate Torque-slip Relations with Injected E.M.F.

The torque of a 3-phase induction motor can be shown* to be

$$TM = (\sqrt{2})TB_mL(D/2)I_2 \cos\phi_2 \text{ newton-metres}$$

If the supply voltage is constant as is normally the case the flux in the air gap may also be assumed constant and an approximate expression obtained for the torque-slip curves. As in the plain induction motor, with the same assumption made, this expression gives reasonable accuracy for the normal working range but becomes inaccurate as the motor approaches standstill conditions.

The above expression for the torque may be written

$$TM = K \,.\, I_2 \cos\phi_2$$

From Fig. 4.3 which shows the secondary circuit quantities with the e.m.f. E_i injected at an angle β, it can be seen that—

$$\phi_2 = \gamma - \theta$$

$$\therefore \quad \cos\phi_2 = \cos\gamma\cos\theta + \sin\gamma\sin\theta$$

* Say, *The Performance and Design of A.C. Machines*, Ch. XII (Pitman).

Also $\sin \gamma = \dfrac{E_j \sin (180 - \beta)}{I_2 z_{2s}} = \dfrac{E_j \sin \beta}{I_2 z_{2s}}$

and $\cos \gamma = \sqrt{(1 - \sin^2 \gamma)} = \sqrt{\left\{1 - \left(\dfrac{E_j \sin \beta}{I_2 z_{2s}}\right)^2\right\}}$

Hence

$$I_2 \cos \phi_2 = I_2 \left[\sqrt{\left\{1 - \left(\frac{E_j \sin \beta}{I_2 z_{2s}}\right)^2\right\}} \times \frac{r_2}{z_{2s}} + \frac{E_j \sin \beta}{I_2 z_{2s}} \times \frac{x_{L2s}}{z_{2s}}\right]$$

$$= (1/z_{2s}^2)\{r_2\sqrt{(I_2^2 z_{2s}^2 - E_j^2 \sin^2 \beta)} + x_{L2s} E_j \sin \beta\} \quad (4.3)$$

Also from Fig. 4.3,

$$I_2^2 z_{2s}^2 = (sE_2)^2 + E_j^2 - 2sE_2E_j \cos (180 - \beta)$$
$$= (sE_2)^2 + E_j^2 + 2sE_2E_j \cos \beta \quad . \quad . \quad (4.4)$$

Constant Injected E.M.F. If E_j is constant, the expression for $I_2^2 z_{2s}^2$ may be substituted in eq. (4.3), and this in turn substituted in the expression for the torque giving—

$$TM = (K/z_{2s}^2)[r_2\sqrt{\{(sE_2)^2 + E_j^2 + 2sE_2E_j \cos \beta - E_j^2 \sin^2 \beta\}} + x_{L2s}E_j \sin \beta]$$
$$= (K/z_{2s}^2)\{r_2(sE_2 + E_j \cos \beta) + x_{L2s}E_j \sin \beta\}$$
$$= K\left(\frac{sE_2 r_2}{z_{2s}^2} + E_j \frac{r_2 \cos \beta + x_{L2s} \sin \beta}{z_{2s}^2}\right) \text{ newton-metres} \quad (4.5)$$

It can be seen that if $E_j = 0$ this reduces to the expression for an ordinary induction motor. It should be noted, however, that the secondary-circuit reactance x_{L2s} is usually made up of two parts, one of which is proportional to slip and the other which is constant; thus $x_{L2s} = sx_{2m} + x_{2a}$. In general x_{2m} corresponds to the reactance of the induction motor secondary winding and x_{2a} to the reactance of the auxiliary machine although there are exceptions to this. The value of z_{2s} in the above is therefore equal to $r_2 + j(sx_{2m} + x_{2a})$.

Speed-torque curves have been plotted in Fig. 4.8 for a hypothetical machine having the following constants—

$$E_2 = 100$$
$$K = 50$$
$$r_2 = 1{\cdot}0\ \Omega$$
$$x_{2m} = 4{\cdot}0\ \Omega$$
$$x_{2a} = 1{\cdot}0\ \Omega$$

It can be seen that injecting the e.m.f. at $\beta = 0°$ gives speed-torque curves above synchronism with a nearly flat characteristic over the working range; the maximum torque is also very much increased over that for the plain induction motor ($E_j = 0$). Injecting at $\beta = 180°$ gives speeds below synchronism with a rather

steep characteristic and a low maximum torque. Injecting at 90° does not have a great effect on the speed but, as explained previously, will improve the power factor. The value of E_j required to give a reasonable power-factor improvement will, of course, be much smaller than that required to give, say, a 50 per cent change in speed.

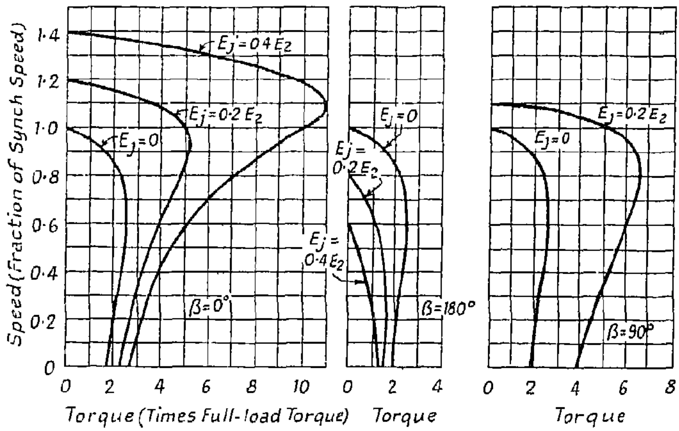


FIG. 4.8. SPEED-TORQUE CURVES FOR INDUCTION MOTOR WITH CONSTANT INJECTED E.M.F.

INJECTED E.M.F. PROPORTIONAL TO CURRENT. In this case the injected e.m.f. may be written $E_j = aI_2$, the angle of injection still being β to the secondary e.m.f., so that eqs. (4.3) and (4.4) become—

$$I_2 \cos \phi_2 = (1/z_{2s}^2)\{r_2\sqrt{(I_2^2 z_{2s}^2 - a^2 I_2^2 \sin^2 \beta)} + aI_2 x_{L2s} \sin \beta\}$$
$$= (I_2/z_{2s}^2)\{r_2\sqrt{(z_{2s}^2 - a^2 \sin^2 \beta)} + a x_{L2s} \sin \beta\}$$

and

$$I_2^2 z_{2s}^2 = (sE_2)^2 + a^2 I_2^2 + 2sE_2 a I_2 \cos \beta$$

so that $$I_2 = \frac{sE_2}{z_{2s}^2 - a^2}\{a \cos \beta \pm \sqrt{(z_{2s}^2 - a^2 \sin^2 \beta)}\}$$

Substituting these in the expression for torque gives—

$$TM = \frac{KsE_2 r_2}{z_{2s}^2}\left\{\frac{\sqrt{(z_{2s}^2 - a^2 \sin^2 \beta)} + a(x_{L2s}/r_2) \sin \beta}{z_{2s}^2 - a^2}\right\}$$
$$\{a \cos \beta \pm \sqrt{(z_{2s}^2 - a^2 \sin^2 \beta)}\}$$
$$= \frac{KsE_2 r_2}{z_{2s}^2(z_{2s}^2 - a^2)}\left[z_{2s}^2 - a^2 \sin^2 \beta + a\sqrt{(z_{2s}^2 - a^2 \sin^2 \beta)}\right.$$
$$\left.\left\{\cos \beta + \frac{x_{L2s}}{r_2} \sin \beta\right\} + a^2 \sin \beta \cos \beta \cdot \frac{x_{L2s}}{r_2}\right] \quad (4.6a)$$

If $\beta = 0$, $\sin\beta = 0$ and $\cos\beta = 1$, so that this becomes—

$$TM = KsE_2r_2/(z_{2s}^2 - z_{2s}a) \qquad . \quad . \quad (4.6b)$$

If $\beta = 180°$, $\sin\beta = 0$ and $\cos\beta = -1$ so that the torque expression becomes—

$$TM = KsE_2r_2/(z_{2s}^2 + z_{2s}a) \qquad . \quad . \quad (4.6c)$$

If $\beta = 90°$, $\sin\beta = 1$ and $\cos\beta = 0$ so that the expression becomes—

$$TM = \frac{KsE_2r_2}{z_{2s}^2}\left(1 + \frac{a(x_{L2s}/r_2)}{\sqrt{(z_{2s}^2 - a^2)}}\right) \quad . \quad . \quad (4.6d)$$

In Fig. 4.9 are plotted torque-slip curves for the same machine as those of Fig. 4.8 but with $E_j = 1{\cdot}0\, I_2$, i.e. $a = 1{\cdot}0$. It can be seen

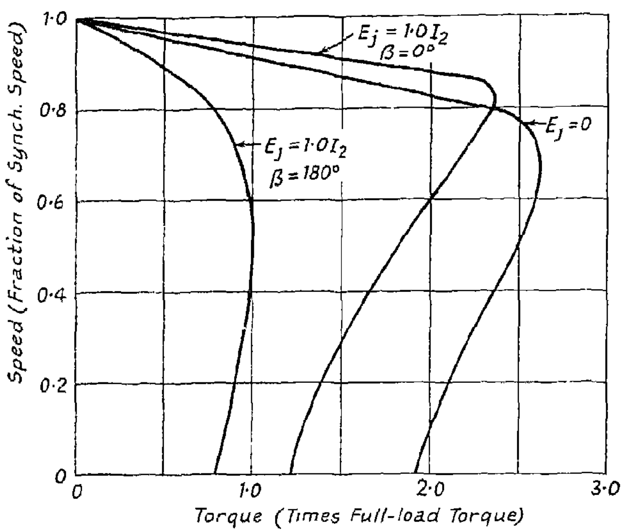


FIG. 4.9. SPEED-TORQUE CURVES FOR INDUCTION MOTOR WITH INJECTED E.M.F. PROPORTIONAL TO ROTOR CURRENT

that, as expected, the no-load speed is not affected, but that if $\beta = 180°$ the speed characteristic over the working range is made more steep while if $\beta = 0°$ it is made less steep; of these two conditions only the former is of practical importance. If the e.m.f. is injected at 90° an improvement in power factor results as previously indicated and also an improvement in maximum torque. As the e.m.f. necessary to give a reasonable improvement of power factor is small the change in the speed-torque curve is not very great.

With the e.m.f. proportional to current it is, in practice, more

convenient to inject it at a definite angle to the rotor current rather than to the secondary e.m.f.—the rotor current and the secondary e.m.f. are, however, usually nearly in phase so that this does not appreciably affect the general relations given above.

Equivalent Circuit

For analytical calculations an equivalent circuit, based on that of the induction motor,* may be used. Such a circuit is simply that of the induction motor with the injected e.m.f. included in the secondary as shown in Fig. 4.10. This diagram is the basic circuit

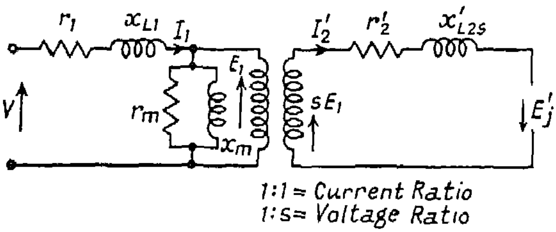


FIG. 4.10. EQUIVALENT CIRCUIT OF INDUCTION MOTOR WITH INJECTED E.M.F. IN SECONDARY

for all machines in which an injected e.m.f. is introduced into the secondary circuit, i.e. those discussed in Parts II and III of this book.

In the diagram the secondary quantities have been referred to the primary in the usual way so that by inspection the following expressions can be written down—

$$\begin{aligned} \mathbf{V} &= \mathbf{I}_1\mathbf{z}_1 + \mathbf{E}_1 \\ \mathbf{I}_0\mathbf{z}_0 &= \mathbf{E}_1 \\ \mathbf{I}_1 &= \mathbf{I}_0 + \mathbf{I}_2' \\ \mathbf{I}_2'\mathbf{z}_{2s}' &= s\mathbf{E}_1 + \mathbf{E}_j' \end{aligned} \qquad (4.7)$$

From (*a*), (*b*) and (*c*)—

$$\begin{aligned} \mathbf{V} &= (\mathbf{I}_0 + \mathbf{I}_2')\mathbf{z}_1 + \mathbf{E}_1 \\ &= \mathbf{E}_1(1 + \mathbf{z}_1/\mathbf{z}_0) + \mathbf{I}_2'\mathbf{z}_1 \\ &= \mathbf{E}_1\mathbf{c} + \mathbf{I}_2'\mathbf{z}_1 \end{aligned}$$

$$\therefore \quad \mathbf{E}_1 = (\mathbf{V} - \mathbf{I}_2'\mathbf{z}_1)/\mathbf{c} \qquad (4.8)$$

From (4.7*d*) and (4.8)—

$$\mathbf{I}_2' = (s\mathbf{V} + \mathbf{c}\mathbf{E}_j')/(s\mathbf{z}_1 + \mathbf{c}\mathbf{z}_{2s}')$$

* Say, *The Performance and Design of A.C. Machines*, Ch. XII (Pitman).

† z_0 is the impedance of the magnetizing branch and may comprise r_0 and x_0 in series or r_m and x_m in parallel.

so that from (4.7*c*)

$$\mathbf{I}_1 = \frac{1}{\mathbf{c}}\left[\frac{\mathbf{V}}{\mathbf{z}_0} + \frac{s\mathbf{V} + \mathbf{c}\mathbf{E}_j'}{s\mathbf{z}_1 + \mathbf{c}\mathbf{z}_{2s}'}\right] \qquad (4.9a)$$

The ratio z_1/z_0 is usually small relative to 1 so that to a first approximation **c** may be regarded as unity and at zero angle; the primary current thus becomes

$$\mathbf{I}_1 = \mathbf{V}/\mathbf{z}_0 + (s\mathbf{V} + \mathbf{E}_j')/(s\mathbf{z}_1 + \mathbf{z}_{2s}') \qquad (4.9b)$$

If $E_j = 0$ this reduces to the expression resulting from the equivalent circuit for an induction motor with the magnetizing branch moved to the supply terminals.

From the above relations the various powers and the torque can be found.

Stator input $= VI_1 \cos\phi_1$ watts/phase

where ϕ_1 is the primary phase angle found from (4.9*b*).

Rotor input (across gap) $= E_1 I_2' \cos\phi_2$ watts/phase

where ϕ_2 is the phase angle between E_1 and I_2'.

Torque = rotor input across gap synch. watts/phase
= (rotor input)/$2\pi n_1$ newton-metres
= rotor input $\times$ 0·117/n_1 Lb-ft

Mechanical output = rotor input $(1 - s)$ watts/phase

Power delivered to or from regulating machine

$$= E_j' I_2' \cos\phi_j \text{ watts/phase}$$

where ϕ_j is the phase angle between E_j' and I_2'.

The behaviour of an induction motor with any of the types of regulating machine can be found from the above for a given value of E_j. Simplifications or modifications can, however, be made in particular cases as described in succeeding chapters.

EXERCISES 4

1. The particulars refer to the rotor of a 2,000-h.p., 6·6-kV, 50-c/s induction motor—

Resistance, 0·02 Ω/ph.; reactance, 0·12 Ω/ph. at 50-c/s.

If the rotor current is 1,000 A per phase and the air-gap flux is constant at a value giving a standstill rotor e.m.f. of 1,000 V per phase find the slip, rotor power factor and the power component of the rotor current under each of the following conditions—

(i) Rotor short-circuited.
(ii) An added rotor resistance of 0·4 Ω/ph.
(iii) An added rotor inductive reactance of 0·25 Ω/ph. at 50-c/s.
(iv) An added capacitance of 15·9 F per phase.
(v) 200 V per phase injected 180° to the rotor e.m.f.
(vi) 200 V per phase injected in phase with the rotor e.m.f.
(vii) 10 V per phase injected at 90° leading the rotor e.m.f.
(viii) 10 V per phase injected at 90° leading the rotor current.
(ix) 10 V per phase injected at 30° leading the rotor current.

In cases (viii) and (ix) an approximate result may be obtained by neglecting the rotor reactance relative to its resistance.

2. At full load a 1,000-h.p., 3·3-kV induction motor runs at 735 r.p.m. with an efficiency of 94 per cent and a power factor of 0·87. The magnetizing and iron-loss current is 40 A at 0·15 power factor and the ratio of stator to rotor turns (both star-connected) is 2 to 1. Estimate the voltage, current and kVA rating of a phase advancer to improve the power factor to 1·0 when delivering full-load torque. Neglect stator impedance and rotor leakage reactance.

3. A 2,000-h.p., 12-pole, 50-c/s induction motor is equipped with an auxiliary machine for giving speed control. If the slip power is returned from the auxiliary machine to the supply and if the stator input power is 1,000 kW find, neglecting all losses, with the motor running at (*a*) 350 r.p.m. and (*b*) 650 r.p.m., the slip power, the mechanical power produced in the rotor, the total mechanical power at the shaft, the shaft torque and the total input from the supply. Draw diagrams showing the power flow in each case.

Repeat with the power from the auxiliary machine added to the shaft power.

4. A 1,500-h.p., 3-phase, 3·3-kV, 24-pole induction motor has a full-load power factor of 0·88, a full-load slip of 0·0125 and a full-load efficiency of 94 per cent. The stator resistance and reactance drops are 0·01 and 0·1 per unit respectively. The ratio of stator to rotor turns (both star-connected) is 2 to 1. Magnetizing current is 70 A and iron-loss current is 10 A. Find the rotor resistance and estimate the rating of a phase advancer to improve the full-load power factor to 0·95 leading. Neglect rotor-circuit reactance.

5. An induction motor drives a load requiring 1,000 kW at synchronous speed and is fitted with a Scherbius machine to give speed control down to standstill. The Scherbius machine may deliver its output (*a*) to the supply or (*b*) to the motor shaft. For each of the types of drive given below plot curves to a base of speed showing shaft output, stator input power and slip power. Neglect losses.

(i) Power constant at all speeds (torque proportional to $1/n$).
(ii) Power proportional to speed (constant torque).
(iii) Power proportional to square of speed (torque proportional to n).
(iv) Power proportional to cube of speed (torque proportional to n^2).

6. A wind-tunnel fan (torque proportional to square of speed) requiring 10,000 h.p. at 600 r.p.m. is driven by an induction motor with a Scherbius machine for giving speed control down to zero. Plot to a base of speed a curve of power carried by the Scherbius machine and show analytically that it will not exceed approximately $\frac{1}{7}$ of 10,000 h.p.

7. The following are the equivalent-star parameters for an induction motor with a slip regulator, the slip power being fed to or from the supply.

$$V = 1{,}150\ \text{V} \qquad z_0 = 5 + j40\ \Omega$$
$$z_1 = 0{\cdot}3 + j1{\cdot}0\ \Omega \qquad E' = 200\underline{/180^\circ}\ \text{V}$$
$$z_{2s}' = 1{\cdot}0 + j(s1{\cdot}0 + 1{\cdot}0)\ \Omega$$

Using the equivalent circuit determine, for a slip of 0·5, the stator input, rotor input, mechanical output, power fed back to the supply and the copper and iron losses.

Repeat with $E_j' = 500\underline{/0^\circ}$ and a slip of $-$ 0·2.

CHAPTER 5

REGULATING MACHINES GIVING INJECTED E.M.F. DEPENDENT ON THE CURRENT ("SERIES" MACHINES)

THE simplest type of auxiliary machine for injecting an e.m.f. into the secondary circuit of an induction motor is that in which the e.m.f. is dependent on, and may often be regarded as being proportional to, the secondary current. The Leblanc and Walker machines shown in Fig. 4.2 (page 46) fall into this category and,

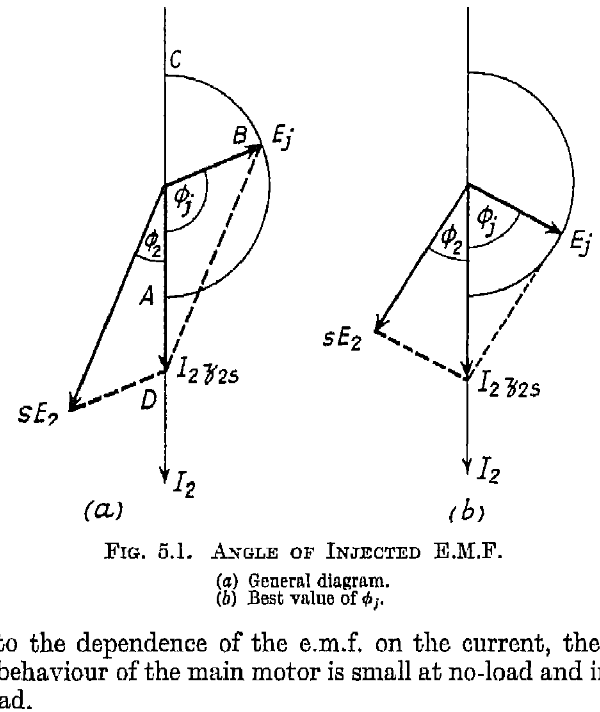


FIG. 5.1. ANGLE OF INJECTED E.M.F.

(a) General diagram.
(b) Best value of ϕ_j.

owing to the dependence of the e.m.f. on the current, their effect on the behaviour of the main motor is small at no-load and increases with load.

ANGLE OF INJECTED E.M.F. With these machines the angle at which the e.m.f. is injected bears a definite relationship to the phase angle of the secondary current and normally leads it. In what follows this angle is referred to as the angle ϕ_j and is positive when leading the current I_2.

The best value for this angle ϕ_j can be seen from the complexor diagrams of Fig. 5.1 in which I_2 represents the secondary current and E_j the injected e.m.f. leading this current by ϕ_j. For a given value of I_2, and therefore of E_j, and various values of ϕ_j the locus of the end of the E_j complexor lies on the semicircle ABC. If

reactance is neglected (it is usually small with this type of machine since the slip is low) the impedance drop I_2z_{2s} may be drawn in phase with I_2 as indicated. For any position of E_j the complexor sE_2 can now be drawn ($\mathbf{sE}_2 = \mathbf{I}_2\mathbf{z}_{2s} - \mathbf{E}_j$); this complexor can also be represented by the line DB. To obtain the maximum angle ϕ_2 of the lead of I_2 ahead of sE_2, i.e. for operation as a phase advancer, it can be seen that DB should be a tangent to the semicircle; E_j should thus be injected at an angle considerably less than 90° ahead of the secondary current. Under these conditions E_j is leading sE_2 by 90° and $\phi_j = 90 - \phi_2$. Thus if it is desired to make $\phi_2 = 60°$,

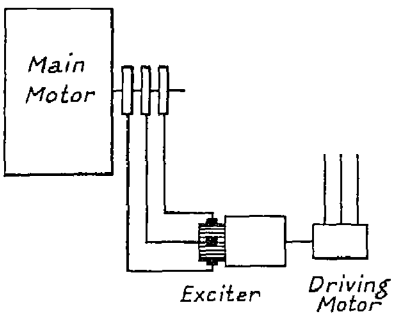


FIG. 5.2. LEBLANC EXCITER

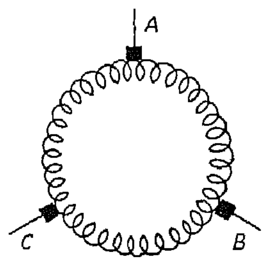


FIG. 5.3. DIAGRAMMATIC REPRESENTATION OF LEBLANC EXCITER

ϕ_j must be 30° and this angle has been found to be a convenient practical value.

If it is desired to use a machine of this type as a slip regulator, i.e. to control the speed, the e.m.f. must be injected more nearly in phase or phase opposition to the current. As mentioned in the previous chapter such control is likely to be required only in order to reduce the speed, so that ϕ_j will have to approach 180°.

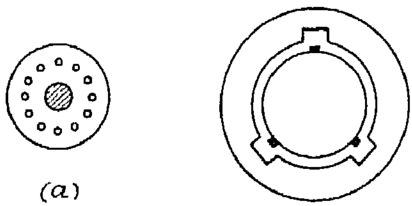
The Leblanc Exciter

This device, suggested by Leblanc in 1895, consists simply of a rotating armature carrying a double-layer winding connected to a commutator, no stator winding or stator being necessary. Three brushes per pole-pair lie on the commutator and are connected to the slip rings of the main motor as shown in Fig. 5.2. The armature is driven by a suitable motor, e.g. a small cage induction motor, at an approximately constant speed.

Suppose firstly that the armature be stationary—the winding will form an ordinary delta-connected circuit fed by the three brushes as shown in Fig. 5.3 and will carry the secondary currents, i.e. currents of slip frequency. These will set up a rotating field moving at slip-frequency speed relative to the brushes and, as explained on page 9, the field axis will be horizontal at the moment when the current in brush A is a maximum; the e.m.f. induced by this field in the conductors will thus lag 90° behind the current and will, neglecting

saturation of the iron, be proportional to the current. The armature thus behaves like a group of three delta-connected reactors.

Suppose now that the armature is driven in the same direction and at the same speed as the field. For the same current supplied to the brushes the speed and magnitude of the field will not be affected since the current in the group of conductors between, say, A and B will be the same although the conductors themselves will be continually changing. The conductors are, however, moving at the same speed as the field and therefore have no e.m.f. induced in



(a) (b)

FIG. 5.4. LEBLANC EXCITER CONSTRUCTIONS

(a) Tunnel slots, no stator.
(b) Increased air gap, opposite coils being commutated

them so that the armature behaves simply as a group of delta-connected resistors.

If the speed of the armature is raised above that of the field the conductors will be moving faster than the field, i.e. the relative speed of conductors and field will have reversed as compared to the stationary condition, so that the e.m.f. induced will also have reversed; it will thus lead the current by 90°. Under these conditions the armature is behaving as a group of delta-connected capacitors and will cause an improvement of the power factor of the main motor.

This 90° phase displacement between current and injected e.m.f. shows that, neglecting losses, no power is consumed or generated by the exciter so that it cannot produce a change in speed.

The Leblanc exciter thus consists of an armature driven at above the synchronous speed corresponding to slip-frequency currents and it gives an e.m.f. approximately proportional to the secondary current of the main motor and leading it by 90°. It can therefore act as a phase advancer and is sometimes so used to improve the power factor of small induction motors of sizes up to about 50 h.p.

Construction of Simple Leblanc Exciter

Although the simple arrangement described above is possible, it is preferable to equip the exciter with a stator in order to provide a low-reluctance path for the flux; the stator need, however, carry no windings. Alternatively the armature winding can be placed in tunnel slots as shown in Fig. 5.4(a). Commutation is a difficulty

with this type of machine and limits its application to the small sizes mentioned. The rotating field sets up an e.m.f. in the coils short-circuited by the brush. A slight improvement can be effected by employing a stator and air gap as shown in Fig. 5.4(*b*) with longer gaps opposite the coils undergoing commutation, so that the flux cutting the short-circuited conductors is reduced.

MODIFICATIONS FOR OBTAINING CONTROL OF E.M.F. The advancer described above, if driven at constant speed, has no means

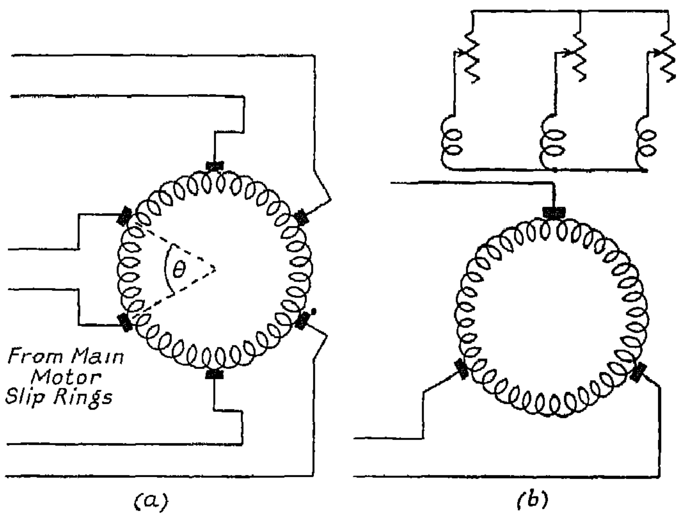


FIG. 5.5. MODIFICATIONS TO GIVE CONTROL OF INJECTED E.M.F.

(*a*) Movable brushes.
(*b*) Stator winding with variable resistor.

for controlling the magnitude of the e.m.f. injected, i.e. for controlling the power factor of the main motor.

If such control is desirable a variable-speed driving motor for the exciter could be used, but this would probably have to be a d.c. motor necessitating a d.c. supply. An alternative suggested by Rudra* is to employ two movable brushes per phase as shown in Fig. 5.5(*a*): by moving the brushes to vary the angle θ any desired e.m.f. can be tapped off the commutator, but, as can be seen, six leads have to be brought from the rotor of the main motor. Another alternative is to add a stator winding to the advancer and connect this to a variable resistor as shown in Fig. 5.5(*b*); by adding resistance the phase angle of the e.m.f. relative to the current is varied, and some particular value of resistance gives the maximum advancing effect as indicated by the complexor diagram of Fig. 5.1. There is, of course, an accompanying small change in slip, and the

* J. J. Rudra, *J. Instn. Elect. Engrs.*, **86**, p. 383 (1940).

corresponding slip power is lost in the resistance. The complication of any of these methods, however, hardly warrants their use on the small sizes of motor for which the simple Leblanc advancer is suitable.

The Walker Series Machine

In 1909 Miles Walker* suggested adding a series winding to the stator of the simple Leblanc exciter and a considerably improved performance was obtained as a result.

Two sets of windings must, in practice, be added, firstly a compensating winding connected in series with the brushes and secondly an exciting winding also in series with the brushes. The compensating winding is designed to neutralize the armature m.m.f. entirely so that were it not for the exciting winding there would be no flux produced and no e.m.f. generated. The exciting winding is thus solely responsible for producing the flux in the machine and, neglecting saturation, the e.m.f. produced is proportional to the current in the winding, i.e. to I_2. The compensating winding must occupy a definite position relative to the brushes since the axis of the armature m.m.f. at any moment is dependent on brush position. The machine can, however, be designed with the exciting winding so arranged that the e.m.f. can be injected at any desired angle, ϕ_j, relative to the current.

As the injected e.m.f. and the rotor current need no longer be at 90° as in the simple Leblanc exciter, power can be absorbed by or delivered from the Walker machine so that a variation of speed can be obtained as well as an improvement of power factor if desired. Any power delivered to or taken from the regulating machine will pass through its driving motor which will act as a generator or motor and return slip energy to or receive it from the supply.

The mechanical construction of the machine is similar to that of the Scherbius machine described on page 82.

Analytical Treatment of Motor and Auxiliary Machine having Injected E.M.F. Proportional to Current

The treatment is based on eqs. (4.7) on page 55; substituting $\mathbf{a}'\mathbf{I}_2'$ for $\mathbf{E}_j'$ where $\mathbf{a}' = a' \cos \delta + \mathrm{j}a' \sin \delta$ gives—

$$\left.\begin{aligned} \mathbf{V} &= \mathbf{I}_1\mathbf{z}_1 + \mathbf{E}_1 && (a)\\ \mathbf{I}_0\mathbf{z}_0 &= \mathbf{E}_1 && (b)\\ \mathbf{I}_1 &= \mathbf{I}_0 + \mathbf{I}_2' && (c)\\ \mathbf{I}_2'\mathbf{z}_{2s}' &= s\mathbf{E}_1 + \mathbf{a}'\mathbf{I}_2' && (d) \end{aligned}\right\} \qquad (5.1)$$

From (d)

$$\mathbf{I}_2' = s\mathbf{E}_1/(\mathbf{z}_{2s}' - \mathbf{a}')$$

* *J. Instn. Elect. Engrs.*, **42**, p. 599 (1908–9).

From (*b*) and (*c*)

$$\mathbf{I}_1 = \mathbf{E}_1/\mathbf{z}_0 + s\mathbf{E}_1/(\mathbf{z}_{2s}' - \mathbf{a}')$$

$$\therefore \qquad \mathbf{E}_1 = \mathbf{I}_1 \frac{1}{1/\mathbf{z}_0 + s/(\mathbf{z}_{2s}' - \mathbf{a}')}$$

From (*a*)

$$\mathbf{V} = \mathbf{I}_1 \left\{ \mathbf{z}_1 + \frac{1}{\dfrac{1}{\mathbf{z}_0} + \dfrac{s}{(\mathbf{z}_{2s}' - \mathbf{a}')}} \right\} = \mathbf{I}_1 \left\{ \mathbf{z}_1 + \frac{1}{\dfrac{1}{\mathbf{z}_0} + \dfrac{1}{(\mathbf{z}_{2s}' - \mathbf{a}')/s}} \right\}$$

The term in the bracket is the effective impedance of the motor and its auxiliary machine as viewed from the stator terminals of the motor; it can be seen to be made up of the primary impedance $\mathbf{z}_1\ (= r_1 + \mathrm{j}x_{L1})$ in series with two branches in parallel, one being the magnetizing impedance $\mathbf{z}_0\ (= r_0 + \mathrm{j}x_0)$ and the other an impedance $(\mathbf{z}_{2s}' - \mathbf{a}')/s$ where $\mathbf{z}_{2s}' = r_2' + r_e' + \mathrm{j}s(x_{L2}' + x_{Le}')$. Putting in the real and quadrature terms of the complex impedances gives—

$$\mathbf{V} = \mathbf{I}_1 \left[r_1 + \mathrm{j}x_{L1} + \frac{1}{\dfrac{1}{r_0 + \mathrm{j}x_0} + \dfrac{1}{(r_2' + r_e' - a'\cos\phi_j)/s + \mathrm{j}\{x_{L2}' + x_{Le}' - (a'\sin\phi_j)/s\}}} \right]$$

The equivalent circuit of the motor and auxiliary machine can thus be represented as in Fig. 5.6(*a*) in which the magnetizing impedance has been represented by the parallel branches r_m and x_m in the usual way, and the reactance $-(a'\sin\phi_j)/s$ has been represented by a variable capacitor as it is negative for all normal values of ϕ_j.

Inspection of the circuit shows that the total rotor input per phase is given by—

$$\text{rotor input} = (I_2')^2(r_2' + r_e' - a'\cos\phi_j)/s \text{ watts}$$

As with the plain induction motor this represents the motor torque in synchronous watts per phase.

The distribution of power in the rotor circuit can be found by writing the above total input as follows—

rotor input

$$= (I_2')^2\{r_2' + r_e' - a'\cos\phi_j + (r_2' + r_e' - a'\cos\phi_j)(1 - s)/s\}$$

$$= (I_2')^2 r_2' + (I_2')^2 r_e' - (I_2')^2 a'\cos\phi_j + (I_2')^2(r_2' + r_e' - a'\cos\phi_j)$$

rotor	reg. m/c	power	mechanical
= copper +	copper +	in +	output
loss	loss	reg. m/c shaft	of main motor

If the angle of injection, ϕ_j, lies between 0 and 90°, power is delivered from the regulating machine to the rotor and if between 90° and 180° it is being delivered from the rotor to the regulating machine.

APPROXIMATE CURRENT LOCUS. An approximate and simpler equivalent circuit is shown in Fig. 5.6(*b*), the magnetizing branch

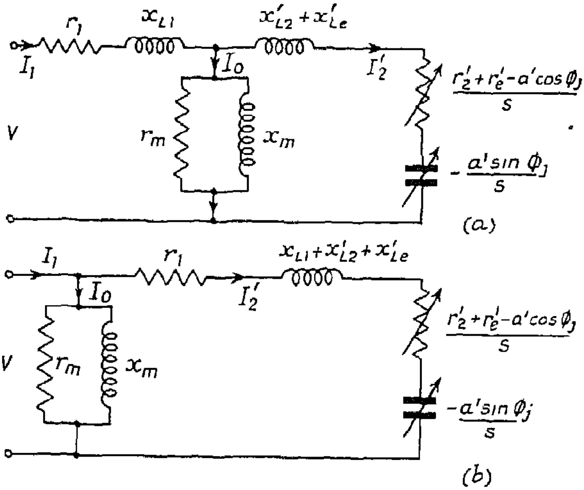


FIG. 5.6. EQUIVALENT CIRCUITS FOR INDUCTION MOTOR WITH "SERIES" REGULATING MACHINE ($E_j \propto I_2$)

(*a*) Complete equivalent circuit.
(*b*) Simplified equivalent circuit

being located at the supply terminals as is commonly done with the induction motor.

Assuming r_2', r_e', and a' to be constant, the main branch of this approximate circuit comprises a constant and a variable resistance and a constant and a variable reactance, the two variables having a constant ratio of $(-a' \sin \phi_j)/(r_2' + r_e' - a' \cos \phi_j)$. It is shown in Appendix 2 that if such a circuit is supplied at constant voltage the locus of the end of the resulting current complexor is a circle having the following co-ordinates for its centre—

$$P_x = \frac{V}{2\{x_{L1} + x_{L2}' + x_{Le}' + r_1(a' \sin \phi_j)/(r_2' + r_e' - a' \cos \phi_j)\}}$$

$$P_y = P_x \cdot (a' \sin \phi_j)/(r_2' + r_e' - a' \cos \phi_j)$$

To draw the approximate current locus the complexor for the magnetizing-branch current is drawn as in Fig. 5.7; the centre of

the circle is then located with reference to point A and the circle drawn with radius PA.

Predetermination of Behaviour

Calculations regarding the behaviour of an induction motor with a regulating machine can be made from the above equivalent circuits either by drawing the locus diagrams for the circuit or by analytical methods. The locus diagrams are preferable for giving an overall picture to illustrate the effect of making changes in various

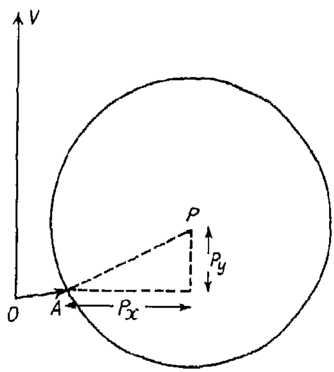


FIG. 5.7. CONSTRUCTION OF APPROXIMATE LOCUS DIAGRAM

parameters, but the analytical method is preferable for predetermination of actual behaviour. Both are illustrated in the following example.

Example 5.1. The following data refer to an induction motor equipped with a series regulating machine. Using the equivalent circuits of Fig. 5.6, predetermine the behaviour of the machine.

Rating: 3,000 h.p., 3·3 kV, 3-ph., 50 c/s, star-connected.

Primary resistance	$r_1 = 0{\cdot}1\ \Omega/\text{ph.}$
Primary leakage reactance	$x_{L1} = 0{\cdot}435\ \Omega/\text{ph.}$
Secondary resistance (referred to primary)	$r_2' = 0{\cdot}1\ \Omega/\text{ph.}$
Secondary leakage reactance (referred to primary)	$x_{L2}' = 0{\cdot}44\ \Omega/\text{ph.}$
Magnetizing equiv. resistance	$r_m = 75\ \Omega/\text{ph.}$
Magnetizing reactance	$x_m = 16\ \Omega/\text{ph.}$
Reg. machine resistance	$r_e' = 0{\cdot}08\ \Omega/\text{ph.}$
Reg. machine leakage reactance	$x_{Le}' = 0{\cdot}4\ \Omega/\text{ph.}$
Injected voltage per ampere	$a' = 0{\cdot}12$

(i) COMPARISON BETWEEN RIGOROUS AND APPROXIMATE EQUIVALENT CIRCUITS. The locus diagram for the circuit of Fig. 5.6(*a*) may be determined by the method of inversion,* the procedure being as follows—

Draw the impedance locus for the right-hand branch with varying values of s; invert this, giving the corresponding admittance locus; add the constant admittance of the magnetizing branch to each point on the admittance locus by moving the origin to the left by the amount of the magnetizing admittance. Invert this new admittance to find the impedance locus for the two parallel branches; add the stator impedance by moving the origin to the right, thus giving the total impedance locus. Inverting this gives the total admittance locus which, to another scale, gives the current locus.

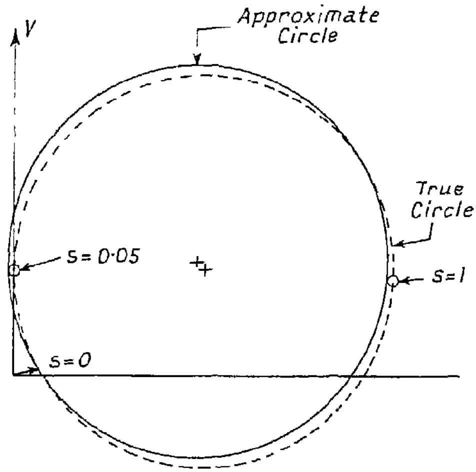


FIG. 5.8. TRUE AND APPROXIMATE CIRCLE DIAGRAMS

Carrying out the above for the machine in question operating with an angle of injection of $\phi_j = 90°$ gives the dotted circular locus of Fig. 5.8. The normal working range is between the point for $s = 0$ and the point where s is approximately 0·05.

The approximate circular locus can be found from the equivalent circuit of Fig. 5.6(*b*), the co-ordinates of the centre of the circle being given by—

$$P_x = \frac{1{,}910}{2\{0{\cdot}435 + 0{\cdot}44 + 0{\cdot}4 + 0{\cdot}1(a' \sin \phi_j)/(0{\cdot}1 + 0{\cdot}08 - a' \cos \phi_j)\}}$$

and

$$P_y = P_x(a' \sin \phi_j)/(0{\cdot}1 + 0{\cdot}08 - a' \cos \phi_j)$$

* Smith and Say, *Engineering Design Class Manual*, Ch. VIII (Chapman and Hall).

With $a' = 0{\cdot}12$ and $\phi_j = 90°$ these become $P_x = 710$ A and $P_y = 473$ A, and the circle is shown by the full line on Fig. 5.8, the origin being taken at the end of the magnetizing-circuit current complexor.

It can be seen that the error resulting from the use of the approximate method is not great, and this method is therefore suitable for observing the effect of changes in the constants.

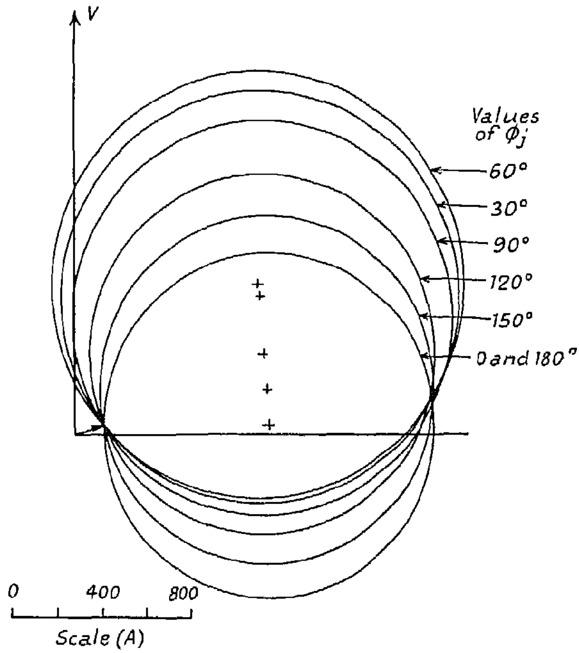


FIG. 5.9. EFFECT OF ANGLE OF INJECTION ON CURRENT LOCUS ($a' = 0{\cdot}12$)

(ii) EFFECT ON LOCUS OF VARYING THE ANGLE OF INJECTION. Using an injected e.m.f. of 0·12 V/A (referred to primary) the expressions for the co-ordinates of the centre become—

$$P_x = \frac{955}{1{\cdot}275 + (0{\cdot}012 \sin \phi_j)/(0{\cdot}18 - 0{\cdot}12 \cos \phi_j)} \quad . \quad (5.2)$$

and $$P_y = P_x(0{\cdot}12 \sin \phi_j)/(0{\cdot}18 - 0{\cdot}12 \cos \phi_j) \quad . \quad . \quad (5.3)$$

and Table 5.1 gives the figures for various values of ϕ_j. The resulting circular loci are plotted in Fig. 5.9 from which it can be seen that the most effective angle of injection is between 30° and 60°, as previously mentioned on page 59.

TABLE 5.1

ϕ_j	$\sin \phi_j$	$0{\cdot}012 \sin \phi_j$	$\cos \phi_j$	$0{\cdot}12 \cos \phi_j$	$0{\cdot}18 - 0{\cdot}12 \cos \phi_j$	$\dfrac{0{\cdot}012 \sin \phi_j}{0{\cdot}18 - 0{\cdot}12 \cos \phi_j}$	$1{\cdot}275 + \dfrac{0{\cdot}012 \sin \phi_j}{0{\cdot}18 - 0{\cdot}12 \cos \phi_j}$	P_x (amperes)	P_y (amperes)
0	0	0	1·0	0·12	0·06	0	1·275	750	0
30	0·5	0·006	0·866	0·104	0·076	0·079	1·354	706	560
60	0·866	0·0104	0·5	0·06	0·12	0·087	1·362	700	607
90	1·0	0·012	0	0	0·18	0·067	1·342	710	473
120	0·866	0·0104	− 0·5	− 0·06	0·24	0·0435	1·318	726	316
150	0·5	0·006	− 0·866	− 0·104	0·284	0·0211	1·296	738	156
180	0	0	− 1	− 0·12	0·30	0	1·275	750	0

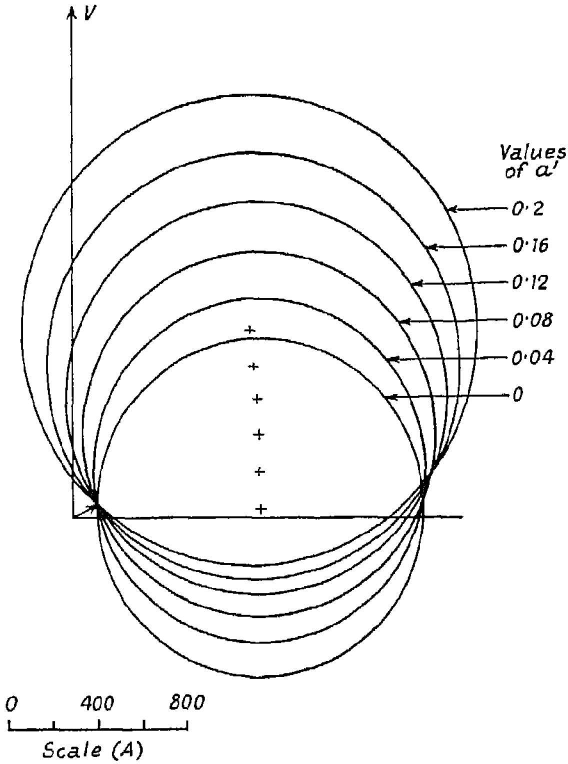


FIG. 5.10. EFFECT OF MAGNITUDE OF INJECTED E.M.F. ON CURRENT LOCUS ($\phi_i = 90°$)

TABLE 5.2

a'	$\dfrac{0{\cdot}1a'}{0{\cdot}18}$	$1{\cdot}275 + \dfrac{0{\cdot}1a}{0{\cdot}18}$	P_x (amperes)	P_y (amperes)
0	0	1·275	747	0
0·04	0·022	1·297	735	163
0 08	0 044	1·319	728	323
0·12	0·067	1 342	710	473
0·16	0·089	1·364	700	622
0·20	0 111	1·386	689	764

TABLE 5.3

ANALYTICAL CALCULATION OF BEHAVIOUR ($a' = 0{\cdot}12$, $s = 0{\cdot}05$)

Angle of Injection ϕ_i	$0°$	$90°$	$180°$
Variable resistance	$\frac{0{\cdot}18 - 0{\cdot}12}{0{\cdot}05} = 1{\cdot}2\ \Omega$	$\frac{0{\cdot}18}{0{\cdot}05} = 3{\cdot}6\ \Omega$	$\frac{0{\cdot}18 + 0{\cdot}12}{0{\cdot}05} = 6{\cdot}0\ \Omega$
Variable reactance	0	$-\frac{0{\cdot}12 \times 1{\cdot}0}{0{\cdot}05} = -2{\cdot}4\ \Omega$	0
Total reactance	$0{\cdot}84\ \Omega$	$1{\cdot}56\ \Omega$	$0{\cdot}84\ \Omega$
Impedance of rotor branch	$1{\cdot}2 + j0{\cdot}84 = 1{\cdot}47\underline{/35}$	$3{\cdot}6 - j1{\cdot}56 = 3{\cdot}91\underline{/-23{\cdot}5}$	$6{\cdot}0 + j0{\cdot}84 = 6{\cdot}55\underline{/8°}$
Admittance of rotor branch Y_2	$0{\cdot}68\underline{/-35} = 0{\cdot}555 - j0{\cdot}389$	$0{\cdot}256\underline{/23{\cdot}5} = 0{\cdot}235 + j0{\cdot}935$	$0{\cdot}153\underline{/-8°} = 0{\cdot}151 - j0{\cdot}0212$
Admittance of magnetizing branch	$0{\cdot}0133 - j0{\cdot}0625$	$0{\cdot}0133 - j0{\cdot}0625$	$0{\cdot}0133 - j0{\cdot}0625$
Total admittance of rotor and magnetizing branches	$0{\cdot}568 - j0{\cdot}451 = 0{\cdot}727\underline{/-38{\cdot}5°}$ ℧	$0{\cdot}248 + j0{\cdot}031 = 0{\cdot}25\underline{/7{\cdot}1°}$ ℧	$0{\cdot}164 - j0{\cdot}0837 = 0{\cdot}184\underline{/-27{\cdot}1°}$ ℧
Total impedance of rotor and magnetizing branches	$1{\cdot}38\underline{/38{\cdot}5°} = 1{\cdot}08 + j0{\cdot}857$	$4{\cdot}0\underline{/-7{\cdot}1°} = 3{\cdot}96 - j0{\cdot}484$	$5{\cdot}43\underline{/27{\cdot}1°} = 4{\cdot}85 + j2{\cdot}47$
Stator impedance	$0{\cdot}1 + j0{\cdot}435$	$0{\cdot}1 + j0{\cdot}435$	$0{\cdot}1 + j0{\cdot}435$

Total impedance Z	$1{\cdot}18 + j1{\cdot}292$ $= 1{\cdot}75\angle 47{\cdot}5^\circ\ \Omega$	$4{\cdot}06 - j0{\cdot}049$ $4{\cdot}08\angle -1^\circ\ \Omega$	$4{\cdot}95 + j2{\cdot}905$ $5{\cdot}75\angle 30{\cdot}8^\circ\ \Omega$
Total admittance Y	$= 0{\cdot}572\angle -47{\cdot}5^\circ$ ℧	$0{\cdot}244\angle 1^\circ$ ℧	$0{\cdot}174\angle -30{\cdot}8^\circ$ ℧
Input current $I_1 = V \times Y$	$1{,}090\angle -47{\cdot}5^\circ$ A	$466\angle 1^\circ$ A	$332\angle -30{\cdot}8$ A
E.m.f. $E = V - I_1 Z$	$1{,}447 - j260$ $= 1{,}470$ V	$1{,}864 - j198$ $1{,}870$ V	$1{,}808 - j106$ $1{,}810$ V
Rotor current $I_2' = EY_2$	$1{,}470 \times 0{\cdot}68 = 1{,}000$ A	$1{,}870 \times 0{\cdot}256 = 477$ A	$1{,}810 \times 0{\cdot}153 = 276$ A
Torque (synch. kW) = rotor input	$\frac{3 \times 1{,}000^2 \times 1{\cdot}2}{1{,}000}$ $= 3{,}600$ kW	$\frac{3 \times 477^2 \times 3{\cdot}6}{1{,}000}$ $= 2{,}470$ kW	$\frac{3 \times 276^2 \times 6{\cdot}0}{1{,}000}$ $= 1{,}370$ kW
Rotor copper loss $3(I_2')^2 r_2'$	$\frac{3 \times 1{,}000^2 \times 0{\cdot}1}{1{,}000}$ $= 300$ kW	$\frac{3 \times 477^2 \times 0{\cdot}1}{1{,}000}$ $= 68{\cdot}5$ kW	$\frac{3 \times 276^2 \times 0{\cdot}1}{1{,}000}$ $= 22{\cdot}8$ kW
Reg. machine copper loss $3(I_2')^2 r_e'$	$\frac{3 \times 1{,}000^2 \times 0{\cdot}08}{1{,}000}$ $= 240$ kW	$\frac{3 \times 477^2 \times 0{\cdot}08}{1{,}000}$ $= 55$ kW	$\frac{3 \times 276^2 \times 0{\cdot}08}{1{,}000}$ $18{\cdot}2$ kW
Power to reg. machine shaft $3(I_2')^2 a' \cos \delta$	$\frac{-3 \times 1{,}000^2 \times 0{\cdot}12}{1{,}000}$ $= -360$ kW	0	$\frac{3 \times 276^2 \times 0{\cdot}12}{1{,}000}$ $= 27{\cdot}5$ kW
Mechanical output at motor shaft	$\frac{3 \times 1{,}000^2 \times 0{\cdot}06 \times 0{\cdot}95}{1{,}000 \times 0{\cdot}05}$ $= 3{,}420$ kW $= 4{,}600$ h.p.	$\frac{3 \times 477^2 \times 0{\cdot}18 \times 0{\cdot}95}{1{,}000 \times 0{\cdot}05}$ $= 2{,}340$ kW $= 3{,}140$ h.p.	$\frac{3 \times 276^2 \times 0{\cdot}3 \times 0{\cdot}95}{1{,}000 \times 0{\cdot}05}$ $= 1{,}300$ kW $= 1{,}750$ h.p.

(iii) EFFECT ON LOCUS OF VARYING THE MAGNITUDE OF THE INJECTED E.M.F. If the angle of injection is assumed to be 90° the expressions (5.2) and (5.3) become

$$P_x = \frac{955}{1{\cdot}275 + 0{\cdot}1a'/0{\cdot}18}$$

$$P_y = P_x(a'/0{\cdot}18)$$

The calculations are shown in Table 5.2 and the resulting loci in Fig. 5.10. As expected the higher values of e.m.f. give greater

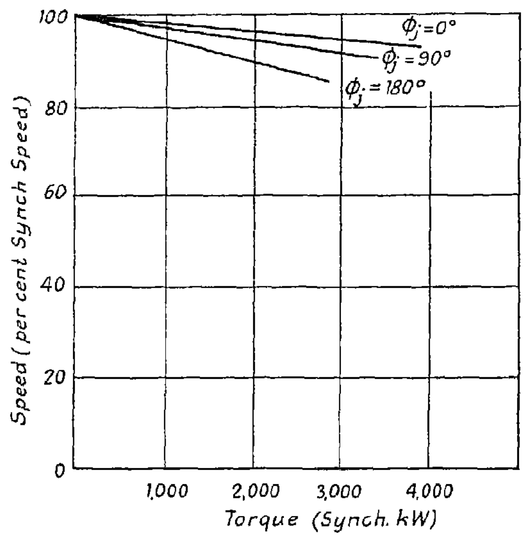


FIG. 5.11. SPEED-TORQUE CURVES

power-factor improvement as well as higher values of maximum power.

(iv) ANALYTICAL CALCULATION. Direct calculations from the equivalent circuit can be made for various values of slip as shown in Table 5.3. An injected e.m.f. corresponding to a value of $a' = 0{\cdot}12$ and values of ϕ_j equal to 0°, 90° and 180° have been chosen and calculations made for a slip of 5 per cent in each case. Torque-slip curves plotted from these and similar calculations are shown in Fig. 5.11.

EFFECT OF SATURATION AND BRUSH-RESISTANCE VARIATION. In the above calculations it has been assumed that saturation of the magnetic circuit could be neglected, i.e. that a' is constant, and that the resistances and reactances could be assumed constant. The assumption of constant reactance is reasonably accurate but, on account of the variable brush-contact resistance, the assumption of

constant resistance may lead to appreciable errors. The voltage drop between the brushes and the slip ring and commutator is generally about 1 V so that the equivalent brush-contact resistance is $2/I_2\ \Omega$ —as the voltage and winding resistances in the secondary circuit

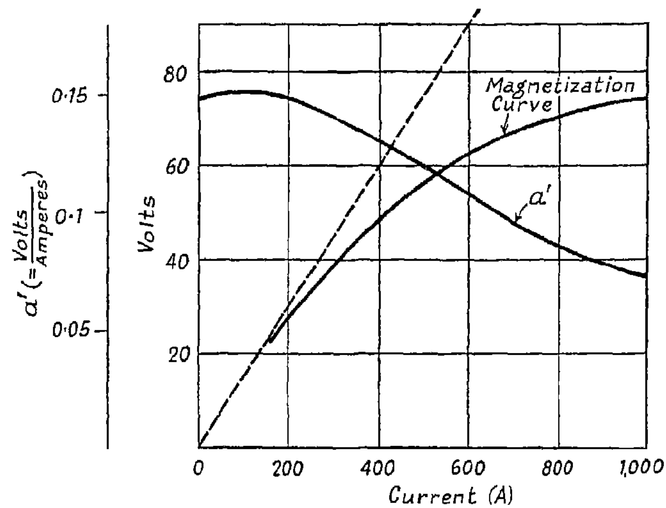


FIG. 5.12. MAGNETIZATION CURVE OF SERIES REGULATING MACHINE

are low this value is comparable with them and causes an appreciable variation in the total secondary-circuit resistance.

A typical magnetization curve for a regulating machine is shown

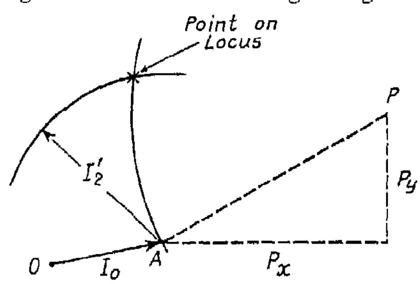


FIG. 5.13. CONSTRUCTION OF CURRENT LOCUS ALLOWING FOR SATURATION AND VARYING BRUSH RESISTANCE

in Fig. 5.12 from which it can be seen that saturation causes a decrease in the value of a' at high currents.

To construct the current loci, making allowance for the above, a step-by-step method must be used. Having drawn the no-load current as shown in Fig. 5.13, a value of secondary current (referred to the primary) within the operating range may be selected and an

arc drawn with this as radius from the no-load point as centre. For the selected value of current the brush resistance and the value of a' may be determined and the co-ordinates of the centre of the circle calculated for these values—where this circle cuts the arc already drawn gives one point on the current locus. The process may be repeated for as many secondary currents as desired.

For the particular machine, the data for which are given on page 65, the secondary and regulating-machine resistances, referred to the primary, are given by—

$$r_2' = 0{\cdot}08 + 2{\cdot}0/I_2'$$
$$r_e' = 0{\cdot}04 + 6{\cdot}2/I_2'$$

and the magnetization curve is that given in Fig. 5.12. Using these figures gives the results shown in Table 5.4, leading to the locus

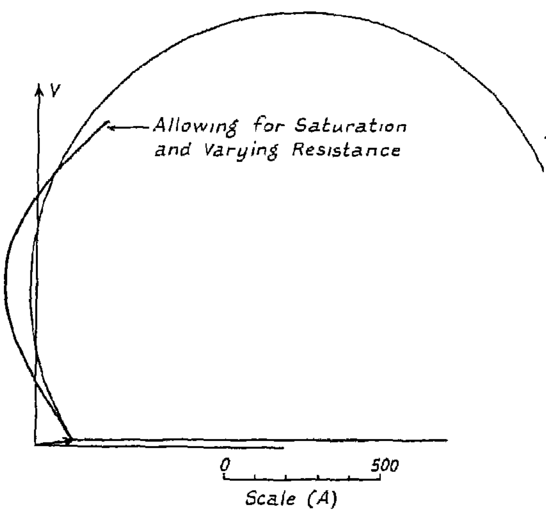


FIG. 5.14. CURRENT LOCI WITH AND WITHOUT ALLOWANCE FOR SATURATION AND VARYING RESISTANCE

shown in Fig. 5.14. It can be seen that the general effect of the saturation is to give a somewhat improved characteristic.

TABLE 5.4

I_2'	a'	r_2'	r_e'	P_x	P_y
100	0·154	0·10	0·102	705	536
200	0·148	0·09	0·071	700	640
400	0·131	0·085	0·056	700	655
600	0·117	0·083	0·050	700	620
800	0·087	0·082	0·048	740	495
1,000	0·073	0·082	0·046	775	440

Applications of Series Regulator

The series regulator is not widely used in practice chiefly because its effect, either for speed control or power-factor improvement, depends on the load of the main motor.

In the case, however, of induction motors driving rolling mills or mine winders where load equalization* is required a machine having a fairly steeply falling speed-torque characteristic is needed. A series regulator then forms a practical alternative to the use of resistors in the rotor circuit—the series regulator scheme will be more expensive in first cost but less in running cost due to the saving of the slip energy and also the improved power factor which is possible. A number of such installations have been built.

EXERCISES 5

1. A 2-pole armature with a commutator winding is provided with three brushes spaced at 120°. The equivalent-star reactance per phase at 50 c/s when the armature is stationary is 5 Ω and the resistance per phase is 0·05 Ω. Three-phase currents of 100 A at 2 c/s are supplied to the brushes. Plot curves showing the magnitude and phase angle relative to the current of the equivalent-star e.m.f. between brushes as the armature speed is varied from 240 r.p.m. against the direction of the rotating field to 320 r.p.m. with it.

2. A 4-pole armature with a commutator and three brushes per pole-pair has a standstill reactance of 4 Ω/ph. (equivalent-star value) when supplied with three-phase currents at 50 c/s. It is connected as a Leblanc exciter to an induction motor and driven at 2,000 r.p.m. If the slip frequency of the induction motor is 3 c/s determine the effective capacitance of the armature.

3. A 20-pole, 50-c/s induction motor has a full-load slip of 3·5 per cent. A 4-pole Leblanc phase advancer is to be used with it. If, at standstill, the advancer injects, with full-load current, a lagging e.m.f. of 2 V into the rotor circuit, at what speed must it be driven to inject 20 V leading, the current being the same?

4. An armature carrying a 2-pole commutator winding has diametric brushes with their axis at 90° to a steady field. The moment of inertia of the armature is 0·013 kg-m². If driven at 1,000 r.p.m. the e.m.f. at the brushes is 100 V. Find the effective capacitance of the armature and the current that would flow if 100 V at 1 c/s were applied to the brushes (the armature not being mechanically driven). Neglect resistance, inductance and friction.

5. A 150-h.p., 2·2-kV induction motor with star-connected rotor and stator has the following parameters—

$$r_1 = 0{\cdot}6\ \Omega/\text{ph.} \qquad r_2 = 0{\cdot}02\ \Omega/\text{ph.}$$
$$x_1 = 3{\cdot}0\ \Omega/\text{ph.} \qquad x_2 = 0{\cdot}26\ \Omega/\text{ph.}$$

Turns ratio, stator/rotor = 235/40.
No-load current = 15 A at 0·1 power factor.

(i) Find the rotor current and the primary current and power factor when running with a slip of 2·1 per cent.

(ii) If the motor is fitted with a series-type of slip regulator giving an injected e.m.f. of 0·015 V per ampere find, for the above rotor current, the angle (relative to the rotor current) at which the e.m.f. must be injected if the

* Taylor, *Utilization of Electric Energy*, Ch. I (English Universities Press).

rotor current is to lead the secondary e.m.f. by 40°. What will be the slip and primary power factor under these conditions?

6. A 3·3-kV, star-connected induction motor equipped with a series-type slip regulator takes a no-load current of 30 A at 0·2 power factor. The equivalent circuit may be assumed to comprise a magnetizing branch in parallel with a branch made up of a 0·3-Ω resistor, a 3-Ω inductor, a variable resistor of $0{\cdot}3/s$ Ω and a variable capacitor of $0{\cdot}5/s$ Ω. Determine the primary current and power factor and the power delivered to the regulator when running with 6 per cent slip.

7. Using the equivalent circuit of Fig. 5.6(*b*), develop an expression for the value of slip at which maximum torque occurs.

CHAPTER 6

REGULATING MACHINES GIVING CONSTANT INJECTED E.M.F. ("SHUNT" MACHINES)

WITH a constant injected e.m.f., i.e. E_j independent of load, the *torque-slip curves for the main motor are approximately as shown* in Fig. 4.8 (page 53). Thus if the e.m.f. is injected at an angle $\beta = 0°$ or $180°$ to the secondary e.m.f. the speed characteristics over the normal working range, for various values of injected e.m.f., are *as shown in Fig. 6.1; they are shunt characteristics giving a speed*

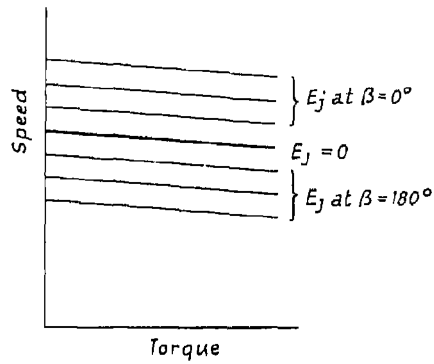


FIG. 6.1. SPEED-TORQUE CHARACTERISTICS WITH INJECTED E.M.F. INDEPENDENT OF LOAD

which remains approximately constant and independent of load for a given setting of the controls.

If the e.m.f. is injected 90° leading the secondary e.m.f. it has little effect on the speed but provides power-factor improvement and gives an approximately constant excitation for the main motor, so that the power factor of the motor tends to lead at low loads and is about unity at full load.

In practice auxiliary machines of this type are used either solely for power-factor improvement of the main motor with a negligible change of its speed, or for speed control over a range below, or above and below, synchronism—in the latter case power-factor improvement may also be included.

The Frequency Converter

The simplest type of regulating machine in this category is the frequency converter, consisting simply of an armature carrying a

commutator winding and having slip rings at the other end as shown in Fig. 4.2(*d*) (page 46). If polyphase currents at a frequency f_s are supplied to the rings a rotating field will be set up moving at a speed of $n_s = f_s/p$ relative to the conductors; if the armature is driven at a speed n_r the speed of the field will be increased or decreased to $n_f = n_s \pm n_r$ depending on the direction in which it is driven. The frequency f_b of the e.m.f.'s appearing at the brushes depends on the speed of the field relative to these fixed brushes and is therefore given by $f_b = p(n_s \pm n_r)$.

Such a machine thus acts as a frequency converter, changing from a frequency f_s at the slip rings to f_b at the brushes, the ratio between f_s and f_b depending on the speed at which it is driven. The ratio between the e.m.f.'s at the slip rings and the brushes can be found from the e.m.f. equations of Chapter 2 [eqs. (2.7*b*) and (2.11)]. Thus for a machine with N equally-spaced slip rings and N' brushes the e.m.f.'s are—

$$\text{E.m.f. between rings} = (\sqrt{2})\pi(f_f \pm f_r)T\Phi k_m \text{ volts}$$

$$\text{E.m.f. between brushes} = (\sqrt{2})(T_a/a)(f_f \pm f_r)\Phi \sin(\theta/2) \text{ volts}$$

Since $T = T_a/Na$, $k_m = \sin(\sigma/2)/(\sigma/2)$ where $\sigma = 2\pi/N$ and $\theta = 2\pi/N'$, the ratio becomes

$$(\text{slip-ring e.m.f.})/(\text{brush e.m.f.}) = \sin(\pi/N)/\sin(\pi/N')$$

and, neglecting impedance voltage drops, the ratio is constant under all conditions.

Maximum e.m.f. between brushes occurs when the axis of the rotating field is midway between the brushes, so that, by moving the brushes round the commutator, the moment at which this occurs can be varied relative to the slip-ring e.m.f.'s, i.e. movement of the brushes varies the phase angle of the brush e.m.f. relative to that of the slip rings.

The frequency converter can be made to provide e.m.f.'s of a frequency appropriate for injection into the secondary circuit of an induction motor and can therefore be used for speed or power-factor control. Two general possibilities are available giving either the constant-torque or the constant-horse-power drives mentioned previously.

CONSTANT-TORQUE DRIVE. If the frequency converter is driven from the main motor shaft as shown in Fig. 6.2 with the slip rings connected to the supply through a suitable variable-ratio transformer, the slip energy will be returned to or taken from the supply giving the constant-torque arrangement referred to on page 48. The frequency of the e.m.f.'s applied to the slip rings is the supply frequency f_1 and the speed of the converter corresponds to a frequency of $f_1(1 - s)$. If the rotation of the field is arranged to oppose that of the shaft, the speed of the field will correspond to $f_1 - f_1(1 - s) = sf_1$, the slip frequency of the main motor. E.m.f.'s

of this frequency thus appear at the brushes and can be injected into the rotor circuit of the main motor. The ratio of the slip-ring to the brush e.m.f. has already been shown to be constant, so that for a given slip-ring voltage the injected e.m.f. is constant and

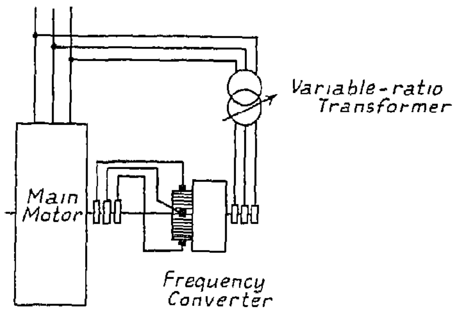


FIG. 6.2. CONSTANT-TORQUE DRIVE WITH FREQUENCY CONVERTER

independent of the load; the particular constant value can, however, be selected by control of the variable-ratio transformer. It has also been shown that the phase of the e.m.f. can be varied by brush shifting so that this arrangement enables a constant e.m.f. of any

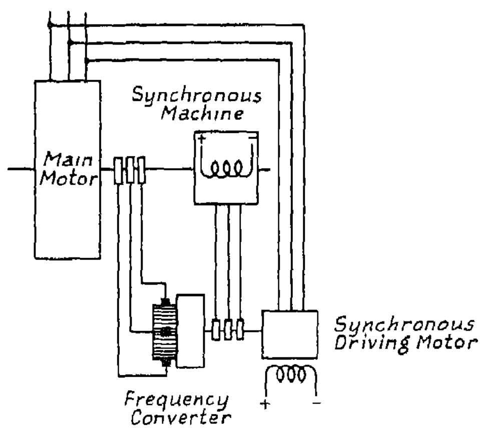


FIG. 6.3. CONSTANT-HORSE-POWER DRIVE WITH FREQUENCY CONVERTER

desired magnitude to be injected into the secondary circuit of the main motor at any desired angle.

CONSTANT-HORSE-POWER DRIVE. This alternative is shown in Fig. 6.3. The frequency converter is driven by a synchronous motor fed from the supply, and the converter slip rings are connected to another synchronous motor mounted on the main motor shaft. The

slip energy passing through the converter is thus returned to or taken from the motor shaft giving a constant-horse-power drive with a large torque at low speeds.

The synchronous machine on the shaft must have the same number of poles as the main motor and it will then generate e.m.f.'s of a frequency $f_1(1 - s)$. These e.m.f.'s are applied to the rings of the converter which is itself being driven at a speed corresponding to f_1. If it is driven in the opposite direction to the field the speed of the field will then be $f_1 - f_1(1 - s) = sf_1$ as in the previous case so that the e.m.f.'s produced at the brushes are again appropriate for injection into the main motor secondary circuit.

The magnitude of the injected e.m.f. can in this case be controlled by varying the excitation of the synchronous machine, and the phase angle at which it is injected can be controlled by shifting the brushes as before.

CONSTRUCTION OF THE FREQUENCY CONVERTER. Although a stator is not essential to the operation of the frequency converter it is desirable in order to provide a low-reluctance path for the flux as in the Leblanc exciter. Such a stator consists simply of unslotted laminations carrying no windings.

It can be seen that in both types of drive there is a machine mounted on the main motor shaft—this usually runs at a fairly low speed in the large sizes for which schemes of this nature are desired so that the physical size of the auxiliary machine is considerable. With the constant-h.p. drive the d.c. excitation required for the synchronous machine is an added complication. The frequency converter is therefore not commonly used in practice although it forms a valuable adjunct to the Scherbius scheme described on page 83.

Kramer Control (Synchronous Converter)

A special case of the frequency converter scheme employs a synchronous converter as the auxiliary machine, i.e. a frequency converter changing from slip frequency to zero frequency (direct current). The main connexions for the constant-torque and constant-h.p. arrangements are shown in Fig. 6.4. The d.c. output from the converter can be supplied to a d.c. motor on the main shaft as shown in Fig. 6.4(*a*) giving the constant-h.p. drive, or it can be supplied to a d.c. motor driving an a.c. generator which returns the slip power to the supply as in Fig. 6.4(*b*) giving the constant-torque drive. In both cases the speed of the main motor is varied by adjusting the excitation of the d.c. motor which alters its back e.m.f.—this back e.m.f. is transmitted through the synchronous converter to its slip rings and hence is injected into the secondary circuit of the main motor at slip frequency. In both cases the low-frequency slip power is applied to the rings of the converter, and to facilitate the performance of the latter six instead of three slip rings are used as with normal synchronous converter practice.*

* Say, *The Performance and Design of A.C. Machines*, Ch. XXII (Pitman).

A synchronous converter cannot, however, be designed to give satisfactory operation with less than about 10 c/s at its slip rings so that the slip frequency of the main motor must always be greater than this, i.e. the speed of the main motor must be less than about 80 per cent or greater than 120 per cent of its synchronous speed. A continuous range of speed control both above and below synchronism is thus not practicable but this system is occasionally used where speeds below synchronism are required.

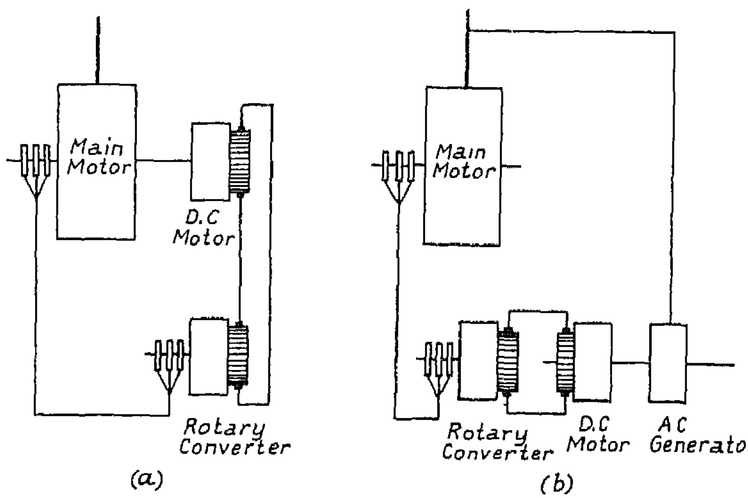


FIG. 6.4. KRAMER CONTROL SYSTEM

(*a*) Constant-h.p. drive.
(*b*) Constant-torque drive.

The power factor of the current taken by the converter can be varied in the usual way by control of the converter excitation and this, therefore, varies the power factor of the main motor.

A modification of the Kramer system which has been used for several large wind-tunnel drives employs a synchronous motor-generator set instead of a rotary converter. As the horse-power of a wind-tunnel fan falls with a decrease in speed, adding the slip power to the shaft would be inappropriate and it is returned to the supply as shown in Fig. 6.5. Control of the speed in this case is effected by adjusting the speed of auxiliary set 1 by varying the excitation of either of the d.c. machines. As with the ordinary Kramer scheme the maximum speed of the main motor should not exceed 80 to 90 per cent of its synchronous speed.

The above arrangement may seem to involve a large number of machines compared, say, to a simple Ward-Leonard scheme; for a fan drive, however, where power is proportional to (speed)^3 it can be shown that the maximum rotor power is only about 1/7th of the

full-speed power so that none of the auxiliary machines need exceed this size.

The Scherbius Machine

Instead of the series winding used in the Walker modification of the simple Leblanc exciter, shunt or separately-excited windings

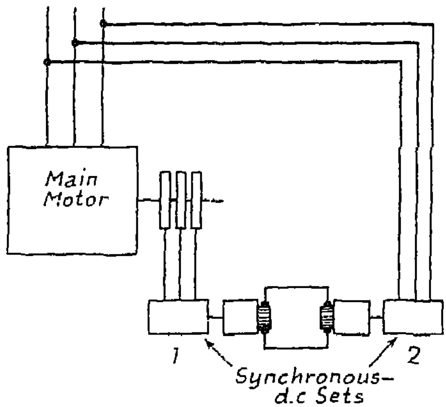


FIG. 6.5. MODIFIED KRAMER CONTROL

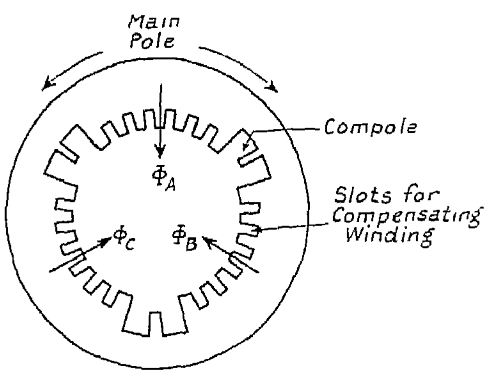


FIG. 6.6. STATOR OF SALIENT-POLE SCHERBIUS MACHINE

may be used; in such windings the exciting current is not dependent on the rotor current and auxiliary machines of this type are known as Scherbius machines.* The exciting current must, of course, be of slip frequency.

The exciting windings may be placed in slots but, to assist commutation, they are more usually wound on salient poles as shown in Fig. 6.6. Compensating windings must also be used in all cases

* Some writers refer to the Walker machine as a Scherbius machine with series windings.

to neutralize the armature m.m.f. so that the air-gap flux is solely due to the exciting winding. The armature coils have a span of approximately 120° so that both sides can be under the influence of a compole during commutation. The flux in a pole is in phase with the exciting current in the pole winding, and the e.m.f. between the brushes on each side of the pole is also in phase with the current, the equivalent-star e.m.f. being, of course, 30° displaced from this.

The flux distribution in the salient-pole arrangement is more complicated than in the slotted arrangement, but the fluxes rise and fall in sequence so that the equivalent of a rotating field is produced.

Commutation difficulties limit the frequency of the rotor currents to a maximum of about 20 c/s so that speeds down to zero cannot be obtained.

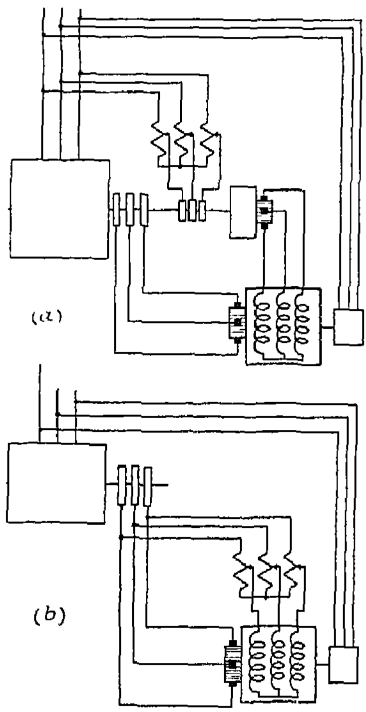


FIG. 6.7. EXCITATION OF SCHERBIUS MACHINE (CONSTANT-TORQUE DRIVES)
(a) By frequency converter.
(b) By shunt-connected winding.

METHODS OF EXCITATION. Separate excitation may be obtained from a frequency converter mounted on the main motor shaft as described in the previous section and as shown in Fig. 6.7(a). The magnitude of the e.m.f. applied to the exciting winding can be adjusted by the regulator in the slip-ring circuit of the converter but remains constant for any given setting. The brushes deliver slip-frequency currents as required, and the reactance of the exciting winding is therefore proportional to slip; the exciting current is thus dependent on slip, a typical curve being shown in Fig. 6.8(a) for an exciting winding having a 50 c/s reactance of 100 times its resistance and fed at a constant voltage. It can be seen that the current varies widely in the neighbourhood of synchronous speed; this variation can, however, be reduced to any desired amount by connecting swamping resistors in series with the exciting winding. The phase angle of the exciting current relative to the applied voltage also varies, tending to zero at synchronous speed. The phase of the applied voltage, can, of course, be adjusted by shifting the brushes of the frequency converter.

Instead of obtaining the variable voltage from the frequency converter by means of the regulator in the slip-ring circuit it can be obtained by the use of two sets of movable brushes as in the Schrage

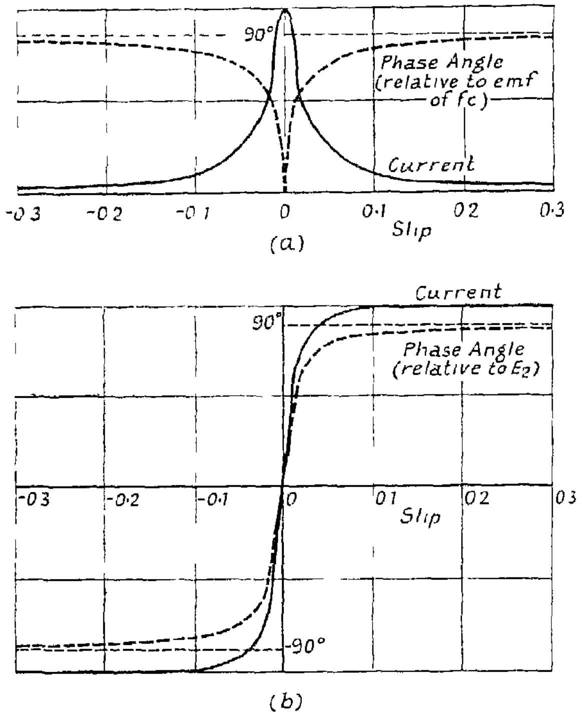


FIG. 6.8. VARIATION OF EXCITING CURRENT WITH SLIP

(a) Excitation from frequency converter
(b) Shunt excitation.

motor (page 102); any desired e.m.f. can thus be collected from the commutator and some degree of phase-angle control can also be effected.

Shunt excitation is shown in Fig. 6.7(b) from which it is seen that the exciting voltage is the slip-ring voltage, i.e. sE_2, and is therefore proportional to slip. The exciting current is therefore as shown in Fig. 6.8(b) and is seen to be almost independent of slip except in the neighbourhood of synchronous speed. This gives the desired shunt speed characteristics but leads to difficulties, discussed later, when speeds above and below synchronism are required, necessitating passing through synchronous speed. The phase angle of the current is again seen to be approximately 90° to the applied e.m.f. except near synchronous speed.

The use of the separate excitation with the frequency converter is limited to speeds within about 20 per cent of synchronism but has the advantage that the phase-angle control mentioned above enables the power factor of the main motor to be held at approximately unity over the whole speed range.

The shunt excitation is, however, more common on account of its simplicity.

It should be noted that if the machine is designed to give an injected e.m.f. having a leading component at, say, sub-synchronous speeds, then, at super-synchronous speeds the component will lag due to the reversal of the slip e.m.f. Power-factor improvement both above and below synchronous speed is thus impracticable and it is usual in such cases to inject the e.m.f. at approximately 0° or 180° to the secondary e.m.f.

To obtain particular characteristics both separately-excited and shunt excitation may be used on the same machine; series windings, as used for the Walker machine, may also be included.

SPEED VARIATION ABOVE AND BELOW SYNCHRONISM (OHMIC DROP EXCITER). If a speed range of, say, 30 per cent of normal speed is required it may be preferable to design the equipment to give a variation of 15 per cent above and 15 per cent below synchronous speed instead of 30 per cent below. The size of the auxiliary machine will then need to be only about 15 per cent of the size of the main motor instead of 30 per cent. A set of this type is referred to as a *double-range* equipment as compared to the *single-range* equipment when only speeds below synchronism are obtainable. To obtain speeds both above and below synchronism, however, involves certain complications and additional equipment.

Suppose the main motor is running below synchronous speed with the e.m.f. from the auxiliary machine injected in phase opposition to the secondary e.m.f. As this injected e.m.f. E_j is decreased the speed rises towards synchronism and the frequency decreases towards zero. When E_j is made zero (by moving the tapping switch to the neutral point of the auto-transformer), the slip is governed by $sE_2 = I_2 z_{2s}$ where z_{2s} is the secondary circuit impedance, which, at the low frequency now obtaining, is almost entirely resistive.

Any further increase in speed requires an e.m.f. E_j in the opposite direction and, to produce this, the direction of the current in the exciting winding will have to be reversed. Owing to the ohmic drop $I_2 z_e$ in the regulating machine, which is approximately in phase with I_2, the e.m.f. impressed across the auto-transformer cannot be reduced quite to zero so that the field current cannot be reversed—no further decrease in slip can therefore take place unless some additional apparatus is employed.

The device generally used consists of a small frequency converter mounted on the same shaft as the main motor as shown in Fig. 6.9. The brushes are connected to the exciting circuit as shown so that some exciting current will be produced even though the tapping

switch on the auto-transformer is at the neutral point. The phase of the e.m.f. from the frequency converter is adjusted to be in phase with the secondary e.m.f. E_2, and therefore approximately with I_2z_{2s}. It is thus opposed to E_j when this is in such a direction as to give speeds below synchronism. The exciting current due to the

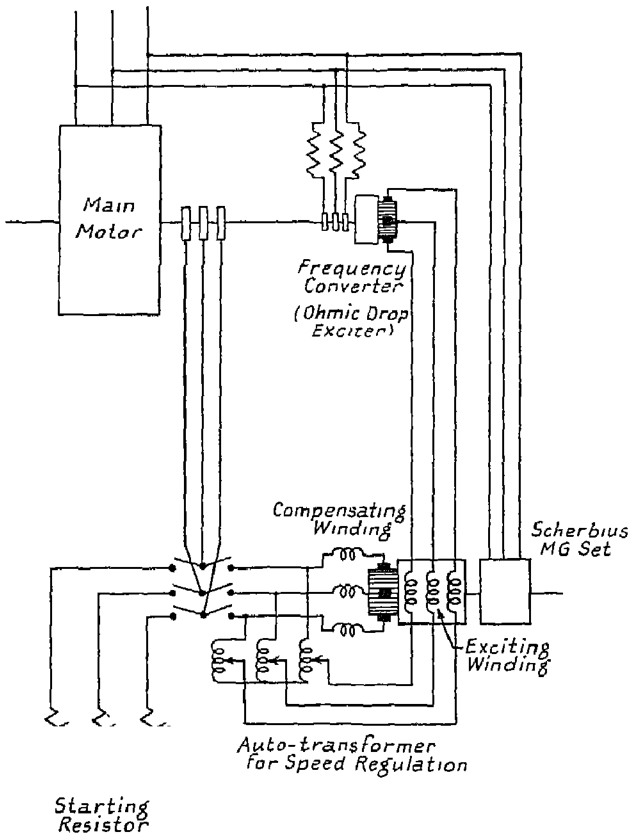


FIG. 6.9. INDUCTION MOTOR WITH DOUBLE-RANGE SCHERBIUS CONTROL

e.m.f. from the frequency converter may be considered to set up an e.m.f. E_{jf} in the secondary circuit of the main motor so that the total injected e.m.f. will thus be $E_j - E_{jf}$. This will thus be zero before the tapping switch on the auto-transformer reaches the neutral point. Further movement of the switch beyond this zero point will make the total injected e.m.f. negative and enable it to overcome the ohmic drop I_2z_{2s} so that synchronous speed can be passed; sE_2, and therefore the field current, is now reversed so that increasing the tapped voltage from the auto-transformer will now

raise the speed. At the moment of passing through synchronous speed the frequency converter acts as a synchronous converter and direct current is passed through the field windings. As the main function of the frequency converter is to overcome the ohmic drop of the secondary circuit it is commonly called an *ohmic drop exciter*.

COMMUTATION. If the Scherbius type of machine is used to give speed control to the main motor over a fairly wide speed range, the frequency of the rotor currents may be one-third or more of the supply frequency. Considerable e.m.f.'s are thus induced in the coils short-circuited by the brushes when undergoing commutation, and it is necessary to fit compoles to prevent sparking. These compoles must produce an appropriate local flux in the air gap opposite the coil-sides being commutated, and the smooth rotating field that would otherwise obtain is destroyed.

It can be seen from Fig. 6.6 that a coil undergoing commutation is linked with the whole flux of one pole and therefore has induced in it a transformer e.m.f. of

$$E_{tb} = (\sqrt{2})\pi f T_c \Phi \text{ volts} \qquad . \quad . \qquad (6.1)$$

and this e.m.f. lags the flux by 90°.

A rotational e.m.f. to counteract this must be produced by the rotation of the conductors in the compole flux. The maximum e.m.f. induced in one side of a coil undergoing commutation is

$$e_c = B_c L v T_c \text{ volts}$$

The compoles will naturally be excited by currents 120° apart, so that the e.m.f. induced in the other side of the coil will be displaced from it by 120° and the total coil e.m.f. will be the difference between these two. The r.m.s. e.m.f. per coil will thus be

$$E_c = (\sqrt{3}/\sqrt{2}) B_c L v T_c \text{ volts}$$

If neutralization is to be complete this must equal the transformer e.m.f. of eq. (6.1) in magnitude and phase so that, considering the magnitude,

$$(\sqrt{2})\pi f T_c \Phi = (\sqrt{3}/\sqrt{2}) B_c L v T_c$$

$$\therefore \qquad B_c = \{2\pi/(\sqrt{3})Lv\} f \Phi$$

It has already been seen that the flux Φ is approximately constant for a given speed setting so that a compole flux proportional to slip frequency is required; this is not easy to obtain and, in practice, a constant flux must be used which will give proper neutralization at one speed only. Such a flux can, however, conveniently be obtained by exciting the compoles in parallel with the main poles so that B_c will vary as Φ is changed for different speed settings.

The phase of the compole e.m.f. must be opposite to that of the

transformer e.m.f. For the coil linked with pole A this is as shown in the diagram of Fig. 6.10. The e.m.f.'s induced in each coil-side by the compole flux are in phase with those fluxes and these are in phase with the corresponding main fluxes since the compole windings are in parallel with the main windings. It can thus be seen that the compoles on each side of pole A must have fluxes in phase with

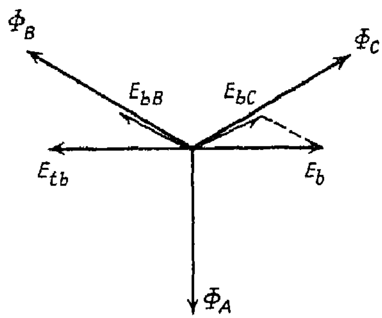


FIG. 6.10. E.M.F.'S INDUCED IN SHORT-CIRCUITED COILS

Φ_B and Φ_C respectively, i.e. to produce the correct phase relation *each compole must be excited by a current in phase with that of* the main pole opposite to it.

The above exciting winding on the compole will produce a flux that will neutralize the transformer e.m.f. but will have no effect on the reactance e.m.f. It has been shown on page 31 that this e.m.f. is proportional to the current entering a brush, so that to provide a neutralizing e.m.f. for it a compole flux proportional to the current is required. This cannot, however, be produced simply by adding series turns to the compole since the compole flux must be sufficient to set up in the shunt winding a back e.m.f. equal to the applied e.m.f., and any series turns would simply cause an additional current to flow in the shunt winding giving an m.m.f. to neutralize the series-turn m.m.f. To make the series turns effective they would have to be wound on a separate pole, i.e. the compole would have to be divided axially into two parts each with a separate winding; this complication is not usually warranted and only compensation for the transformer e.m.f. is provided.

Analytical Treatment of Motor and Regulating Machine Giving Constant Injected E.M.F.

If the injected e.m.f. is constant it may be written

$$\mathbf{E}_j = \mathbf{b}\mathbf{E}_2$$

or, referred to primary,

$$\begin{aligned}\mathbf{E}_j' &= \mathbf{b}\mathbf{E}_1 \\ &= \mathbf{E}_1(b \cos \beta + \mathrm{j}b \sin \beta)\end{aligned}$$

The general equations of page 55 thus become, all quantities being referred to the primary—

$$\left.\begin{aligned} \mathbf{V} &= \mathbf{I}_1\mathbf{z}_1 + \mathbf{E}_1 & (a)\\ \mathbf{I}_0\mathbf{z}_0 &= \mathbf{E}_1 & (b)\\ \mathbf{I}_1 &= \mathbf{I}_0 + \mathbf{I}_2' & (c)\\ \mathbf{I}_2'\mathbf{z}_{2s}' &= s\mathbf{E}_1 + \mathbf{b}\mathbf{E}_1 = \mathbf{E}_1(s+\mathbf{b}) & (d) \end{aligned}\right\} \quad . \quad (6.2)$$

From (d)

$$\mathbf{I}_2' = \mathbf{E}_1(s+\mathbf{b})/\mathbf{z}_{2s}'$$

From (c) and (b)

$$\mathbf{I}_1 = \mathbf{E}_1/\mathbf{z}_0 + \mathbf{E}_1(s+\mathbf{b})/\mathbf{z}_{2s}'$$

Therefore

$$\mathbf{E}_1 = \mathbf{I}_1\frac{1}{1/\mathbf{z}_0 + (s+\mathbf{b})/\mathbf{z}_{2s}'}$$

Substituting in (a)

$$\mathbf{V} = \mathbf{I}_1\left\{\mathbf{z}_1 + \frac{1}{1/\mathbf{z}_0 + (s+\mathbf{b})/\mathbf{z}_{2s}'}\right\}$$

The term in the bracket thus represents the impedance of the motor as viewed from the stator terminals. Substituting for the complex quantities* gives—

$$\mathbf{V} = \mathbf{I}_1\left\{r_1 + \mathrm{j}x_{L1} + \frac{1}{\dfrac{1}{r_0+\mathrm{j}x_0} + \dfrac{s+b\cos\beta}{r_2'+r_e'+\mathrm{j}(sx_{L2}'+x_{Le}')} + \mathrm{j}\,\dfrac{b\sin\beta}{r_2'+r_e'+\mathrm{j}(sx_{L2}'+x_{Le}')}}\right\}$$

$$= \mathbf{I}_1\left\{r_1 + \mathrm{j}x_{L1} + \frac{1}{\dfrac{1}{r_0+\mathrm{j}x_0} + \dfrac{1}{\dfrac{r_2'+r_e'}{s+b\cos\beta} + \mathrm{j}\,\dfrac{sx_{L2}'+x_{Le}'}{s+b\cos\beta}} + \dfrac{1}{\dfrac{sx_{L2}'+x_{Le}'}{b\sin\beta} - \mathrm{j}\,\dfrac{r_2'+r_e'}{b\sin\beta}}}\right\}$$

* If the regulating machine is a frequency converter the terms r_e' and x_{Le}' include the resistance and reactance of the regulating transformer, the latter being the supply-frequency value and independent of slip.

The quantity in the brackets thus represents an impedance $r_1 + jx_{L1}$ in series with three circuits in parallel, viz. $r_0 + jx_0$,

$$(r_2' + r_e')/(s + b\cos\beta) + j(sx_{L2}' + x_{Le}')/(s + b\cos\beta)$$

and $$(sx_{L2}' + x_{Le}')/(b\sin\beta) - j(r_2' + r_e')/(b\sin\beta)$$

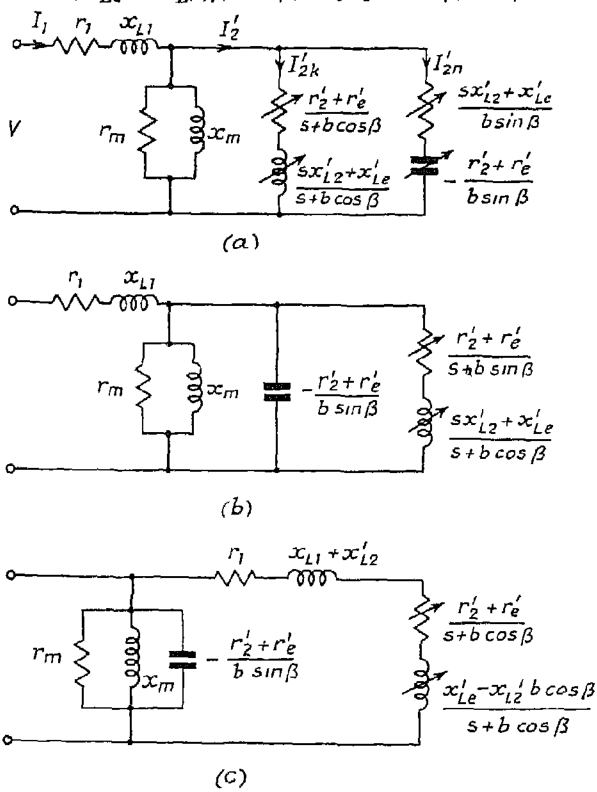


FIG. 6.11. EQUIVALENT CIRCUITS FOR INDUCTION MOTOR WITH "SHUNT" REGULATING MACHINE

(a) Complete equivalent circuit.
(b) Equivalent circuit assuming $(sx_{L2}' + x_{Le}')$ small relative to $(r_2' + r_e')$
(c) Approximate equivalent circuit.

The equivalent circuit is thus as shown in Fig. 6.11(*a*); the magnetizing impedance of r_0 and x_0 in series has, for convenience, been replaced by r_m and x_m in parallel and the reactive term

$$-j(r_2' + r_e')/(b\sin\beta)$$

has been represented by a capacitor since it is negative for all normal values of β.

Approximate Equivalent Circuits. If, as may be the case, the resistive term in the capacitive branch can be neglected relative to the capacitive term, the circuit can be simplified by considering this branch as part of the magnetizing branch as shown in Fig. 6.11(*b*).

A still further simplification can be made by shifting the magnetizing branch to the supply terminals, as is common with the induction motor, and also by rewriting the impedance

$$(sx_{L2}' + x_{Le}')/(s + b \cos \beta)$$

as

$$x_{L2}' + (x_{Le}' - x_{L2}' b \cos \beta)/(s + b \cos \beta)$$

The equivalent circuit thus becomes as shown in Fig. 6.11(*c*).

The right-hand portion of this circuit contains a variable resistance in series with a variable reactance, the ratio between the two being constant. As shown in Appendix 2 the current locus for such a circuit is a circle and, with the above values, the co-ordinates of the centre are given by—

$$P_x = \frac{V}{2(x_{L1} + x_{L2}')\left(1 - \dfrac{x_{Le}' - x_{L2}'b \cos \beta}{r_2' + r_e'} \cdot \dfrac{r_1}{x_{L1} + x_{L2}'}\right)}$$

$$P_y = - P_x \,.\, (x_{Le}' - x_{L2}'b \cos \beta)/(r_2' + r_e')$$

The second part of the denominator in the expression for P_x is usually almost equal to unity so that $P_x \simeq V/\{2(x_{L1} + x_{L2}')\}$.

It may be noted from the complete equivalent circuit that, if the e.m.f. is injected at 0° or 180°, i.e. for speed control only without power-factor improvement, the impedance of the right-hand branch becomes infinite since $\sin \beta = 0$. If injection is at 90° for power-factor improvement only, there will be current in both branches.

The total rotor input, which is proportional to torque, is given by the I^2r loss due to the currents I_{2k}' and I_{2n}' in the two branches of the equivalent circuit—

$$(I_{2k}')^2 \frac{r_2' + r_e'}{s + b \cos \beta} + (I_{2n}')^2 \frac{sx_{L2}' + x_{Le}'}{s + b \cos \beta} \text{ watts/phase}$$

Rotor copper loss $= (I_2')^2 r_2$

Regulating-machine copper loss $= (I_2')^2 r_e$

Mechanical output = rotor input $(1 - s)$

Power to or from regulating-machine shaft
= rotor input — copper losses — mechanical output.

Iron, friction and windage losses in the regulating machine are neglected.

Predetermination of Behaviour

As with the series type of regulating machine the behaviour can be predicted either by drawing the locus diagram or analytically.

Example 6.1. The following data refer to an induction motor equipped with a shunt regulating machine (the same motor as in Example 5.1). Using the equivalent circuit of the previous section predetermine the behaviour of the machine.

Rating: 3,000 h.p. (synch. speed), 3·3 kV, 3-ph., 50 c/s, star-connected.

Primary resistance	$r_1 = 0{\cdot}1\ \Omega$/ph.
Primary leakage reactance	$x_{L1} = 0{\cdot}435\ \Omega$/ph.
Secondary resistance (referred to primary)	$r_2' = 0{\cdot}1\ \Omega$/ph.
Secondary leakage reactance (referred to primary)	$x_{L2}' = 0{\cdot}44\ \Omega$/ph.
Magnetizing equiv. resistance	$r_m = 75\ \Omega$/ph.
Magnetizing equiv. reactance	$x_m = 16\ \Omega$/ph.
Regulating machine resistance	$r_e' = 0{\cdot}18\ \Omega$/ph.
Regulating machine reactance	$x_{Le}' = 0{\cdot}1\ \Omega$/ph.

(i) REGULATING MACHINE USED FOR PHASE ADVANCING WITH NO SPEED CONTROL. The e.m.f. required will be quite small and may be taken as 1/10 of the standstill secondary e.m.f. ($b' = 0{\cdot}1$).

The capacitive term in the right-hand branch is $0{\cdot}28/(s + b\cos\beta)$ and the resistive term will be $(s \times 0{\cdot}44 + 0{\cdot}1)/(s + b\cos\beta)$. If s is small, as will be the case for power-factor improvement only, the resistive term will be less than half the capacitive term so that the approximate circuit of Fig. 6.11(c) may be used.

The co-ordinates of the centre of the circle forming the current locus are—

$$P_x = \frac{1{,}910}{2(0{\cdot}435 + 0{\cdot}44)\left(1 - \dfrac{0{\cdot}1 - 0{\cdot}44 \times b\cos\beta}{0{\cdot}28}\cdot\dfrac{0{\cdot}1}{0{\cdot}435 + 0{\cdot}44}\right)}$$

$$= \frac{955}{0{\cdot}875\{1 - (0{\cdot}1 - 0{\cdot}44\,b\cos\beta)0{\cdot}407\}}$$

$$P_y = -P_x \times (0{\cdot}1 - 0{\cdot}44\,b\cos\beta)/0{\cdot}28$$

With $b = 0{\cdot}1$ and for various values of β the centres are obtained as in Table 6.1.

TABLE 6.1

β	$\cos\beta$	$0{\cdot}44\,b\cos\beta$	P_x	P_y
0°	1	0·044	1,120	− 224
45°	0·707	0·031	1,125	− 276
90°	0	0	1,138	− 405
135°	− 0·707	− 0·031	1,152	− 540
180°	− 1	− 0·044	1,156	− 595

The no-load current is approximately equal to the current in the shunt branch of the equivalent circuit, i.e. its power component is 1,910/75 = 25·5 A and the reactive component is

$$I_{xm} = \frac{1{,}910}{16} - \frac{1{,}910}{0{\cdot}28/(0{\cdot}1 \sin\beta)}$$
$$= 120 - (1{,}910 \times 0{\cdot}1 \sin\beta)/0{\cdot}28$$

and is therefore as shown in Table 6.2.

TABLE 6.2

β	$0{\cdot}1 \sin\beta$	$\frac{1{,}910 \times 0{\cdot}1 \sin\beta}{0{\cdot}28}$	I_{xm}
0°	0	0	120 (lag)
45°	0·0707	483	− 363 (lead)
90°	0·1	682	− 562 (lead)
135°	0·0707	483	− 363 (lead)
180°	0	0	120 (lag)

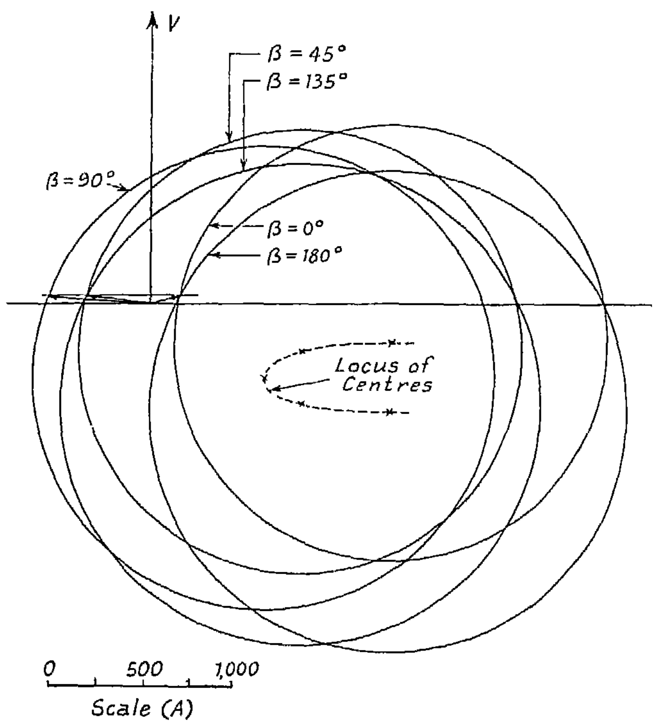


FIG. 6.12. CURRENT LOCI WITH VARIOUS ANGLES OF INJECTION

From the position of the centres and the no-load points the circles for the current loci can be drawn as shown in Fig. 6.12.

It can be seen that the general effect is to move the circles towards the left as β approaches $90°$.

TABLE 6.3

β	$0°$	$180°$
Cos β	1·0	− 1·0
Slip	− 0·48	0·52
Variable resistance R	$\dfrac{0{\cdot}28}{-0{\cdot}48+0{\cdot}5} = 14\,\Omega$	$\dfrac{0{\cdot}28}{0{\cdot}52-0{\cdot}5} = 14\,\Omega$
Variable reactance	$\dfrac{-0{\cdot}48\times0{\cdot}44+0{\cdot}1}{0{\cdot}02} = -5{\cdot}5\,\Omega$	$\dfrac{0{\cdot}52\times0{\cdot}44+0{\cdot}1}{0{\cdot}02} = 16{\cdot}4\,\Omega$
Impedance of variable branch	$14 - j5{\cdot}5 = 15\underline{/-21{\cdot}5°}$	$14 + j16{\cdot}4\,\Omega = 21{\cdot}6\underline{/49{\cdot}5°}$
Admittance of variable branch Y_2	$0{\cdot}0667\underline{/21{\cdot}5°} = 0{\cdot}062 + j0{\cdot}0244$	$0{\cdot}0462\underline{/-49{\cdot}5°} = 0{\cdot}030 - j0{\cdot}0352$
Admittance of magnetizing branch	$0{\cdot}013 - j0{\cdot}062$	$0{\cdot}013 - j0{\cdot}062$
Admittance of magnetizing and rotor branches	$0{\cdot}075 - j0{\cdot}0376 = 0{\cdot}084\underline{/-26{\cdot}5°}$	$0{\cdot}043 - j0{\cdot}0972 = 0{\cdot}106\underline{/-66{\cdot}1°}$
Impedance of magnetizing and rotor branches	$11{\cdot}9\underline{/26{\cdot}5°} = 10{\cdot}6 + j5{\cdot}3$	$9{\cdot}44\underline{/66{\cdot}1°} = 3{\cdot}8 + j8{\cdot}6$
Stator impedance	$0{\cdot}1 + j0{\cdot}435$	$0{\cdot}1 + j0{\cdot}435$
Total impedance	$10{\cdot}7 + j5{\cdot}73 = 12{\cdot}1\underline{/28{\cdot}1°}$	$3{\cdot}9 + j9{\cdot}035 = 9{\cdot}8\underline{/66{\cdot}5°}$
Total admittance	$0{\cdot}0825\underline{/-28{\cdot}1°}$	$0{\cdot}102\underline{/-66{\cdot}5°}$
Primary current I_1	$158\underline{/-28{\cdot}1°}$ A	$195\underline{/-66{\cdot}5°}$ A
$E = 1{,}910 - I_1z_1$	$1{,}910 - 158\underline{/-28{\cdot}1°} \times 0{\cdot}45\underline{/77°}$ $= 1{,}910 - 71\underline{/48{\cdot}9°}$ $= 1{,}864 - j53{\cdot}5$ $= 1{,}870$ V	$1{,}910 - 195\underline{/-66{\cdot}5°} \times 0{\cdot}45\underline{/77°}$ $= 1{,}910 - 87{\cdot}6\underline{/10{\cdot}5°}$ $= 1{,}824 - j16$ $= 1{,}824$ V
$I_2' = EY_2$	155 A	187 A
Rotor input $3(I_2')^2R$	$\dfrac{3\times(155)^2\times14}{1{,}000} = 1{,}000$ kW	$\dfrac{3\times(187)^2\times14}{1{,}000} = 1{,}460$ kW
Rotor copper loss	$\dfrac{3\times(155)^2\times0{\cdot}1}{1{,}000} = 7{\cdot}2$ kW	$\dfrac{3\times(187)^2\times0{\cdot}1}{1{,}000} = 10{\cdot}5$ kW
Regulating machine copper loss	$\dfrac{3\times(155)^2\times0{\cdot}18}{1{,}000} = 13$ kW	$\dfrac{3\times(187)^2\times0{\cdot}18}{1{,}000} = 19{\cdot}0$ kW
Mechanical output	$1{,}000 \times 1{\cdot}48$ = 1,480 kW = 1,980 h.p.	$1{,}460 \times 0{\cdot}48$ = 700 kW = 940 h.p.
Power to regulating machine	− 500 kW	730 kW

(ii) Speed Control Only. Suppose $b = 0{\cdot}5$ and the e.m.f. is injected at 0° and at 180°. As $\sin \beta = 0$ the right-hand branch has an infinite impedance and carries no current. As $\cos \beta = 1$ for $\beta = 0°$ and -1 for $\beta = 180°$ the denominator of the left-hand rotor branch becomes $s \pm b$.

At no torque (no rotor current) the denominator $s \pm b$ must be zero so that $s = \pm b$, i.e. $s \pm 0{\cdot}5$. The no-load speed is thus 50 per cent or 150 per cent of synchronous speed.

There will be a drop in speed as the load is added of, say, 2 per cent, so that typical calculations can be made with $s = 0{\cdot}52$ and $-0{\cdot}48$. The results are given in Table 6.3.

Applications of "Shunt" Machines

Where speed control is required for drives requiring above about 500 h.p., an induction motor fitted with a shunt, usually a Scherbius, type of regulator is the only practicable solution and has the merit of giving a continuously variable speed over the available range.

The frequency converter is not generally used as it must be directly coupled to the induction motor and is therefore large and expensive for its output. The Kramer and the modified Kramer systems are limited to speeds not exceeding about 80 per cent of synchronous speed but are employed in a number of cases for fan drives in wind-tunnel installations where control down to very low speeds is required.

The most common scheme is the Scherbius single- or double-range regulator and this is used for rolling-mill motors, mine-winding equipment, large fan drives, etc. Several sets have been built in Europe and America as part of motor-generator sets for coupling power networks of different frequencies. Scherbius machines for power-factor improvement without speed control are also used in a number of installations.

For drives requiring less than about 500 h.p. self-contained commutator motors as described in Part III are generally more economical.

EXERCISES 6

1. Find the ratios of voltage and frequency at the commutator and slip rings of a 4-pole frequency converter, the rings being connected to a 50-c/s supply; the converter has 6 slip rings and 3 brushes per pole-pair and is driven, in the opposite direction to the rotating field, at (*a*) 1,000 r.p.m. and (*b*) 1,500 r.p.m.

2. A star-connected 3·3-kV, 700-h.p. induction motor has the following particulars—

Stator: resistance = $0{\cdot}5\ \Omega$/ph.; reactance = $4{\cdot}7\ \Omega$/ph.
Rotor: resistance = $0{\cdot}006\ \Omega$/ph.; reactance = $0{\cdot}06\ \Omega$/ph.
Turns ratio = 9:1.
No-load current = 25 A at 0·15 p.f.

(*a*) Find the primary current, power factor and h.p. when running at 3 per cent slip.

(*b*) The motor is equipped with a series slip regulator having a resistance of $0{\cdot}005\ \Omega$/ph., a reactance of $0{\cdot}03\ \Omega$/ph. and giving an injected e.m.f. of $0{\cdot}01$ V/ampere $45°$ ahead of the rotor current.

Find the primary current, power factor, h.p. output and power delivered from the regulator when running at 3 per cent slip.

(*c*) The motor is equipped with a shunt phase advancer having a resistance of $0{\cdot}003\ \Omega$/ph., a reactance of $0{\cdot}007\ \Omega$/ph. and giving a constant injected e.m.f. of $6{\cdot}4$ V.

Find the primary current and power factor and the h.p. output when running at 3 per cent slip.

3. The open-circuit voltage between the slip rings of a 3-ph., 500-h.p., 20-pole, 50-c/s induction motor is 800 V when the motor is stationary and the stator is excited at normal voltage. The motor is equipped with a Scherbius machine for speed control. If the normal slip at full load is 1 per cent, calculate the approximate voltage and current of the Scherbius machine when the induction motor is running at speeds of 225 and 350 r.p.m. and is developing full-load torque. Assume that the reactance per phase at 50-c/s of the star-connected rotor winding of the induction motor is 20 times the resistance per phase and that the Scherbius machine operates at unity power factor.

PART III: THREE-PHASE COMMUTATOR MOTORS

CHAPTER 7

TYPES OF MOTOR

WITH the exception of the three-phase series motor, all the types of three-phase commutator motor that have been developed are modifications of the ordinary induction motor; the necessary commutator for the production of slip-frequency e.m.f.'s is incorporated in the motor itself instead of forming part of a separate machine as described in the previous chapter. Self-contained motors of this type may be built to give speed and power-factor control or only power-factor control, the latter being commonly referred to as *compensated induction motors*.

In most cases commutation difficulties limit the size for which these motors can be built to about 500 h.p. Up to this size, however, the self-contained motor is generally superior in behaviour to and lower in cost than the ordinary induction motor fitted with an auxiliary machine to give similar characteristics.

A considerable number of different types of motor have been devised, and the more important are briefly mentioned in this chapter. Only a few have, however, been commercially successful, these being discussed more fully in the succeeding chapters.

The Heyland Motor

The earliest of the compensated motors was developed by Heyland in 1901. The stator is similar to that of an ordinary induction motor, and the rotor carries a commutator winding with three brushes per pole-pair as shown, for a two-pole machine, in Fig. 7.1(*a*). The rotating field in the air gap moves past the brushes at supply frequency, so that the commutator converts from supply frequency at the brushes to slip frequency in the rotor conductors. An e.m.f. for injection into the secondary circuit can thus be obtained from tappings on the primary as indicated, and the phase at which this e.m.f. is injected can be varied by shifting the brushes.

Modifications of this motor are—

(i) A machine in which an auxiliary winding is used instead of tappings on the stator winding for providing the injected e.m.f., the arrangement being as shown in Fig. 7.1(*b*).

(ii) A machine in which the rotor carries an ordinary cage or slip-ring winding and a separate commutator winding, as shown in Fig. 7.1(*c*), the brushes being connected to tappings on the stator winding or to a separate winding as before. In this case the

commutator winding has to carry only the magnetizing current and not the whole secondary current.

With each of these motors the magnitude of the injected e.m.f. is fixed although its phase is variable by brush shifting. They are thus suitable for giving an improved power factor but not for speed control.

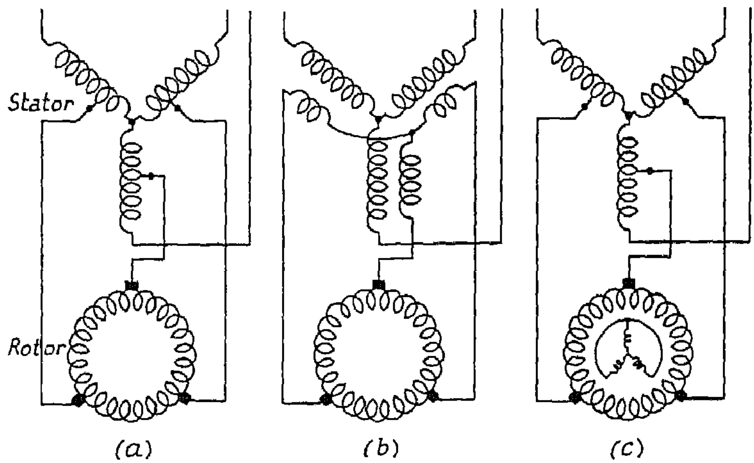


FIG. 7.1. FORMS OF HEYLAND MOTOR

(a) Tappings on stator winding.
(b) Separate stator winding.
(c) Separate commutator winding.

Eichberg Motor

This may be regarded as a further modification of the Heyland motor in which speed as well as power-factor control is obtainable. Instead of a single tapping on each phase of the stator winding there are a number of tappings connected to a tapping switch, as shown in Fig. 7.2(*a*), so that various values of injected e.m.f. can be obtained. If the brushes are adjusted so that these e.m.f.'s are injected at about 0° or 180° to the secondary e.m.f. a corresponding number of no-load speeds are obtained. Since the e.m.f. injected will be constant the speed-torque characteristics will be as shown in Fig. 7.3(*a*).

The injected e.m.f. can be varied from its maximum value down to zero by moving the tapping switch but it cannot be reversed, so that only a range of speeds *either* above *or* below synchronism can be obtained.

An alternative arrangement employs a separate tapped winding as shown in Fig. 7.2(*b*). By connecting the mid-points of these windings in star, variation of speed both above and below synchronism can be obtained.

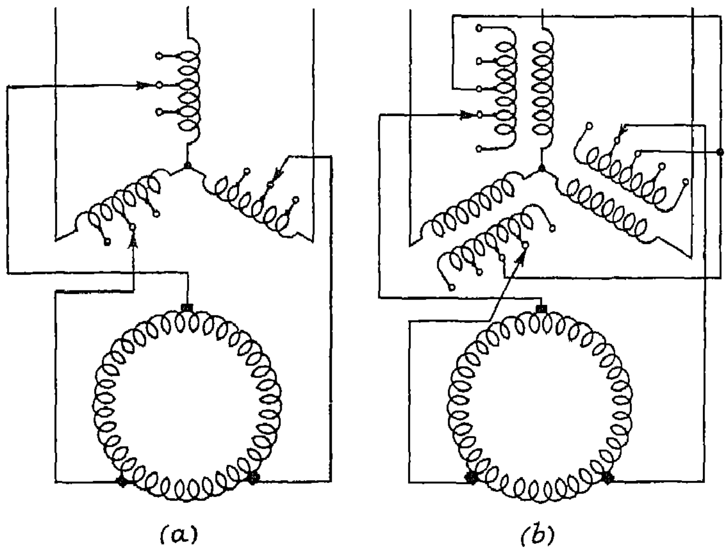


FIG. 7.2. EICHBERG MOTORS

(a) Tapped stator winding.
(b) Tapped auxiliary winding.

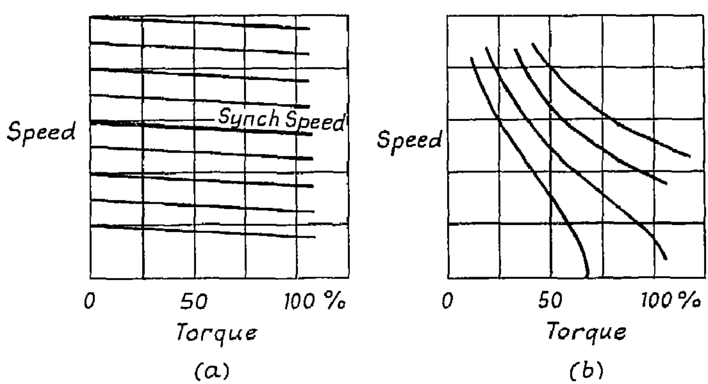


FIG. 7.3. SPEED-TORQUE CURVES

(a) Eichberg, doubly-fed, Schrage motors
(b) Series motor.

Doubly-fed Motor

Instead of obtaining the injected e.m.f. from tappings on the stator winding as in the Eichberg motor, a separate variable-ratio transformer can be used as shown in Fig. 7.4. By connecting the mid-points, instead of the ends, of the windings in star, reversal of the injected e.m.f., and therefore speeds above and below synchronism, can be obtained.

As an alternative to the tapped transformer an induction regulator can be used and has the advantage of giving a smooth, instead of a stepped, speed variation. The induction regulator, however, on account of its air gap, takes a greater magnetizing current than the transformer.

The variable-ratio transformer has to deal with the slip power, and, as shown on page 47, for a wide range of speed this will be nearly as great as the motor power. The transformer or regulator may

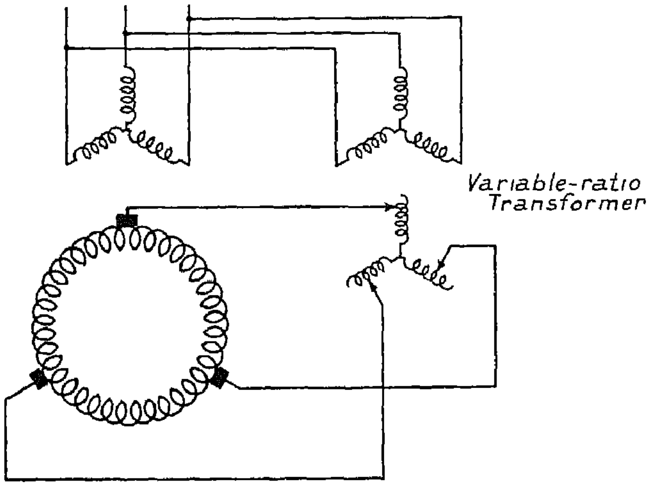


FIG. 7.4. DOUBLY-FED MOTOR

therefore be as large as the motor and the unit is no longer self-contained; the auxiliary item is, however, a static piece of apparatus and this type of motor, in practice, has proved to be commercially successful. It is discussed more fully in the next chapter.

The Osnos (No-lag) Motor

This motor, first proposed in 1902, is designed to operate at unity or leading power factor and may therefore be classed as a compensated induction motor. Its construction differs from the ordinary induction motor in that the primary winding is on the rotor, fed through slip rings from the supply, and the secondary winding is on the stator.

In its simplest form the rotor (primary) winding is also connected to a commutator from which three brushes per pole-pair collect an e.m.f. and inject it into the secondary (stator) winding as shown in Fig. 7.5(*a*). With a supply voltage of 400 such an arrangement would give too high a voltage at the commutator, and it is therefore more convenient to have the primary and commutator (tertiary) windings electrically separate as shown in Fig. 7.5(*b*). The two windings are in the same slots, the commutator winding usually being uppermost.

The primary-winding currents set up a rotating field moving at synchronous speed relative to the rotor conductors. The rotor itself runs in the opposite direction to this field due to ordinary induction motor action so that the speed of the field relative to the fixed brushes is slip speed. E.m.f.'s of slip frequency thus appear at the

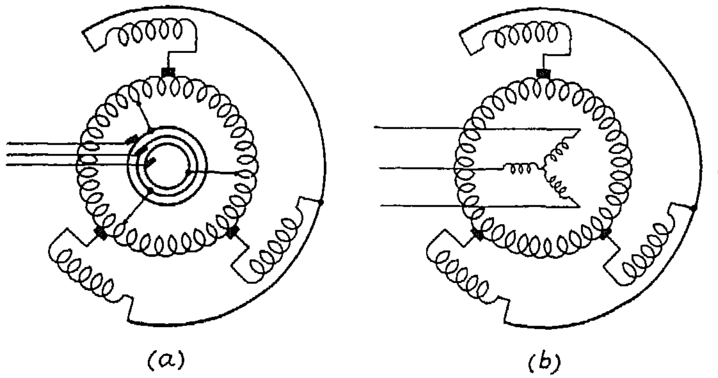


FIG. 7.5. OSNOS MOTOR

(a) Single rotor winding.
(b) Separate primary and commutator windings.

brushes and can be injected into the secondary circuit. The magnitude of these e.m.f.'s, for a given air-gap flux, is constant but their phase angle may be varied by shifting the brushes. As normally

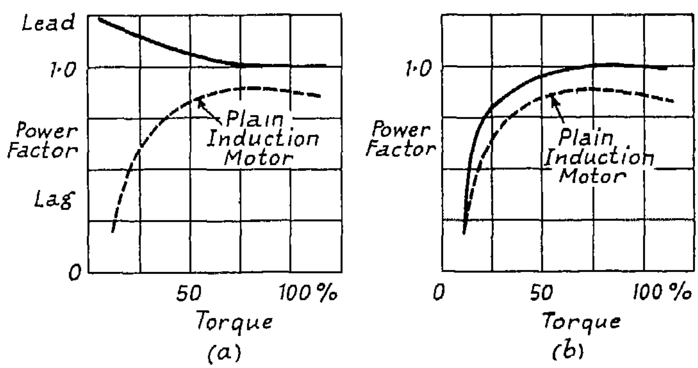


FIG. 7.6. POWER-FACTOR CURVES

(a) Osnos, Fynn-Weichsel motors.
(b) Torda motor.

built the brushes are so placed as to inject an e.m.f. at an angle β of approximately 90° to give power-factor improvement and the speed is approximately synchronous speed. A typical power-factor characteristic, compared to that of an ordinary induction motor,

is shown in Fig. 7.6(*a*). The motor has proved commercially practicable under the trade name of "No-lag Motor," and is more fully described in Chapter 10.

The Schrage Motor

This is a modification of the Osnos motor, suggested in 1914 in order to enable speed variation to be effected. The construction is similar to that of the Osnos motor except that each secondary phase has two sets of brushes per pole-pair connected as shown, for a two-pole machine, in Fig. 7.7. The connecting leads between the brushes

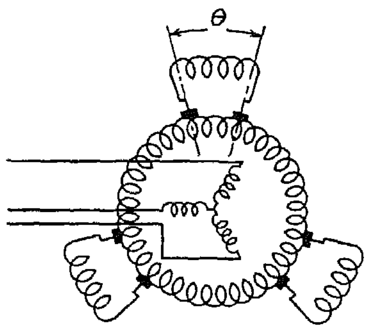


FIG. 7.7. SCHRAGE MOTOR

and the stator winding are flexible, and the brush rockers are designed so that they can be moved in order to vary the angle θ. A variable e.m.f. can thus be collected from the commutator and injected into the secondary circuit. By allowing the brushes to pass each other on the commutator the injected e.m.f. can be reversed and speeds both above and below synchronism obtained. This motor has also proved very satisfactory in practice and is more fully described in Chapter 9.

The Torda Motor

This machine, at one time manufactured under the trade name "All-watt Motor," is an ordinary induction motor with a simple Leblanc exciter incorporated within it. The secondary winding is arranged in slots around the outer periphery of the rotor in the usual way and brought out to three slip rings for starting, and for connexion to the fixed brushes of the commutator winding. A commutator winding is arranged in slots on the inner periphery of the rotor core plates as shown in Fig. 7.8(*a*). The two windings have different numbers of poles so that there is no mutual induction between them, and the commutator winding which is, in fact, a Leblanc exciter acts quite independently of the secondary winding except that its speed is the same; it is connected in series with the rotor winding as in Fig. 7.8(*b*).

The behaviour of the motor can be illustrated by considering the particular case of a 6-pole, 50-c/s, induction motor running with a slip of 5 per cent and having a commutator winding arranged for four poles. With six poles the synchronous speed is 1,000 r.p.m. and at 5 per cent slip the actual speed is 950 r.p.m., and the frequency of the rotor currents is 2·5 c/s. These currents are supplied to the brushes of the commutator winding, and the speed of the rotating magnetic field set up by them is $(2{\cdot}5/2) \times 60 = 75$ r.p.m. The actual speed of the conductors is, however, 950 r.p.m., i.e. much faster than the speed of the field so that, as in the ordinary Leblanc

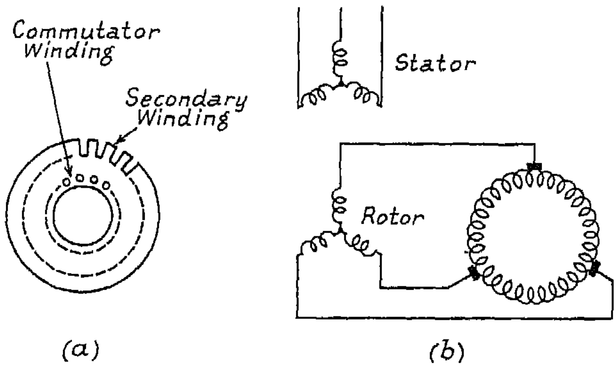


FIG. 7.8. TORDA MOTOR
(a) Rotor core section.
(b) Connexion diagram.

exciter, the commutator winding acts as a capacitor and injects into the secondary circuit an e.m.f. leading the current by 90° and approximately proportional to the current in magnitude.

A typical power-factor characteristic for such a motor is shown in Fig. 7.6(b). There is, as with the ordinary Leblanc exciter, little improvement of power factor at low loads. The commutator winding need not be in circuit at starting and therefore does not have to carry the heavy starting currents.

The Fynn-Weichsel Motor

This motor, at one time popular in America and also manufactured in this country under the trade name "True-watt Motor," has the primary winding on the rotor with a commutator winding placed in the same slots as is done in the Osnos and Schrage motors. There are two secondary phases, as shown in Fig. 7.9, one being connected to two brushes on the commutator and the other being short-circuited on itself. The motor is started as a plain induction motor by the resistance in each of the secondary phases—as the resistance is cut out and the motor reaches nearly synchronous speed a synchronizing torque is set up as in the synchronous-induction motor

and the motor pulls itself into synchronism and continues to run as a synchronous machine excited by direct current obtained from the commutator. The short-circuited secondary winding now acts as a damper winding. The power-factor characteristic of the motor is very similar to that obtained with the Osnos motor (shown in Fig. 7.6(*a*)).

The Three-phase Series Motor

If a series characteristic is required from a three-phase motor, the stator and rotor windings may be connected in series as shown in Fig. 7.10, the rotor winding being connected to a commutator.

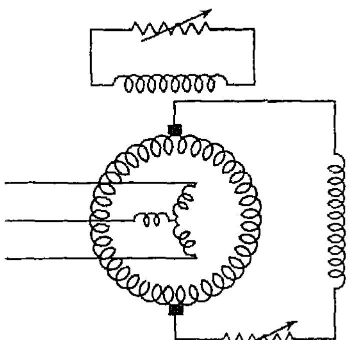


FIG. 7.9. FYNN-WEICHSEL MOTOR

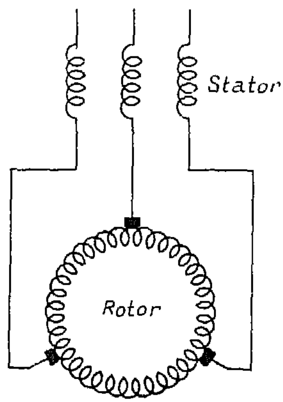


FIG. 7.10. THREE-PHASE SERIES MOTOR

The characteristics are as shown in Fig. 7.3(*b*). Such motors have been used to a considerable extent in Europe but have not so far found much favour in this country or the U.S.A. They are, however, of interest and are more fully discussed in Chapter 11.

EXERCISES 7

1. A doubly-fed motor has a stator input of 50 kW, and the injected e.m.f. is adjusted so that the machine runs at one-third of synchronous speed. Neglecting losses, determine the shaft torque in synchronous watts, the power carried by the auxiliary transformer, the total power taken from the supply and the mechanical output.

2. A 6-pole, 50-c/s, Schrage motor has a standstill secondary e.m.f. of 100 V. What must be the e.m.f. induced between the brushes to give a no-load speed of 1,500 r.p.m.? At what speed will the air-gap field be rotating under these conditions? If the diametric voltage of the commutator winding is 80 V, what will be the angular displacement between the brushes (in mechanical degrees)?

3. In a 50-c/s, 6-pole, Torda motor the commutator winding has 2 poles and a reactance of 2 Ω/ph. at 50 c/s and with the rotor stationary. Calculate the reactance when the motor is running with a slip of 10 per cent.

CHAPTER 8

THE DOUBLY-FED MOTOR

It was explained in the preceding chapter that commercial forms of the doubly-fed motor have been developed in which the e.m.f. for injection into the secondary circuit is obtained from the supply through a variable-ratio transformer or an induction regulator.

The motor comprises a stator similar to that of an ordinary induction motor and a rotor carrying a commutator winding and

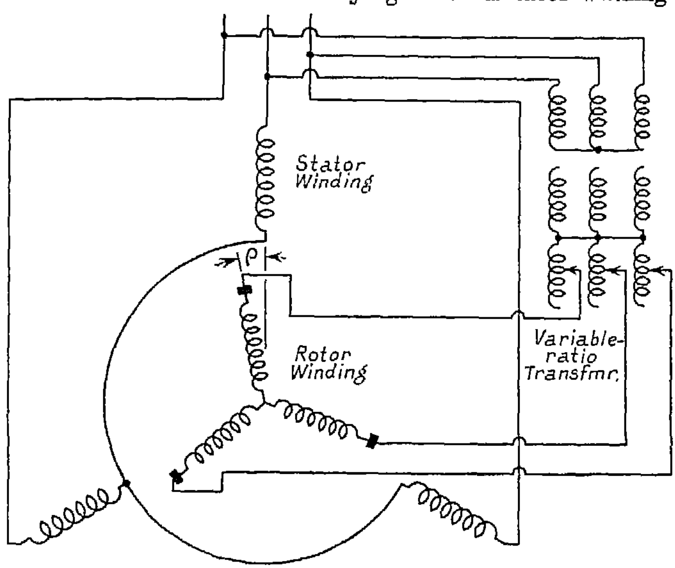


Fig. 8.1. Schematic Diagram of Doubly-fed Motor

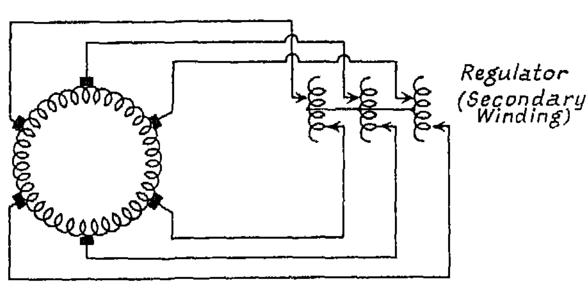


Fig. 8.2. Secondary Circuit with Diametrically-connected Brushes

commutator with three sets of symmetrically-spaced brushes per pole-pair. The commutator converts the injected e.m.f. obtained from the transformer from supply frequency to slip frequency as described on page 25. A schematic connexion diagram for a machine with three brushes per pole-pair is shown in Fig. 8.1 in which the rotor winding is represented by its equivalent-star circuit. In Fig. 8.2 is an alternative arrangement of the secondary circuit with six diametrically-connected brushes per pole-pair.

M.M.F.'s and Flux

The three-phase currents in the stator winding set up a rotating m.m.f. which has a fundamental of constant magnitude and moving at synchronous speed as described on page 6. The rotor carries three-phase currents supplied to it through the commutator, and, since the frequency of the currents at the brushes is supply frequency, they also set up a rotating m.m.f. moving at synchronous speed irrespective of the speed of the rotor; the connexions are so made that these two m.m.f.'s move in the same direction. The axes of the two m.m.f.'s are, however, not coincident but depend on the position of the brushes and on the phase angle between the stator and rotor currents. Whatever their position, however, their resultant is a rotating m.m.f. which sets up the main rotating flux in the air gap moving at synchronous speed; for most purposes, it may be assumed to be sinusoidally distributed over a double-pole pitch.

A difference between the action of the doubly-fed and of the induction motor may thus be noted—in the doubly-fed motor the rotor m.m.f. moves at synchronous speed as a consequence of the fixed brush position; in the ordinary induction motor the rotor m.m.f. moves at slip speed relative to the rotor conductors, but as these are moving at a speed corresponding to $(1 - s)$ the total speed of the m.m.f. relative to the stator adds up to synchronous speed. In each case the result is the same although arrived at by different internal phenomena.

The maximum values of the two fundamental m.m.f.'s are—

Stator m.m.f.

$$F = \{3(\sqrt{2})/\pi\}I_1T_1' \text{ ampere-turns}$$

Rotor m.m.f.

$$F = \{3(\sqrt{2})/\pi\}I_2T_2' \text{ ampere-turns}$$

where I_2 is the brush current and T_2' is the effective equivalent-star turns per pole-pair for the three-brush arrangement and the effective turns per pole-pair in series between diametric brushes for the six-brush arrangement.

E.M.F.'s

The e.m.f. induced in the stator winding by the rotating flux of Φ webers is (from eq. (2.7a), page 21)—

$$E_1 = (\sqrt{2})\pi f_1 T_1 \Phi k_w \text{ volts per phase}$$

The e.m.f. between brushes is (from eq. (2.11), page 24)—

$$E = (\sqrt{2})(T_a/a)(f_1 - f_r)\Phi \sin(\theta/2) = (\sqrt{2})(T_a/a)sf_1\Phi \sin(\theta/2) \text{ volts}$$

The equivalent-star value for the three-brush arrangement ($\theta = 120°$) is thus

$$E = (1/\sqrt{2})(T_a/a)sf_1\Phi \text{ volts}$$

and for the six-brush diametric arrangement ($\theta = 180°$) the value is

$$E = (\sqrt{2})(T_a/a)sf_1\Phi \text{ volts}$$

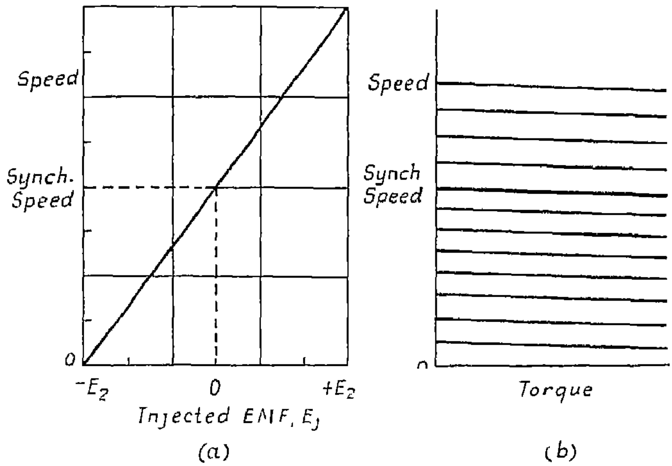


FIG. 8.3. APPROXIMATE SPEED CHARACTERISTICS OF DOUBLY-FED MOTOR

(a) No-load speed.
(b) Speed-torque characteristics.

Approximate Speed Characteristics

An approximate idea of the speed characteristics can be obtained if it is assumed that the e.m.f. from the transformer is injected at 0° or 180° to the secondary e.m.f., as is usually the case with this type of motor.

NO-LOAD SPEED. At no load the secondary current will be small and the secondary impedance drop may be neglected. The e.m.f. equation for the secondary circuit thus becomes

$$sE_2 = \pm E_j$$

so that $$s = \pm E_j/E_2$$

and $$n_r = n_1(1 \pm E_j/E_2)$$

i.e. the slip is proportional to the injected e.m.f. as shown in Fig. 8.3(a). There is no difficulty in making E_j have any desired value from zero up to a value equal to the secondary standstill e.m.f. E_2 so

that no-load speeds from zero up to about twice synchronous speed can be obtained.

SPEED-TORQUE CHARACTERISTICS. Loading the machine does not influence the value of E_j but tends to cause a slight drop in speed and therefore a corresponding increase of sE_2. In accordance with the general rotor-circuit equation, $sE_2 = \pm E_j + I_2z_{2s}$, the value of I_2z_{2s} and therefore of I_2 will increase. This gives rise to an increase of torque tending to maintain the speed which will therefore become stable at a value slightly less than the no-load speed.

From the above equation

$$\begin{aligned} s &= \pm E_j/E_2 + I_2z_{2s}/E_2 \\ &= \text{no-load slip} + I_2z_{2s}/E_2 \end{aligned}$$

Assuming torque to be approximately proportional to current, the speed-torque curves over the normal working range of torque can be plotted as shown in Fig. 8.3(*b*). It is thus seen that the motor can give shunt speed characteristics over a wide range of speed.

COMPLETE CURVES. The expression worked out on page 51 for an induction motor with an injected e.m.f. in the secondary is valid for the doubly-fed motor, and, if constant flux is assumed, families of speed-torque curves can be drawn similar to those of Fig. 4.8.

STARTING TORQUE. At starting ($s = 1$) the above-mentioned expression reduces to

$$TM_s = K\{E_2r_2/z_{2s}^2 + E_j(r_2 \cos\beta + x_{L2} \sin\beta)/z_{2s}^2\} \text{ newton-metres}$$

where $z_{2s}^2 = r_2^2 + x_{L2}^2$.

If $\beta = 0°$ or $180°$ this can be written

$$TM_s = \frac{K}{x_{L2}}\left(\frac{r_2/x_{L2}}{1 + r_2^2/x_{L2}^2}\right)(E_2 \pm E_j)$$

where the negative sign refers to sub-synchronous speeds.

For a given value of E_j, therefore, the starting torque depends on the rotor resistance in exactly the same way as for an induction motor, with the maximum torque when $r_2 = x_{L2}$. It can also be seen that the starting torque is dependent on E_j. If E_j is large and negative, i.e. $\beta = 180°$, *corresponding to a transformer setting for* sub-synchronous speeds the starting torque will be low. With a high-speed setting ($\beta = 0°$) the starting torque will be high.

The secondary starting current, however, is also proportional to $(E_2 \pm E_j)$ so that it will be low with low-speed settings and high with high-speed settings.

Since motors of this type rarely have to start against heavy loads it is usual to switch them direct on to the line with the transformer tappings adjusted to the lowest-speed setting, an interlock between the tapping switch and the main switch ensuring that this condition is fulfilled. Under these conditions with motors having a 3:1 speed range starting currents up to about twice full-load current are taken

and give starting torques of 1·5 times full-load torque. Large motors may require the addition of some resistance in the rotor circuit both to increase the starting torque and to cut down the current. Small motors may, however, safely be started by switching direct on to the line even if not on the low-speed setting.

Slip Power. The conditions relating to the slip power are similar to those investigated generally on page 47 and, for this particular machine, may be written—

$$\text{Motor input} = \begin{matrix}\text{stator}\\ \text{loss}\end{matrix} + \begin{matrix}\text{rotor}\\ \text{resistance}\\ \text{loss}\end{matrix} \pm \begin{matrix}\text{slip power to}\\ \text{or from}\\ \text{supply}\end{matrix} + \begin{matrix}\text{mechanical}\\ \text{output}\end{matrix}$$

The mechanical output is $(1 - s)$ times the total secondary input so that at speeds below synchronism (s positive) the slip power is

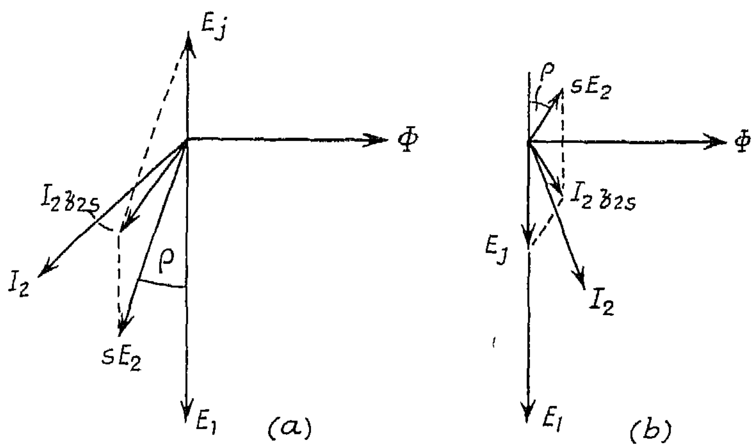


Fig. 8.4. Rotor Quantities Viewed from Rotor Terminals

(a) Sub-synchronous speeds.
(b) Super-synchronous speeds.

returned through the transformer to the supply. At speeds above synchronism the slip power is negative and is fed to the motor through the transformer. Under these circumstances power is fed to both stator and rotor, giving rise to the name "doubly-fed" motor. For a given stator input (neglecting losses) the torque at the shaft is constant for all speeds.

Complexor Diagram of Rotor Quantities

Owing to the phase displacement that may be introduced by shifting the brushes, two methods of drawing the complexor diagram are possible, depending on whether conditions are viewed from the rotor conductors or from outside the rotor.

Outside View. The flux complexor can be used as a reference

as in the ordinary induction motor, and the primary induced e.m.f. E_1 will lag 90° behind it as shown in Fig. 8.4. The injected e.m.f. from the transformer, neglecting small displacements due to voltage drops, will be in phase or in phase opposition to E_1. At sub-synchronous speeds the injected e.m.f. leads the flux by 90° as shown in Fig. 8.4(*a*) and the e.m.f. between brushes sE_2, if these brushes are shifted by an angle ρ from their neutral position in the direction of the rotating flux, will be displaced by the angle ρ as shown. The

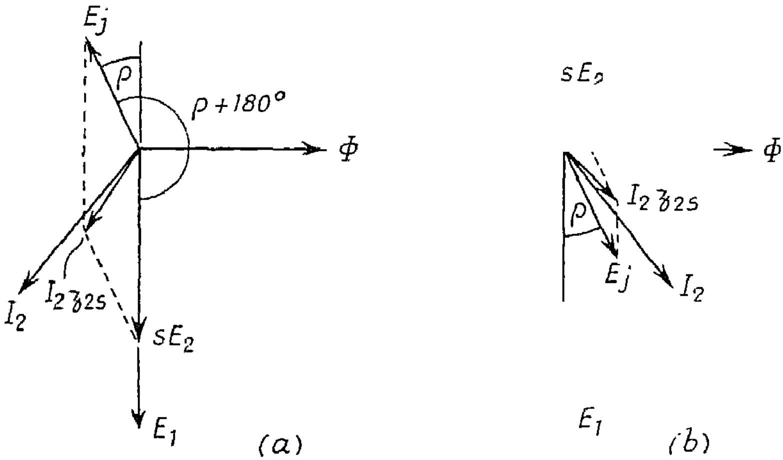


FIG. 8.5. ROTOR QUANTITIES VIEWED FROM ROTOR CONDUCTORS

(*a*) Sub-synchronous speeds.
(*b*) Super-synchronous speeds.

resultant of these two e.m.f.'s overcomes the impedance drop of the secondary circuit (including the transformer) $I_2 z_{2s}$ as shown. The secondary current I_2 can be drawn lagging $I_2 z_{2s}$ by the angle $\tan^{-1}(r_2/x_{L2s})$ where r_2 and x_{L2s} are the resistance and reactance of the secondary circuit, the reactance being variable with slip as explained below. At super-synchronous speeds the injected e.m.f. lags the flux by 90° so that s is negative and the diagram is as shown in Fig. 8.4(*b*).

INSIDE VIEW. Viewed from the rotor conductors the e.m.f. induced in them by the air-gap flux at sub-synchronous speeds lags the flux by 90° as in the ordinary induction motor and as shown in Fig. 8.5(*a*). The injected e.m.f. in this case is ahead of its position in Fig. 8.4 by the angle ρ. The complexor diagrams for sub- and super-synchronous speeds are thus similar to those of Fig. 8.4 except that the quantities are advanced by the angle ρ.

Since torque production depends on the interaction between rotor currents and flux, i.e. upon conditions inside the machine, the inside diagram is generally more convenient and is used in what follows.

The doubly-fed motor is normally built with its brushes set in a

definite fixed position. It should therefore be noted that if the brush displacement is ρ the phase of the injected e.m.f. will be ρ for super-synchronous speeds and $\rho + 180°$ for sub-synchronous speeds.

It can be seen also that, if the brushes are given a shift so that the injected e.m.f. has a component leading E_1 by 90° at super-synchronous speeds, it will have a component lagging by 90° at sub-synchronous speeds and vice versa.

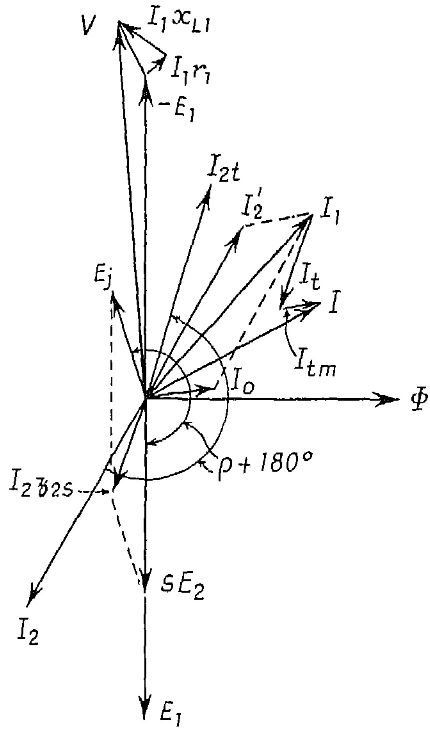


FIG. 8.6. COMPLETE COMPLEXOR DIAGRAM: SUB-SYNCHRONOUS SPEEDS

SECONDARY-CIRCUIT RESISTANCE AND REACTANCE. The secondary-circuit resistance includes that of the transformer and, neglecting variability of brush drop with current, is independent of speed.

The transformer carries currents that are always of supply frequency, and its leakage reactance is therefore constant and given by x_{Lt}. As explained in Chapter 3 (page 39) the impedance of the commutator winding, z_{2s}, is due chiefly to the effect of the currents in the rotor conductors, which are at slip frequency; neglecting the effect of the current change during commutation the total secondary-circuit reactance is thus $sx_{L2} + x_{Lt}$.

Complete Complexor Diagram

The diagrams of Fig. 8.5 can be extended to include the stator quantities as shown in Fig. 8.6 for sub-synchronous speeds. To set up the flux a magnetizing current I_m and a core-loss component I_c, giving a total current I_0 for the magnetic circuit, are required.

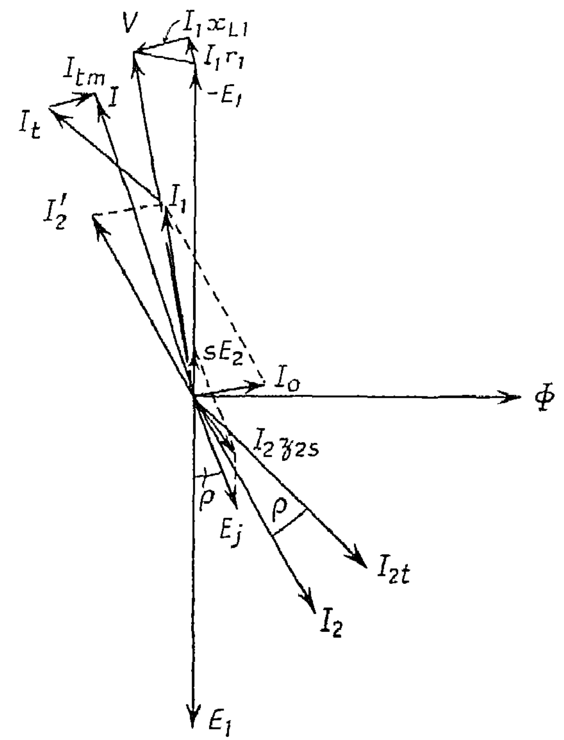


FIG. 8.7. COMPLETE COMPLEXOR DIAGRAM: SUPER-SYNCHRONOUS SPEEDS

The rotor current I_2 sets up an m.m.f. which must be counterbalanced by a corresponding m.m.f. set up by a stator current I_2' such that $I_2T_2' = I_2'T_1'$. The total stator current I_1 is thus $I_0 + I_2'$. The applied voltage V can be found by adding the stator resistance and leakage reactance drops I_1r_1 and I_1x_{L1} to the back e.m.f. $-E_1$ in the usual way as shown.

The total supply current I is made up of the stator current I_1 and the current fed to the rotor through the transformer. If T_p and T_s are the primary and secondary turns on the transformer this current is related in magnitude to the rotor current I_2 by

$$I_t = I_2(T_s/T_p)$$

Due to the brush shift the current in the transformer secondary leads the rotor current by the angle $(\rho + 180°)$ so that I_{2t} can be drawn leading I_2 by the angle $(\rho + 180°)$.

The current I_t will be opposite in phase to this and can be added to I_1 as shown. There will be a magnetizing current I_{tm} for the transformer which can also be added giving the total current taken from the primary I as shown.

It will be observed that the current I_t has a power component opposed to the applied voltage, indicating that power is being

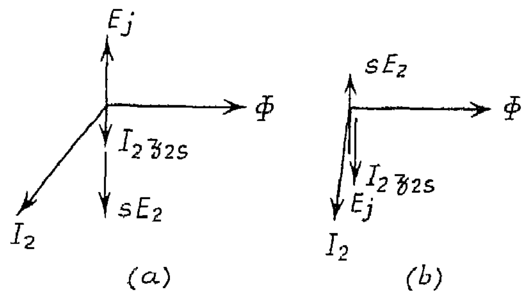


FIG. 8.8. COMPLEXOR DIAGRAMS WITH E_j AT 0° OR 180° TO E_2

(a) Sub-synchronous speeds.
(b) Super-synchronous speeds.

returned to the supply as would be expected when the motor is running below synchronous speed.

A similar diagram can be drawn for super-synchronous speeds as in Fig. 8.7—in this case it will be observed that the current I_t has a component in phase with the applied voltage, indicating that power is being supplied to the rotor as well as to the stator.

Power Factor

It can be seen from the complexor diagrams that, if the brushes are adjusted to give a slightly leading injected e.m.f. at high speeds, a high power factor at these speeds is obtained; on reversing the e.m.f., however, to obtain low speeds β, the effective angle of injection of E_j, becomes greater than 180° and a very low power factor results. If β is made slightly negative at high speeds the power factor can be made good at low speeds but poor at high speeds. The magnetizing current of the variable-ratio transformer, of course, tends to reduce the overall power factor at all speeds.

It is thus desirable to operate with the brushes approximately in their neutral position, i.e. with the injected e.m.f. in phase or phase opposition to E_2 as in Fig. 8.8. At sub-synchronous speeds, s will be large, giving rise to a fairly large secondary leakage reactance in the rotor winding—the secondary current will thus lag I_2z_{2s} as in Fig. 8.8(*a*). At super-synchronous speeds the slip becomes negative, i.e. the rotor tends to act as a capacitance and the angle of lag of

the secondary current behind I_2z_{2s} will be small or may even lead. Even with brushes on neutral, therefore, the low-speed power factors tend to be considerably lower than those at high speeds.

An improvement of power factor at all speeds can be effected by injecting into the secondary circuit an e.m.f. E_{jq} leading E_2 by 90°, the complexor diagram then becoming as in Fig. 8.9. Two convenient

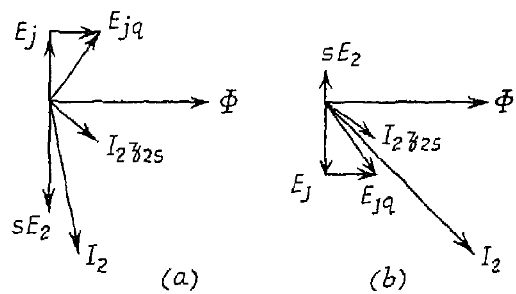


FIG. 8.9. COMPLEXOR DIAGRAMS WITH ADDITIONAL E.M.F. INJECTED AT 90°

(a) Sub-synchronous speeds. (b) Super-synchronous speeds.

methods of obtaining this auxiliary e.m.f. are described below while on page 117 are described special types of regulating transformer for giving the same effect.

AUXILIARY TRANSFORMER. A transformer with its primary connected across, say, phases B and C will give a secondary e.m.f. at 90° to that of phase A, and the secondary winding of such a transformer can therefore be connected in phase A of the secondary circuit as shown in Fig. 8.10(a).

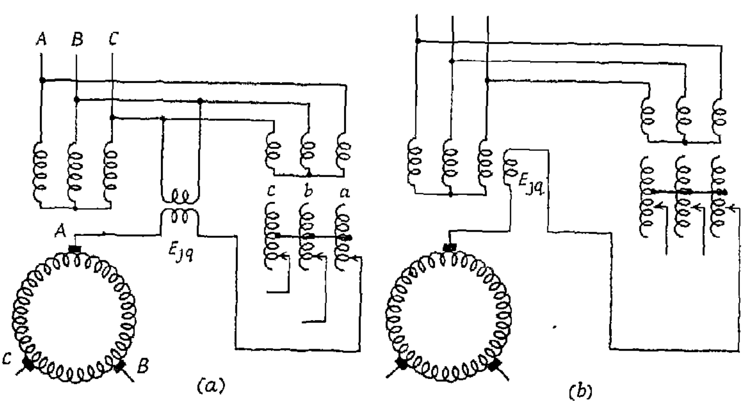


FIG. 8.10. CONNEXIONS FOR POWER-FACTOR IMPROVEMENT (SHOWING ONLY ONE SECONDARY PHASE)

(a) Auxiliary transformer. (b) Auxiliary stator winding.

AUXILIARY STATOR WINDING. An alternative source of the leading e.m.f. is an additional winding (Fig. 8.10(*b*)) on the stator in the same slots as the primary winding but with its axis located at 90° to the main-winding axis. The amount of lead imparted to the total injected e.m.f. for a given value of E_i can be increased slightly by injecting at about 60° instead of at 90°; this, of course, reduces the speed slightly since there is a component of E_{iq} in phase with E_2.

The auxiliary stator winding is used for the majority of medium-sized 420-V machines, but for large or high-voltage motors the auxiliary transformer is preferable while for small motors up to 5 or 10 h.p. the special form regulating transformer is available.

As a result of these arrangements the power factors of large machines tend to be higher than those of an induction motor of corresponding size but for small machines it is lower, due chiefly to the large magnetizing current taken by the voltage regulator.

Commutation

The coils short-circuited by the brushes during commutation have induced in them an e.m.f. as a result of their being cut by the rotating field at a speed equal to the difference between the speeds of the field and of the rotor. The e.m.f. is thus, from eq. (3.2*a*), page 32, given by

$$E = (\sqrt{2})\pi(f_1 \pm f_r)T_c\Phi \text{ volts per coil}$$
$$= (\sqrt{2})\pi s f_1 T_c\Phi \text{ volts per coil}$$

It is thus proportional to slip and is high at starting, zero at synchronous speed and increases again at super-synchronous speeds.

Conditions at starting thus impose a limit to the flux per pole as shown in Table 3.2, page 34. Conditions near to synchronous speed are the most satisfactory so that, although motors can be built for speed ranges as wide as 1–10, it is usual to limit the range to between 0·5 and 1·4 times synchronous speed.

In addition to the above e.m.f. there is also the reactance e.m.f. which cannot be neutralized by means of compoles; one of the discharge windings described in Chapter 3 is therefore usually fitted on all machines above about 5 h.p.

Voltage (Speed) Regulators

The variable injected e.m.f. may be obtained from a tapped transformer or from one of several forms of induction regulator, the latter being more usual in spite of the higher magnetizing current which they take on account of their air gap.

TAPPED TRANSFORMER. Although essentially simple, a tapped transformer has two important disadvantages which preclude its general use: (i) as there must be a finite number of tappings, only a fixed number of speeds can be obtained, and (ii) change of tapping must be made without interrupting the circuit, leading to the usual difficulties due to short-circuiting some turns of the winding when moving from one tapping to the next.

SINGLE INDUCTION REGULATOR. An ordinary single-wound induction regulator gives a secondary voltage which is constant in magnitude but variable in phase relative to the primary voltage as shown in Fig. 8.11(*a*). It can thus produce a component of injected

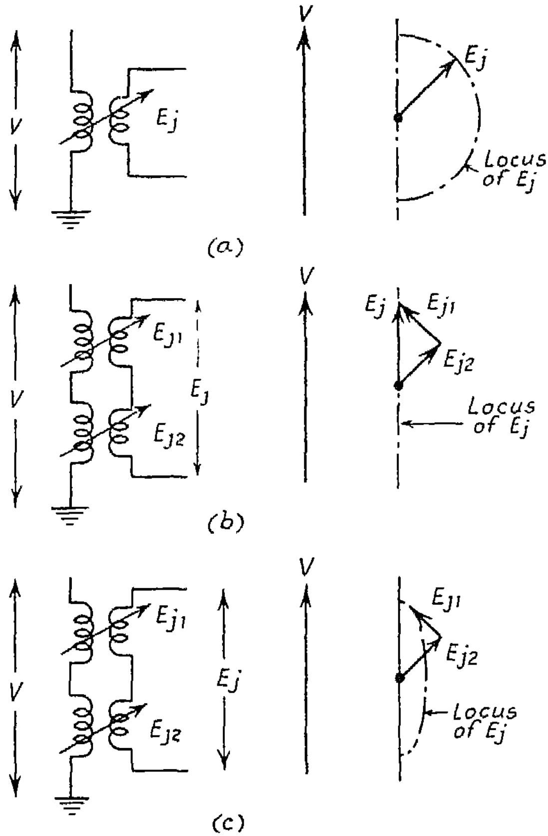


FIG. 8.11. TYPES OF INDUCTION REGULATOR

(*a*) Single regulator.
(*b*) Double regulator.
(*c*) Biased double regulator.

e.m.f. which is in phase with E_2 and variable between $+E_j$ and $-E_j$, but associated with this is a leading component which varies between 0 and E_j. Although a leading component is desirable, as explained on page 114, it requires to be only a few per cent of the in-phase component, and the value produced from the single regulator is much too large. Such a regulator is therefore impracticable.

DOUBLE INDUCTION REGULATOR. By using two regulators with their secondaries in series, each giving half the required voltage,

arranged so that their phase displacements cancel each other as shown in Fig. 8.11(*b*), an injected e.m.f. variable between $+E_j$ and $-E_j$ and having no lagging or leading component is obtained. Such a regulator gives a continuously variable speed and is commonly used except for very small machines; an auxiliary transformer or stator winding gives the necessary power-factor improvement as described on page 114.

Biased Double Induction Regulator. A small quadrature component of e.m.f. can be introduced for power-factor improvement by using a double induction regulator in which the secondary e.m.f.'s are unequal. The complexor diagram thus becomes as shown in Fig. 8.11(*c*), and the end of the injected e.m.f. complexor follows the dotted elliptical locus. Maximum power-factor improvement is thus effected at the middle portion of the speed range but not at the extreme limits—a satisfactory amount of improvement is, however, obtained over most of the range. The differing secondary e.m.f.'s could be obtained by different numbers of turns on the secondary, but as the total number of secondary turns is usually small it is generally more convenient to have different numbers of turns on the primaries, thus giving different fluxes.

Modified Single Induction Regulator. Various modified forms of single regulator, combined with the auxiliary stator winding or separate transformer described on page 114, are described by Schwarz.* The merits of these special single regulators are a smaller quantity of material used, lower loss and lower leakage reactance.

Determination of Characteristics

Approximate determinations of behaviour can be made from the complexor diagram of the rotor circuit, or more exact calculations from the equivalent circuit of the whole machine.

Approximate Determination from Complexor Diagram. The behaviour over the normal working range can be found from a construction based on the complexor diagrams of Fig. 8.9. It is assumed that the brushes are placed in the neutral position and that an auxiliary e.m.f. E_{jq} is injected from an auxiliary stator winding or other suitable source.

Referring to Fig. 8.12 and taking O as the origin the injected e.m.f.'s E_j and E_{jq} can be drawn giving OD as the total injected e.m.f. The power component of the current, which is approximately proportional to the torque, is drawn vertically downwards, and the secondary-circuit resistance and reactance† drops $I_{2w}r_2$ and $I_{2w}x_{L2}$ drawn giving OP as the secondary impedance drop due to the power component of current. The drop due to the reactive component of current I_{2a} (at present unknown) will be at right-angles to OP and

* B. Schwarz, "The Stator-fed A.C. Commutator Machine with Induction Regulator Control," *Proc. Instn. Elect. Engrs.*, **96**, Pt. II, p. 755 (1949).

† Variation of secondary-circuit reactance with slip is neglected; this is justifiable since the transformer reactance preponderates.

will therefore lie along the line XY. The horizontal component of the total impedance drop must, however, equal the horizontal component of OD since there are no other horizontal components of voltage in the secondary circuit; the end of the $I_{2q}z_2$ vector must thus lie on the vertical through D, i.e. at point C. DC therefore represents the secondary induced e.m.f. sE_2. Knowing E_2 and measuring sE_2 from the diagram enables s and hence the speed to

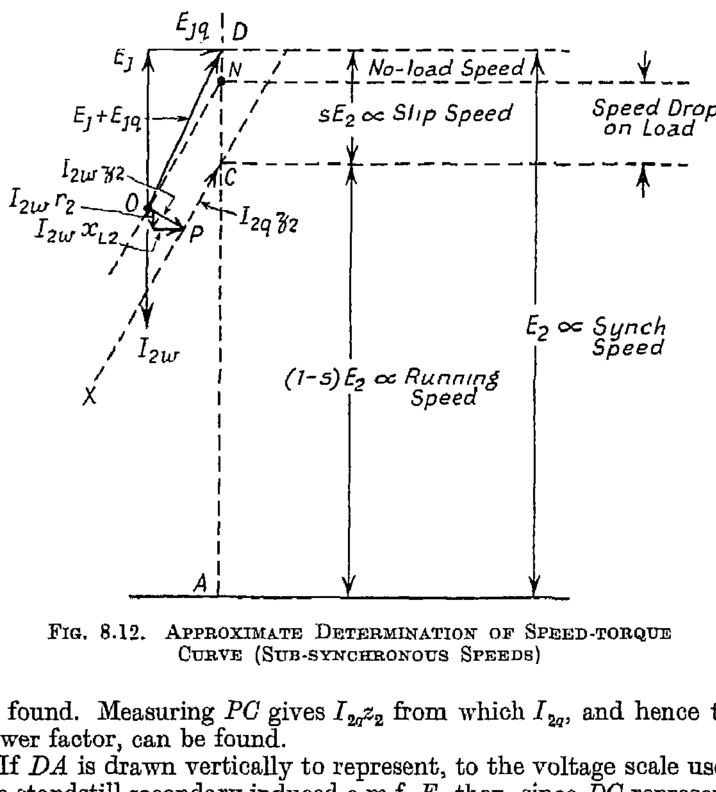


FIG. 8.12. APPROXIMATE DETERMINATION OF SPEED-TORQUE CURVE (SUB-SYNCHRONOUS SPEEDS)

be found. Measuring PC gives $I_{2q}z_2$ from which I_{2q}, and hence the power factor, can be found.

If DA is drawn vertically to represent, to the voltage scale used, the standstill secondary induced e.m.f. E_2 then, since DC represents sE_2, CA will represent $(1 - s)E_2$. To another scale these quantities also represent speeds as shown.

On no load the power component of current is approximately zero, and the voltage drop due to the reactive component then lies along a line through O and parallel to XY. DN therefore represents the slip under these conditions and, to the speed scale used above, AN represents the no-load speed. The speed drop corresponding to the load current I_2 is NC.

For super-synchronous speeds a similar diagram can be drawn as shown in Fig. 8.13.

Equivalent Circuit

The equivalent circuit developed for the induction motor with a constant injected e.m.f. (page 90) applies to the doubly-fed motor and is shown in Fig. 8.14(*a*). The secondary circuit resistance and reactance now include that of the regulating transformer; this

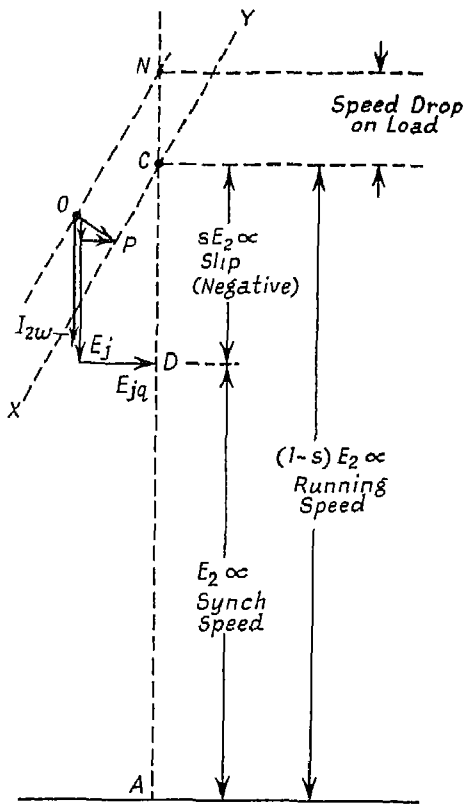


FIG. 8.13. APPROXIMATE DETERMINATION OF SPEED-TORQUE CURVE (SUPER-SYNCHRONOUS SPEEDS)

carries supply-frequency currents so that its reactance is not dependent on slip. The input current to the circuit, I_1, is the stator current of the motor and does not include the current fed from the supply to the regulator.

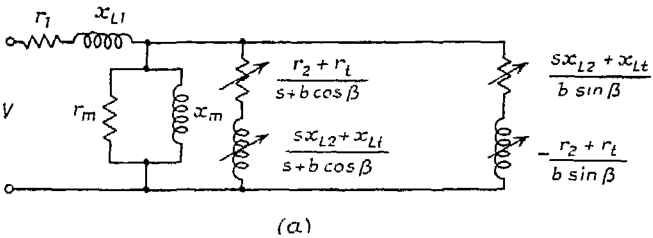
An alternative circuit is shown in Fig. 8.14(*b*), the regulator here being represented by a transformer having a variable ratio and a phase displacement between primary and secondary e.m.f.'s.

Example 8.1. The data refer to a 3-ph., 440-V, 180/60-h.p.,

975/325-r.p.m., 8-pole, doubly-fed motor having a star-connected stator winding.

$$r_1 = 0{\cdot}024\ \Omega \qquad r_2' = 0{\cdot}048\ \Omega \qquad r_t' = 0{\cdot}04\ \Omega$$
$$x_{L1} = 0{\cdot}122\ \Omega \qquad x_{L2}' = 0{\cdot}12\ \Omega \qquad x_{Lt}' = 0{\cdot}072\ \Omega$$
$$I_0 \text{ (motor)} = 7{\cdot}5 - j58\text{ A} \qquad I_0 \text{ (regulator)} = 3{\cdot}5 - j22{\cdot}5\text{ A}$$

All parameters are referred to the motor primary winding.



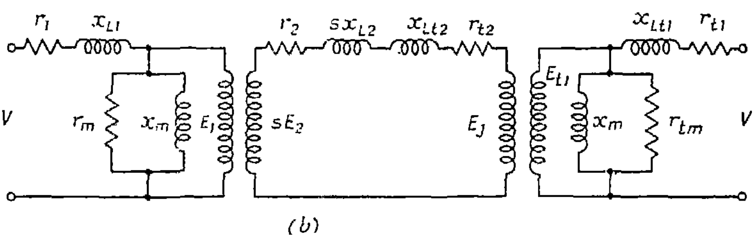


FIG. 8.14. EQUIVALENT CIRCUITS

Using the equivalent circuit of Fig. 8.14(a) determine the behaviour of the motor under the following conditions—

(a) $E_j = 0$	$s = 0{\cdot}09$ (induction-motor operation)
(b) $E_j' = 125\underline{/0^\circ}$	$s = -0{\cdot}45$ (high-speed)
(c) $E_j' = 125\underline{/180^\circ}$	$s = 0{\cdot}75$ (low-speed)
(d) $E_j' = 125\underline{/170^\circ}$	$s = 0{\cdot}52$ (low-speed with power-factor improvement)

The magnetizing impedances of the motor calculated from the no-load current and neglecting stator impedance drop are—

$$r_m = 250/7{\cdot}5 = 33{\cdot}4\ \Omega \qquad x_m = 250/58 = 4{\cdot}32\ \Omega$$

Hence the admittances are—

$$g_m = 0{\cdot}03 \text{ mho} \qquad b_m = -0{\cdot}232 \text{ mho}$$

CASE (d). This case is worked in full, the results for this and the other three cases being given in Table 8.1.

Slip $= 0{\cdot}52$ so that the speed is $750(1 - 0{\cdot}52) = 360$ r.p.m. Since $\beta = 170°$, $\cos\beta = -0{\cdot}985$ and $\sin\beta = 0{\cdot}174$. Secondary e.m.f. per phase (referred to primary) $= 440/\sqrt{3} = 250$ V (approx.).

$$b = E/E_2' = 125/250$$

$$b\sin\beta = 0{\cdot}5 \times 0{\cdot}174 = 0{\cdot}087$$

$$s + b\cos\beta = 0{\cdot}52 - 0{\cdot}5 \times 0{\cdot}985 = 0{\cdot}0275$$

Hence

$$-(r_2' + r_t')/(b\sin\beta) = -0{\cdot}088/0{\cdot}087 = -1{\cdot}011$$

$$(sx_{L2}' + x_{Lt}')/(b\sin\beta) = (0{\cdot}52 \times 0{\cdot}12 + 0{\cdot}072)/0{\cdot}087 = 1{\cdot}54$$

$$\text{Impedance of right-hand branch} = 1{\cdot}54 - j1{\cdot}011 = 1{\cdot}85\underline{/-33{\cdot}4°}$$

$$\text{Admittance of right-hand branch} = 0{\cdot}54\underline{/33{\cdot}4°} = 0{\cdot}45 + j0{\cdot}296$$

$$(r_2' + r_t')/(s + b\cos\beta) = 0{\cdot}088/0{\cdot}0275 = 3{\cdot}2$$

$$(sx_{L2}' + x_{Lt}')/(s + b\cos\beta) = (0{\cdot}52 \times 0{\cdot}12 + 0{\cdot}072)/0{\cdot}0275 = 4{\cdot}9$$

$$\text{Impedance of left-hand rotor branch} = 3{\cdot}2 + j4{\cdot}9 = 5{\cdot}85\underline{/56{\cdot}8°}$$

$$\text{Admittance of left-hand rotor branch} = 0{\cdot}171\underline{/-56{\cdot}8°} = 0{\cdot}094 - j0{\cdot}143$$

Total admittance of rotor and magnetizing branches

$$= (0{\cdot}45 + j0{\cdot}296) + (0{\cdot}094 - j0{\cdot}143) + (0{\cdot}03 - j0{\cdot}232)$$
$$= 0{\cdot}574 - j0{\cdot}079 = 0{\cdot}58\underline{/-7{\cdot}9°}$$

$$\text{Impedance} = 1{\cdot}725\underline{/7{\cdot}9°} = 1{\cdot}705 + j0{\cdot}227$$

Total impedance (including stator winding)

$$= (1{\cdot}705 + j0{\cdot}227) + (0{\cdot}024 + j0{\cdot}122) = 1{\cdot}729 + j0{\cdot}349 = 1{\cdot}76\underline{/11{\cdot}4°}$$

$$\text{Motor current} = 250/(1{\cdot}76\underline{/11{\cdot}4°}) = 142\underline{/-11{\cdot}4°} = 139 - j28\ \text{A}$$

$$\text{Power factor} = 0{\cdot}98 \text{ lagging}$$

Back e.m.f. (corresponding to air-gap flux)

$$= 250 - (139 - j28)(0{\cdot}024 + j0{\cdot}122)$$
$$= 243{\cdot}2 - j16{\cdot}33$$
$$= 244\underline{/-3{\cdot}8°}$$

$$\text{Left-hand rotor branch current} = 244\underline{/-3{\cdot}8^\circ} \times 0{\cdot}171\underline{/-56{\cdot}8^\circ}$$
$$= 41{\cdot}7\underline{/-60{\cdot}6^\circ}$$
$$= 20{\cdot}5 - j36{\cdot}4 \text{ A}$$

$$\text{Right-hand rotor branch current} = 244\underline{/3{\cdot}8^\circ} \times 0{\cdot}54\underline{/33{\cdot}4^\circ}$$
$$= 131{\cdot}5\underline{/29{\cdot}6^\circ}$$
$$= 113{\cdot}5 + j65 \text{ A}$$

$$\text{Total rotor-circuit current} = (20{\cdot}5 - j36{\cdot}4) + (113{\cdot}5 + j65)$$
$$= 134 + j28{\cdot}6 \text{ A}$$

Rotor-circuit current referred to regulator primary

$$= -(134 + j28{\cdot}6)(125/250) = -67 - j14{\cdot}3$$

Total regulator primary current (including regulator no-load current) $= (-67 - j14{\cdot}3) + (3{\cdot}5 - j22{\cdot}5)$

$$= -63{\cdot}5 - j36{\cdot}8 \text{ A}$$

$$\text{Current from supply} = (139 - j28) + (-63{\cdot}5 - j36{\cdot}8)$$
$$= 75{\cdot}5 - j64{\cdot}8$$
$$= 99{\cdot}5\underline{/-40{\cdot}5^\circ} \text{ A}$$

Power factor $= 0{\cdot}76$ lagging

Power distribution—

Total input	$= 3 \times 75{\cdot}5 \times 250 \times 10^{-3}$	$= 56{\cdot}5$ kW
Motor input	$= 3 \times 139 \times 250 \times 10^{-3}$	$= 104$ kW
Stator copper loss	$= 3 \times 139^2 \times 0{\cdot}024 \times 10^{-3}$	$= 1{\cdot}4$ kW
Stator iron loss	$= 3 \times 244^2/33{\cdot}4 \times 10^{-3}$	$= 5{\cdot}35$ kW
Rotor input	$= 104 - 1{\cdot}4 - 5{\cdot}35$	$= 97{\cdot}25$ kW
Torque	$= 97{\cdot}25(0{\cdot}117/12{\cdot}5) \times 10^3$	$= 910$ Lb-ft
Mechanical output	$= 97{\cdot}25(1 - 0{\cdot}52)$	$= 46{\cdot}7$ kW
		$= 62{\cdot}5$ h.p.

Rotor-circuit copper loss

$$= 3(41{\cdot}7^2 \times 0{\cdot}048 + 131{\cdot}5^2 \times 0{\cdot}04)10^{-3} = 2{\cdot}32 \text{ kW}$$

Power to regulator	$= 97{\cdot}25 - 46{\cdot}7 - 2{\cdot}32$	$= 48{\cdot}2$ kW
Regulator iron loss	$= 3 \times 3{\cdot}5 \times 250 \times 10^{-3}$	$= 2{\cdot}6$ kW
Power returned to supply	$= 48{\cdot}2 - 2{\cdot}6$	$= 45{\cdot}6$ kW

Certain approximations, inherent in the development of the equivalent circuit, have been made in the above; for instance, friction and windage loss is included in the mechanical output, rotor iron loss is neglected and the impedance of the regulator primary winding is included with its secondary-winding impedance x_{L2}'.

The calculations given in Table 8.1 are all for approximately full-load torque. In case (*a*) the machine is running as an induction motor, the only power to the regulator being that to supply the iron loss; the power factor is low, partly due to the magnetizing kVAr of the regulator and partly due to its leakage reactance which is in the motor secondary circuit. Case (*b*) shows the inherently high power factor resulting from high-speed operation. Case (*c*) shows the low power factor resulting from low-speed operation unless some means of improvement is adopted. Case (*d*) is similar to case (*c*) and shows the improvement of power factor resulting from a 10° shift of the injected e.m.f.

TABLE 8.1

BEHAVIOUR OF MOTOR (EXAMPLE 8.1)

	(*a*)	(*b*)	(*c*)	(*d*)
Injected e.m.f. (V)	0	125	125	125
Angle β	–	0°	180°	170°
Slip	0·09	– 0·45	0·75	0·52
Speed (r.p.m.)	682	1,087	187·5	360
Motor current (A)	$213/\underline{-55^\circ}$	$159/\underline{-34^\circ}$	$334/\underline{-67^\circ}$	$142/\underline{-11^\circ}$
Motor power factor	0·57 lag	0·83 lag	0·38 lag	0·98 lead
Regulator primary current (A)	$23/\underline{-81^\circ}$	$77/\underline{-30^\circ}$	$-143/\underline{-61^\circ}$	$-73/\underline{-30^\circ}$
Regulator power factor	0·15 lag	0·86 lag	0·49 lag	0·87 lag
Supply current (A)	$233/\underline{-57^\circ}$	$236/\underline{-33^\circ}$	$214/\underline{-71^\circ}$	$99{\cdot}5/\underline{-40^\circ}$
Supply power factor	0·54 lag	0·84 lag	0·32 lag	0·76 lag
Total input (kW)	94	148	51	56·5
Motor input (kW)	91	98	96	104
Regulator input (kW)	2·6	50	– 45	– 47·5
Torque (Lb-ft)	780	855	805	910
Shaft output (h.p.)	102	177	28	62

Design Features

The basic design constants (gap densities, electric loading, etc.) and the arrangement of the stator winding are similar to those for an ordinary induction motor, although the gap length can be slightly greater since means are available for counteracting the resulting low power factor; also, flux densities may be slightly lower on account of rotor iron loss. The diametric brush arrangement is preferred to the three-brush arrangement since it requires a smaller current per brush for a given m.m.f. and therefore a shorter commutator. Copper losses are also reduced in the ratio of 3/4.

ROTOR WINDING. The main rotor winding is similar to that of a d.c. machine and may be lap- or wave-connected. The low voltage

necessitated by commutation difficulties make the lap winding or even a duplex lap winding desirable for all machines above a few h.p.; such windings, however, require equalizing connexions to ensure equality of current through the various parallel circuits.

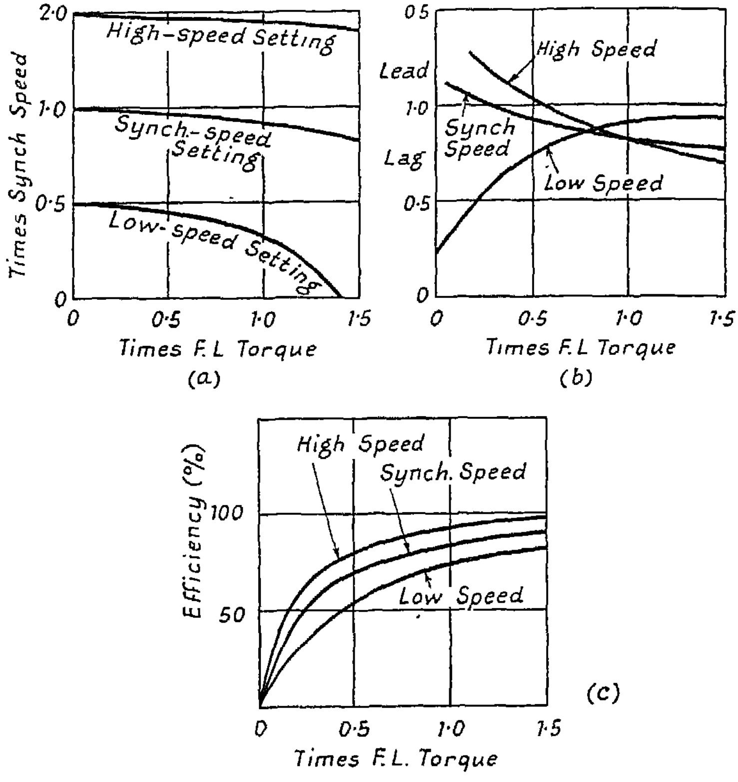


FIG. 8.15. TYPICAL PERFORMANCE CHARACTERISTICS

(a) Speed-torque.
(b) Power factor.
(c) Efficiency.

To give satisfactory commutation a discharge winding as described on page 38 is used on all except very small machines, the Schwarz type being the most usual. This is commonly wave-connected and thus acts as an equalizing winding for the main winding.

FLUX PER POLE. To avoid excessive circulating currents in the coils short-circuited by the brushes the flux per pole must be limited as shown in Table 3.2, page 34. As previously explained, however, the worst conditions obtain only at starting so that, if the required starting torque is low, an auto-transformer can be used to

reduce the voltage at starting: in this way motors for outputs up to 1,000 h.p. can be designed.

VENTILATION. This type of motor can readily be built for speeds down to zero, and care must be taken with the ventilation at very low speeds where natural air circulation is almost non-existent; a separate fan driven by a fractional-horse-power motor is generally fitted.

Applications and Performance

Typical performance curves are shown in Fig. 8.15. The doubly-fed motor is suitable for almost all drives requiring a variable speed with a shunt characteristic, or a particular approximately-constant speed such as 1,300 r.p.m. which is not obtainable with an ordinary induction motor. Common applications include drives for fans, pumps, lifts, cranes and haulage equipment, machine tools and printing machinery.

The Schrage motor, described in the next chapter, has very similar characteristics and is commercially in competition with the doubly-fed motor for many purposes—a tabular comparison between the two motors is given on page 166.

The doubly-fed motor has, however, definite advantages for the following special circumstances—

1. For small machines requiring remote control, e.g. machines mounted in the roof, since the variable-ratio transformer or regulator can be placed at some distance from the motor.

2. For high-voltage motors, above 650 V, which is the practical limit for the Schrage motor.

3. For motors above about 300 h.p. which is about the practical limit in size for the Schrage motor.

4. For totally-enclosed motors, since the enclosure is easier to arrange on motors with fixed brush gear.

Reversal is effected by interchanging two stator supply leads and, if driven above its speed setting, regeneration takes place; in both cases, however, heavy reactive currents flow unless the power-factor compensating device is also reconnected.

A full treatment of the doubly-fed motor by tensor analysis has been published by Jha.*

EXERCISES 8

1. A 4-pole, lap-connected armature with full-pitch coils has 72 slots with 4 conductors per slot and rotates at 1,000 r.p.m. in a rotating flux of 0·01 Wb moving at 1,500 r.p.m. in the same direction.

If 3-phase currents of 50 A (r.m.s.) are fed (*a*) to 3 brushes per pole-pair spaced at 120° (elec.) or (*b*) to 6 brushes per pole-pair diametrically spaced,

* C. S. Jha, "Theory and Equivalent Circuits of the Double Induction Regulator," *Proc. Inst. Elect. Engrs.*, **104**, Pt. C, p. 96 (1957). C. S. Jha, "Theory and Equivalent Circuits of the Stator-fed Polyphase Shunt Commutator Motor," *Proc. Inst. Elect. Engrs.*, **104**, Pt. C, p. 305 (1957).

draw the m.m.f. wave over 2 poles for the instant when the current at one phase is at its peak value and find the maximum value of its fundamental in each case.

Determine also the e.m.f. per phase in each case.

2. A 3-phase commutator winding is to set up a given m.m.f. Compare the copper loss if the commutator is equipped with 3 brushes per pole-pair or 6 brushes per pole-pair (diametric connexion).

3. *A doubly-fed motor has the following particulars*—

Standstill diametric brush e.m.f.	= 100 V (assumed constant).
Rotor circuit standstill reactance	= 0·07 Ω/ph.
Rotor circuit resistance	= 0·02 Ω/ph.
Injected e.m.f.	= + 40 V to − 40 V (no phase displacement).

Draw curves showing the variation of starting torque and starting torque per ampere (*a*) as resistance is added to the rotor circuit (up to 0·2 Ω/ph.) with 40 V injected e.m.f. (low-speed setting) and (*b*) as the injected e.m.f. is varied with no added rotor resistance.

4. A 6-pole, 50-c/s, doubly-fed motor has a standstill brush e.m.f. of 50 V (equivalent-star value), the brushes being in the neutral position. A biased double-induction regulator provides the injected e.m.f. and gives secondary e.m.f.'s of 20 V and 30 V. Plot, to a base of no-load motor speed, the component of the injected e.m.f. which is 90° ahead of the brush voltage. Over what speed range does the quadrature component of injected e.m.f. vary between 8 and 10 V?

5. The data refer to a 400-V, 50-c/s, 6-pole, 480/260-h.p., doubly-fed motor—

Standstill diametric brush e.m.f.	= 200 V
Rotor circuit resistance	= 0·012 Ω
Rotor circuit reactance (assumed constant)	= 0·03 Ω
Injected e.m.f. from stator auxiliary winding (90° leading)	= 15 V.

Draw approximate speed-torque curves with an injected e.m.f. from the regulator of − 100 V, 0 V, and + 100 V. Assume torque to be proportional to the power component of rotor current and that 600 A corresponds to full-load torque. Determine the approximate secondary power factor at 10 per cent and 100 per cent torque in each case. Compare with the values obtained if there were no auxiliary winding on the stator.

6. The data refer to a small 400-V, 4-pole, 50-c/s, doubly-fed motor, all parameters being equivalent-star values referred to the primary winding.

$$r_1 = 2{\cdot}0\ \Omega/\text{ph.} \qquad r_m = 300\ \Omega/\text{ph.}$$
$$x_{L1} = 4{\cdot}4\ \Omega/\text{ph.} \qquad x_m = 80\ \Omega/\text{ph.}$$
$$r_2' = 5{\cdot}9\ \Omega/\text{ph.} \qquad r_t' = 2{\cdot}3\ \Omega/\text{ph.}$$
$$x_{L2}' = 4{\cdot}7\ \Omega/\text{ph.} \qquad x_{Lt}' = 2{\cdot}1\ \Omega/\text{ph.}$$

If the regulator is adjusted to give a ratio of E_j/E_2 of 0·5 with E_j injected at 170° to E_2 and the slip is 0·7, determine the stator current, total input current, power factor, torque, h.p. and the power returned to the supply from the regulator. Draw to scale the complete complexor diagram for the motor and regulator.

CHAPTER 9

THE SCHRAGE MOTOR

LIKE the doubly-fed motor described in the previous chapter this machine is essentially a 3-phase induction motor with an e.m.f. injected into the secondary circuit in order to give speed control. The injected e.m.f. is, in this case, obtained from a commutator (tertiary)* winding mounted on the rotor as briefly described in Chapter 7 (page 102).

Construction

An important constructional difference between the Schrage motor and the ordinary induction motor is that the primary winding of the

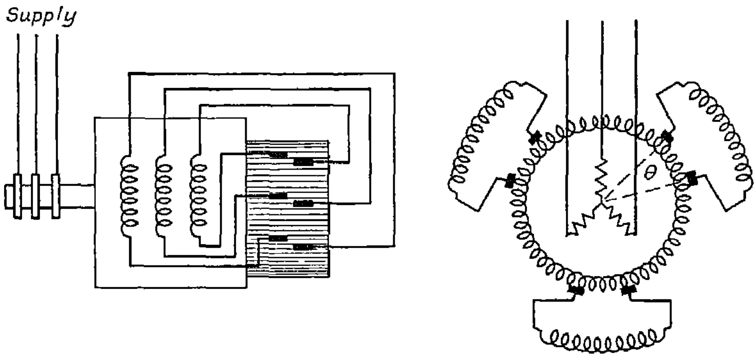


FIG. 9.1. CONNEXIONS OF SCHRAGE MOTOR

former is on the rotor and is fed through slip rings while the secondary winding is on the stator; the arrangement is thus as shown in Fig. 9.1. Each secondary phase is connected to a pair of movable brushes which collect from the commutator the required e.m.f. for injection into the secondary circuit. The tertiary winding is normally wound in the same slots as the primary winding as shown in Fig. 9.2.

THE ROTATING FIELD. If the pairs of brushes connected to each secondary phase are placed adjacent to each other on the same commutator segment, the secondary windings are short-circuited and the machine behaves exactly as an ordinary induction motor. The three-phase currents in the primary and secondary windings set up a resultant rotating m.m.f., and consequently a rotating air-gap flux, moving at synchronous speed (n_1 r.p.s.) relative to the primary

* Also called a *regulating* winding.

(rotor) conductors. According to ordinary induction-motor action, the motor runs in such a direction as to tend to reduce the frequency of the e.m.f.'s set up in the secondary winding, i.e. it will run in the opposite direction to that of the rotating flux at a speed of, say, n_r r.p.s. The speed of the rotating flux relative to the fixed stator is thus $n_1 - n_r$ r.p.s., i.e. slip speed. Slip-frequency e.m.f.'s are thus induced in the secondary (stator) winding. When running as an induction motor the speed n_r is, of course, less than n_1 so that the movement of the flux is in the opposite direction to that of the rotation of the rotor and the slip is positive.

The rotating flux is also cutting the tertiary-winding conductors at a speed of n_1 and induces in them an e.m.f. which is independent of their speed of rotation and which is of the same frequency as the e.m.f. induced in the primary conductors, i.e. supply frequency f_1. The commutator changes the frequency of these e.m.f.'s as described on page 25, to that corresponding to the speed of the field past the brushes, i.e. slip frequency f_2. If the brushes are now separated to a position such as that shown in Fig. 9.1, the e.m.f. appearing between them will therefore always be of the correct frequency for injection into the secondary circuit.

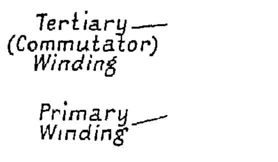


FIG. 9.2. ARRANGEMENT OF WINDINGS IN ROTOR SLOTS

BRUSH MOVEMENT. To give a range of speed control the e.m.f. obtained from the commutator for injection into the secondary circuit must be variable; the brushes must therefore be movable so that the angle θ, known as the *brush separation*, can be varied, the range of movement required, together with the relevant complexor diagrams, being illustrated in Fig. 9.3.

In Fig. 9.3(a) the brushes connected to each secondary phase are standing on the same commutator segment so that induction-motor operation is obtained with the motor running at approximately synchronous speed; the complexor of the secondary e.m.f., sE_2, is drawn vertically downwards as usual. In (b) the brushes are separated so that an e.m.f., E_j, is obtained from the section of the tertiary winding between them. If the centre-line of this group of conductors is coincident with the centre-line of the corresponding secondary phase as shown, the injected e.m.f. and the standstill secondary e.m.f. E_2 will be induced by the same flux but, so far as the secondary circuit is concerned, they can be seen to be in phase opposition, i.e. the angle of injection, β, is 180°. Reference to Fig. 4.1 (page 44) shows that an injected e.m.f. in this direction results in a speed below synchronism, i.e. sE_2 will be vertically downwards since, neglecting impedance drops, sE_2 must be equal and opposite to E_j. If the brushes are moved to the position of Fig. 9.3(c) the injected e.m.f., E_j, is reversed relative to E_2, i.e. the angle of injection is $\beta = 0°$, and sE_2 must also be reversed; this can only arise as a

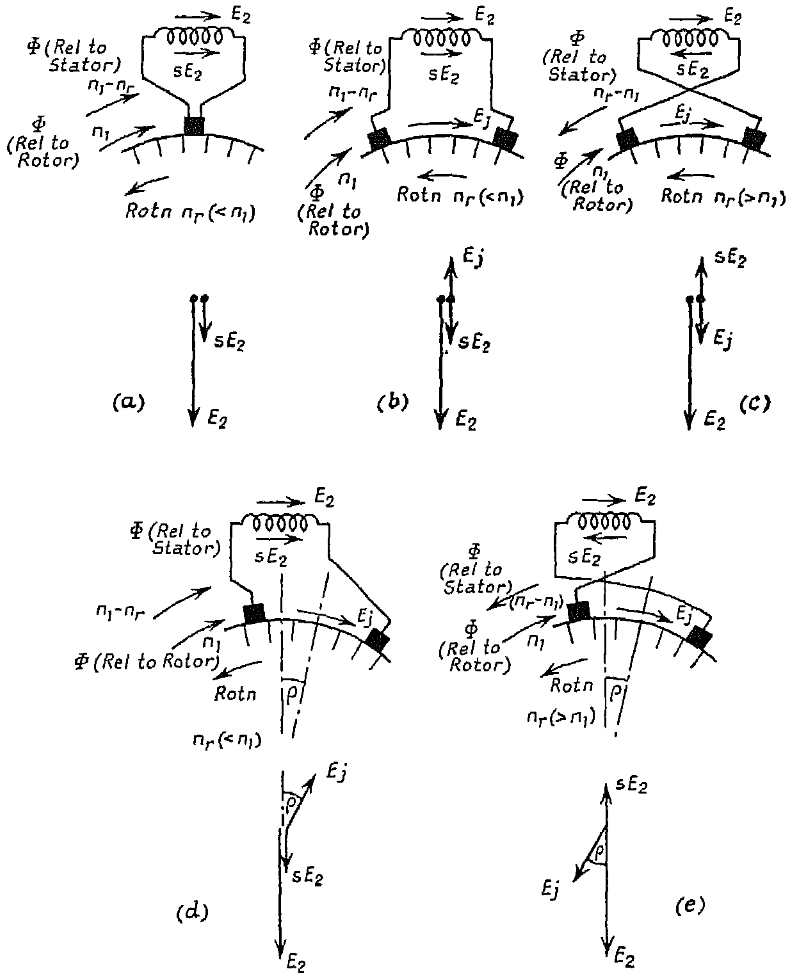


FIG. 9.3. FLUXES AND E.M.F.'S WITH VARIOUS BRUSH POSITIONS

(a) Induction-motor operation.
(b) Sub-synchronous speeds.
(c) Super-synchronous speeds.
(d) Sub-synchronous speeds } with brush shift of ρ against rotation (ρ negative).
(e) Super-synchronous speeds } with brush shift of ρ against rotation (ρ negative).

result of s becoming negative so that a speed above synchronism will be obtained.

A further possibility is to give the brushes, in addition to their separation, a simultaneous movement in the same direction. For speeds below synchronism this is shown in Fig. 9.3(d) where the centre-line of the brush separation has been moved an angle ρ against the direction of rotation (ρ negative). The flux is moving at n_1 r.p.s. relative to the primary conductors and in the opposite direction to the rotation; it thus cuts the group of tertiary conductors between the brushes at a later moment than if ρ were zero as in (b). E_j therefore lags, by the angle ρ, the position it had in (b), i.e. $\beta = 180 - \rho$; it can be seen that this results in a component of E_j leading E_2 by 90° and will give an improved power factor.

If, without altering ρ, the brushes are now reversed to give a speed above synchronism conditions are as in Fig. 9.3(e). The injected e.m.f. again lags the position it previously held with ρ equal to zero (Fig. 9.3(c)) and takes up the position shown—it can be seen that in this case there is a component lagging E_2 by 90° which will tend to give a lagging power factor. In this case the angle of injection, β, is equal to $-\rho$.

The angle ρ is known as the *brush shift* and, in practice, it may be between $-5°$ and $-10°$ but is only useful provided the machine always runs in the same direction. In some cases it is desirable to have different brush shifts at high and low speeds so that the shift as well as the separation is continuously variable.

Mechanical Arrangement of Brush Gear

Apart from the fact that the primary winding is on the rotor and the secondary on the stator, the chief special constructional feature of the Schrage motor is the arrangement of the movable brush gear.

The tertiary winding is, except on small machines, generally lap-wound and three pairs of brushes per pole-pair are employed. All those connected, say, to the starts of the secondary phases are mounted on one brush rocker and those connected to the finishes on another rocker. The two rockers are fitted with toothed racks over a portion of their periphery, the teeth projecting inwards and engaging respectively with the two sides of a pinion mounted between them on a vertical shaft. Variation of brush separation, and therefore control of speed, are obtained by turning the pinion either by means of a hand-wheel or, if remote control is required, by a fractional-horse-power motor. In order to simplify the mechanical construction it is possible, on small motors, to omit a proportion of the brushes if the machine is wave-wound or if equalizing connexions on a lap-wound machine are designed to carry current continuously.

Variation of brush shift (ρ) can be effected by moving one brush rocker faster than the other—two pinions are then required, one for each rocker, having different numbers of teeth.

Small machines sometimes have a link motion for moving the rockers but this becomes rather clumsy when the length of brush travel exceeds a few inches.

Electromotive Forces

The primary and secondary e.m.f.'s E_1 and E_2 are calculated exactly as for an induction motor from—

$$\text{Primary e.m.f. } E_1 = (\sqrt{2})\pi f_1 T_1'\Phi \text{ volts}$$
$$\text{Secondary standstill e.m.f. } E_2 = (\sqrt{2})\pi f_1 T_2'\Phi \text{ volts}$$

TERTIARY E.M.F. The e.m.f. obtained from the commutator winding depends on the brush separation θ, and, from eq. (2.11), page 24, is—

$$E_t = (\sqrt{2})(T_a/a)f_1\Phi \sin(\theta/2) \text{ volts} \qquad (9.1a)$$

where T_a is the total number of turns in the commutator winding.

If T_3 is the number of turns in series between brushes for a brush separation of θ radians

$$T_3 = (T_a/a)(\theta/2\pi)$$
$$\therefore \quad E_t = (\sqrt{2})(2\pi T_3/\theta)f_1\Phi \sin(\theta/2)$$
$$= (\sqrt{2})\pi T_3 f_1\Phi \sin(\theta/2)/(\theta/2)$$
$$= (\sqrt{2})\pi T_3' f_1\Phi \text{ volts} \qquad (9.1b)$$

where $T_3' = T_3 \times$ (distribution factor for spread of θ)
= effective tertiary turns between brushes.

Approximate Speed Characteristics

An approximate idea of the speed characteristics can be obtained by assuming $\rho = 0°$. As this angle rarely exceeds 10° it has very little effect on the speed of the machine.

NO-LOAD SPEED. At no load the secondary current will, if $\rho = 0°$, be very small so that, neglecting impedance drops,

$$sE_2 = E_t$$
$$s = E_t/E_2$$

Hence

$$\text{speed } n_r = n_1(1 - s) = n_1(1 - E_t/E_2)$$
$$= n_1\left\{1 - \frac{(\sqrt{2})(T_a/a)f_1\Phi \sin(\theta/2)}{(\sqrt{2})\pi f_1\Phi T_2'}\right\}$$
$$= n_1\{1 - (T_a/a\pi T_2')\sin(\theta/2)\} \text{ r.p.s.} \qquad (9.2)$$

Taking a particular example of a 12/28-h.p., 500-V, 50-c/s, 6-pole machine having 28 secondary turns per phase and a total of 162 tertiary turns, the no-load speed may be found as follows.

The secondary winding may be assumed to have a phase spread

of 60° ($k_m = 0{\cdot}96$), giving an effective turns T_2' of 27 per phase The tertiary winding may be assumed to be lap-connected ($a = 3$) so that—

$$N_r = 60n_r = 1{,}000\{1 \pm (162/3\pi \times 27) \sin(\theta/2)\} \text{ r.p.m.}$$
$$= 1{,}000\{1 - 0{\cdot}64 \sin(\theta/2)\} \text{ r.p.m.}$$

This curve is plotted in Fig. 9.4(a) and is of similar shape for any size of motor. The usual speed range is between about 50 per cent

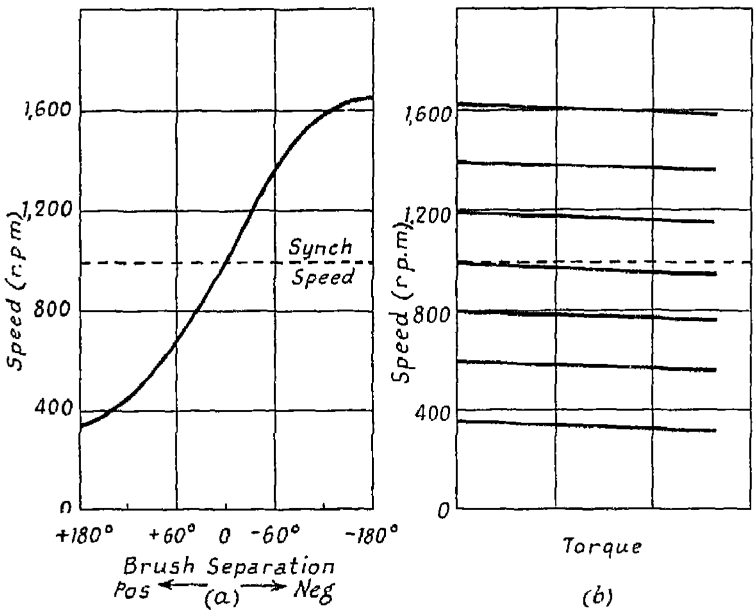


FIG. 9.4. SPEED CHARACTERISTICS
(a) No-load speed.
(b) Speed-torque curves.

below and 50 per cent above synchronism (3 to 1 range), and it can be seen that between these limits the speed is approximately proportional to brush separation.

SPEED-TORQUE CHARACTERISTICS. When, with the brushes adjusted to a particular separation, the machine is loaded, the speed drops causing sE_2 to exceed E_j so that a current flows, limited by the impedance drop of the secondary circuit in accordance with eq. (4.1), page 45, $sE_2 = \pm E_j + I_2 z_{2s}$. The current produces a torque which tends to maintain the speed, and this becomes stable at a value given by

$$s = \pm E_j/E_2 + I_2 z_{2s}/E_2$$
$$= \text{no-load slip} + I_2 z_{2s}/E_2$$

As the latter is small, and assuming torque to be approximately proportional to current, speed-torque curves over the normal working range can be plotted as shown in Fig. 9.4(*b*). The motor thus gives shunt speed characteristics over a range of speed from about 50 per cent below to about 50 per cent above synchronism, although wider ranges can be obtained if desired. Due to the lower secondary-circuit impedance, the slope of the speed-torque curve is usually less than that of a doubly-fed motor of corresponding size.

Slip Power. The slip power appearing at the secondary-winding terminals is transmitted to or from the tertiary commutator winding and is therefore added to or subtracted from the power transmitted directly to the shaft. A constant-horse-power type of drive is thus obtained as described on page 51, the horse-power obtainable for a given primary input power being independent of the speed. High torques are thus obtained at low speeds so that, with the brushes in the low-speed position, starting torques of about twice full-load torque with about $1\frac{1}{2}$ times full-load current are obtainable.

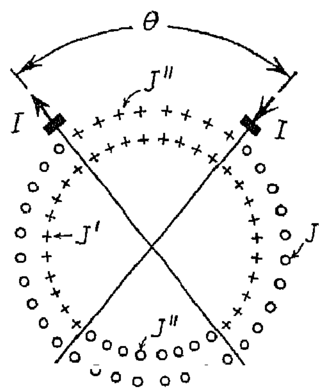


Fig. 9.5. Current Distribution in Tertiary Winding with one Pair of Brushes

The Tertiary Winding

A special feature of the Schrage motor is the tertiary winding, and conditions therein are complicated by the fact that the amount of the winding included in the secondary circuit of the motor varies with the brush separation. The current distribution, m.m.f., and the circuit constants (resistance and leakage reactance) for this winding are thus all functions of θ.

Current Distribution. The distribution can be found by considering each phase separately and superimposing the results. In Fig. 9.5 is shown the distribution for a single phase; if the total secondary current per phase is I_2 the current at any brush will be $I = I_2/a$ and this will divide into two parts J' and J'' as shown. For a machine having three secondary phases, the three sets of conditions shown in Figs. 9.6A, 9.6B and 9.6C must be considered, the winding being divided into zones by lines passing through the brush positions. In Fig. 9.6A the brush separation is less than 60°; in Fig. 9.6B the brush separation begins to overlap into the zones corresponding to the belt of bottom conductors associated with the other phases—this occurs between $\theta = 60°$ and $\theta = 120°$. In Fig. 9.6C, where θ exceeds 120°, the separation overlaps into the zone of the brushes of the adjacent phases.

The diagrams of Fig. 9.6 thus show the current distribution for

TABLE 9.1

CURRENT DISTRIBUTION IN TERTIARY WINDING WITH 3-PHASE SECONDARY: $\theta = 0$–$60°$

Zone	No. of Slots	Top Layer				Bottom Layer			
		Current due to Phase			Total Current	Current due to Phase			Total Current
		A	B	C		A	B	C	
1– 2	$\frac{s}{p}\cdot\frac{\theta}{2\pi}$	J_A''	$-\lambda^2 J_A'$	$-\lambda J_A'$	I_A	J_A'	$\lambda^2 J_A'$	$\lambda J_A'$	0
2– 3	$\frac{s}{6p}\left(1-\frac{3\theta}{\pi}\right)$	$-J_A'$	$-\lambda^2 J_A'$	$-\lambda J_A'$	0	J_A'	$\lambda^2 J_A'$	$\lambda J_A'$	0
3– 4	$\frac{s}{p}\cdot\frac{\theta}{2\pi}$	$-J_A'$	$-\lambda^2 J_A'$	$-\lambda J_A'$	0	J_A'	$\lambda^2 J_A'$	$-\lambda J_A''$	$-\lambda I_A$
4– 5	$\frac{s}{6p}\left(1-\frac{3\theta}{\pi}\right)$	$-J_A'$	$-\lambda^2 J_A'$	$-\lambda J_A'$	0	J_A'	$\lambda^2 J_A'$	$\lambda J_A'$	0
5– 6	$\frac{s}{p}\cdot\frac{\theta}{2\pi}$	$-J_A'$	$\lambda^2 J_A''$	$-\lambda J_A'$	$\lambda^2 I_A$	J_A'	$\lambda^2 J_A'$	$\lambda J_A'$	0
6– 7	$\frac{s}{6p}\left(1-\frac{3\theta}{\pi}\right)$	$-J_A'$	$-\lambda^2 J_A'$	$-\lambda J_A'$	0	J_A'	$\lambda^2 J_A'$	$\lambda J_A'$	0
7– 8	$\frac{s}{p}\cdot\frac{\theta}{2\pi}$	$-J_A'$	$-\lambda^2 J_A'$	$-\lambda J_A'$	0	$-J_A''$	$\lambda^2 J_A'$	$\lambda J_A'$	$-I_A$
8– 9	$\frac{s}{6p}\left(1-\frac{3\theta}{\pi}\right)$	$-J_A'$	$-\lambda^2 J_A'$	$-\lambda J_A'$	0	J_A'	$\lambda^2 J_A'$	$\lambda J_A'$	0
9–10	$\frac{s}{p}\cdot\frac{\theta}{2\pi}$	$-J_A'$	$-\lambda^2 J_A'$	$\lambda J_A''$	λI_A	J_A'	$\lambda^2 J_A'$	$\lambda J_A'$	0
10–11	$\frac{s}{6p}\left(1-\frac{3\theta}{\pi}\right)$	$-J_A'$	$-\lambda^2 J_A'$	$-\lambda J_A'$	0	J_A'	$\lambda^2 J_A'$	$\lambda J_A'$	0
11–12	$\frac{s}{p}\cdot\frac{\theta}{2\pi}$	$-J_A'$	$-\lambda^2 J_A'$	$-\lambda J_A'$	0	J_A'	$-\lambda^2 J_A''$	$\lambda J_A'$	$-\lambda^2 I_A$
12– 1	$\frac{s}{6p}\left(1-\frac{3\theta}{\pi}\right)$	$-J_A'$	$-\lambda^2 J_A'$	$-\lambda J_A'$	0	J_A'	$\lambda^2 J_A'$	$\lambda J_A'$	0

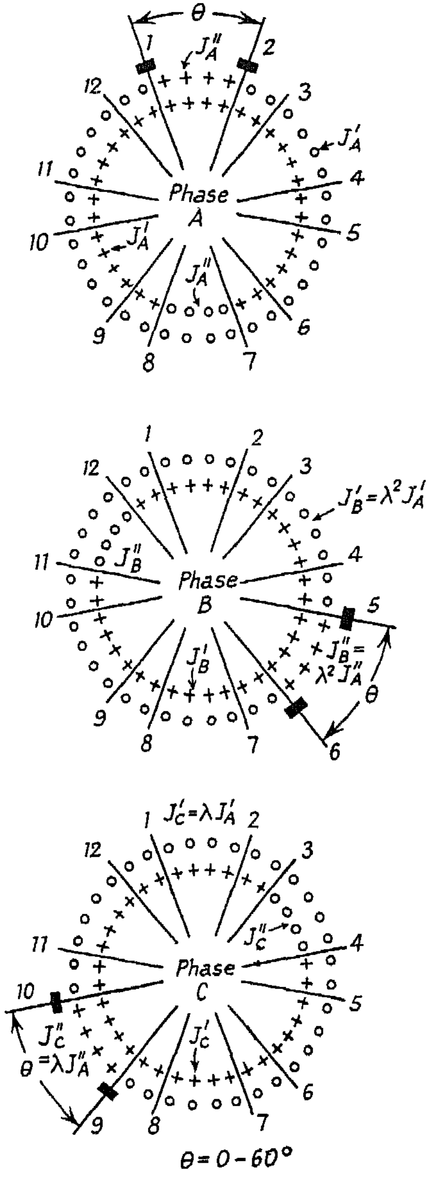


FIG. 9.6A. CURRENT DISTRIBUTION IN TERTIARY WINDING WITH 3-PHASE SECONDARY: $\theta = 0 - 60°$

TABLE 9.2

CURRENT DISTRIBUTION IN TERTIARY WINDING WITH 3-PHASE SECONDARY: $\theta = 60^\circ$–120°

Zone	No. of Slots	Top Layer: Current due to Phase A	Top Layer: Current due to Phase B	Top Layer: Current due to Phase C	Top Layer: Total Current	Bottom Layer: Current due to Phase A	Bottom Layer: Current due to Phase B	Bottom Layer: Current due to Phase C	Bottom Layer: Total Current
1–2	$\frac{s}{6p}\left(\frac{3\theta}{\pi}-1\right)$	J_A''	$-\lambda^2 J_A'$	$-\lambda J_A'$	I_A	J_A'	$\lambda^2 J_A''$	$\lambda J_A'$	$-\lambda^2 I_A$
2–3	$\frac{s}{3p}\left(1-\frac{3\theta}{2\pi}\right)$	J_A''	$-\lambda^2 J_A'$	$-\lambda J_A'$	I_A	J_A'	$\lambda^2 J_A'$	$\lambda J_A'$	0
3–4	$\frac{s}{6p}\left(\frac{3\theta}{\pi}-1\right)$	J_A''	$-\lambda^2 J_A'$	$-\lambda J_A'$	I_A	J_A'	$\lambda^2 J_A'$	$-\lambda J_A'$	$-\lambda I_A$
4–5	$\frac{s}{3p}\left(1-\frac{3\theta}{2\pi}\right)$	$-J_A'$	$-\lambda^2 J_A'$	$-\lambda J_A'$	0	J_A'	$\lambda^2 J_A'$	$-\lambda J_A'$	$-\lambda I_A$
5–6	$\frac{s}{6p}\left(\frac{3\theta}{\pi}-1\right)$	$-J_A'$	$\lambda^2 J_A''$	$-\lambda J_A'$	$\lambda^2 I_A$	J_A'	$\lambda^2 J_A'$	$-\lambda J_A''$	$-\lambda I_A$
6–7	$\frac{s}{3p}\left(1-\frac{3\theta}{2\pi}\right)$	$-J_A'$	$\lambda^2 J_A''$	$-\lambda J_A'$	$\lambda^2 I_A$	J_A'	$\lambda^2 J_A'$	$\lambda J_A'$	0
7–8	$\frac{s}{6p}\left(\frac{3\theta}{\pi}-1\right)$	$-J_A'$	$\lambda^2 J_A''$	$-\lambda J_A'$	$\lambda^2 I_A$	$-J_A''$	$\lambda^2 J_A'$	$\lambda J_A'$	$-I_A$
8–9	$\frac{s}{3p}\left(1-\frac{3\theta}{2\pi}\right)$	$-J_A'$	$-\lambda^2 J_A'$	$-\lambda J_A'$	0	$-J_A'$	$\lambda^2 J_A'$	$\lambda J_A'$	$-I_A$
9–10	$\frac{s}{6p}\left(\frac{3\theta}{\pi}-1\right)$	$-J_A'$	$-\lambda^2 J_A'$	$\lambda J_A''$	λI_A	$-J_A''$	$\lambda^2 J_A'$	$\lambda J_A'$	$-I_A$
10–11	$\frac{s}{3p}\left(1-\frac{3\theta}{2\pi}\right)$	$-J_A'$	$-\lambda^2 J_A'$	$\lambda J_A'$	λI_A	J_A'	$\lambda^2 J_A'$	$\lambda J_A'$	0
11–12	$\frac{s}{6p}\left(\frac{3\theta}{\pi}-1\right)$	$-J_A'$	$-\lambda^2 J_A'$	$\lambda J_A''$	λI_A	J_A'	$-\lambda^2 J_A''$	$\lambda J_A'$	$-\lambda^2 I_A$
12–1	$\frac{s}{3p}\left(1-\frac{3\theta}{2\pi}\right)$	$-J_A'$	$-\lambda^2 J_A'$	$-\lambda J_A'$	0	J_A'	$-\lambda^2 J_A''$	$\lambda J_A'$	$-\lambda^2 I_A$

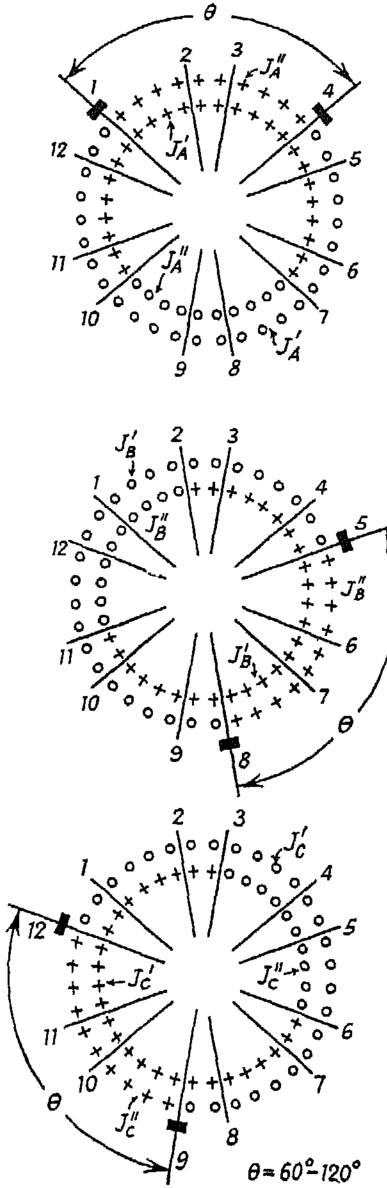


FIG. 9.6B. CURRENT DISTRIBUTION IN TERTIARY WINDING WITH 3-PHASE SECONDARY: $\theta = 60° - 120°$

Table 9.3

Current Distribution in Tertiary Winding with 3-phase Secondary: $\theta = 120$ –180

Zone	No. of Slots	Top Layer: Current due to Phase A	Top Layer: Current due to Phase B	Top Layer: Current due to Phase C	Top Layer: Total Current	Bottom Layer: Current due to Phase A	Bottom Layer: Current due to Phase B	Bottom Layer: Current due to Phase C	Bottom Layer: Total Current
1– 2	$\frac{s}{p}\left(\frac{\theta}{2\pi}-\frac{1}{3}\right)$	J_A''	$-\lambda^2 J_A'$	$\lambda J_A''$	$-\lambda^2 I_A$	J_A'	$-\lambda^2 J_A''$	$\lambda J_A'$	$-\lambda^2 I_A$
2– 3	$\frac{s}{p}\left(\frac{1}{2}-\frac{\theta}{2\pi}\right)$	J_A''	$-\lambda^2 J_A'$	$-\lambda J_A'$	I_A	J_A'	$-\lambda^2 J_A''$	$\lambda J_A'$	$-\lambda^2 I_A$
3– 4	$\frac{s}{p}\left(\frac{\theta}{2\pi}-\frac{1}{3}\right)$	J_A''	$-\lambda^2 J_A'$	$-\lambda J_A'$	I_A	J_A'	$-\lambda^2 J_A''$	$-\lambda J_A''$	I_A
4– 5	$\frac{s}{p}\left(\frac{1}{2}-\frac{\theta}{2\pi}\right)$	J_A''	$-\lambda^2 J_A'$	$-\lambda J_A'$	I_A	J_A'	$\lambda^2 J_A'$	$-\lambda J_A''$	$-\lambda I_A$
5– 6	$\frac{s}{p}\left(\frac{\theta}{2\pi}-\frac{1}{3}\right)$	J_A''	$\lambda^2 J_A''$	$-\lambda J_A'$	$-\lambda I_A$	J_A'	$\lambda^2 J_A'$	$-\lambda J_A''$	$-\lambda I_A$
6– 7	$\frac{s}{p}\left(\frac{1}{2}-\frac{\theta}{2\pi}\right)$	$-J_A'$	$\lambda^2 J_A''$	$-\lambda J_A'$	$\lambda^2 I_A$	J_A'	$\lambda^2 J_A'$	$-\lambda J_A''$	$-\lambda I_A$
7– 8	$\frac{s}{p}\left(\frac{\theta}{2\pi}-\frac{1}{3}\right)$	$-J_A'$	$\lambda^2 J_A''$	$-\lambda J_A'$	$\lambda^2 I_A$	$-J_A''$	$\lambda^2 J_A'$	$-\lambda J_A''$	$\lambda^2 I_A$
8– 9	$\frac{s}{p}\left(\frac{1}{2}-\frac{\theta}{2\pi}\right)$	$-J_A'$	$\lambda^2 J_A''$	$-\lambda J_A'$	$\lambda^2 I_A$	$-J_A''$	$\lambda^2 J_A'$	$\lambda J_A'$	$-I_A$
9–10	$\frac{s}{p}\left(\frac{\theta}{2\pi}-\frac{1}{3}\right)$	$-J_A'$	$\lambda^2 J_A''$	$\lambda J_A''$	$-I_A$	$-J_A''$	$\lambda^2 J_A'$	$\lambda J_A'$	$-I_A$
10–11	$\frac{s}{p}\left(\frac{1}{2}-\frac{\theta}{2\pi}\right)$	$-J_A'$	$-\lambda^2 J_A'$	$\lambda J_A''$	λI_A	$-J_A''$	$\lambda^2 J_A'$	$\lambda J_A'$	$-I_A$
11–12	$\frac{s}{p}\left(\frac{\theta}{2\pi}-\frac{1}{3}\right)$	$-J_A'$	$-\lambda^2 J_A'$	$\lambda J_A''$	λI_A	$-J_A''$	$-\lambda^2 J_A''$	$\lambda J_A'$	λI_A
12– 1	$\frac{s}{p}\left(\frac{1}{2}-\frac{\theta}{2\pi}\right)$	$-J_A'$	$-\lambda^2 J_A'$	$\lambda J_A''$	λI_A	J_A'	$-\lambda^2 J_A''$	$\lambda J_A'$	$-\lambda^2 I_A$

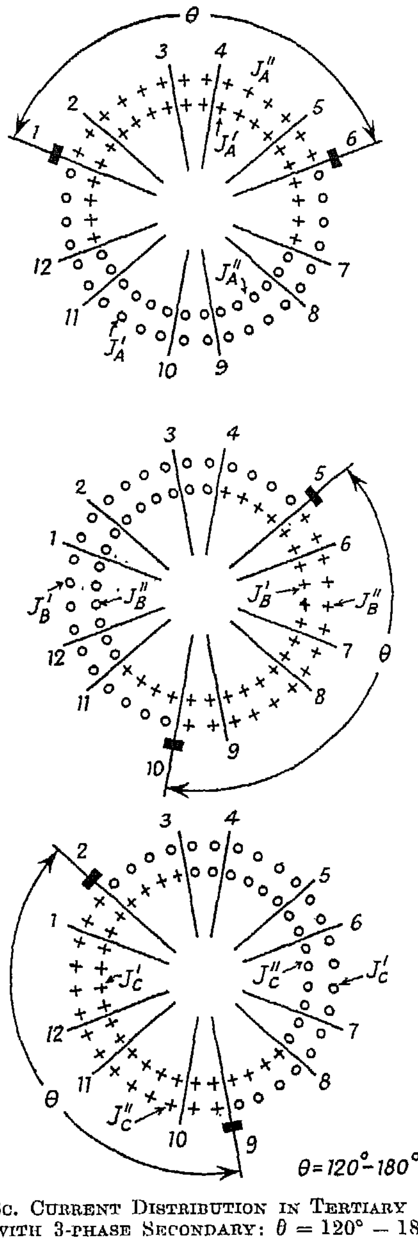


FIG. 9.6C. CURRENT DISTRIBUTION IN TERTIARY WINDING WITH 3-PHASE SECONDARY: $\theta = 120° - 180°$

each of the three phases for each of the above conditions. By writing down the currents in each zone in tabular form and adding them the total current per conductor in any group of conductors can be found. Thus, considering the separation to be between 0 and 60°—in zone 1–2 the current in the top layer is J_A'' due to phase A, $-J_B'(=-\lambda^2 J_A')$* due to phase B and $-J_C'(=-\lambda J_A')$ due to phase C. The total current per conductor in the top-layer conductors of zone 1–2 is thus

$$J_A'' - \lambda^2 J_A' - \lambda J_A' = J_A'' - J_A'(\lambda^2 + \lambda) = J_A'' + J_A'$$
$$= I$$

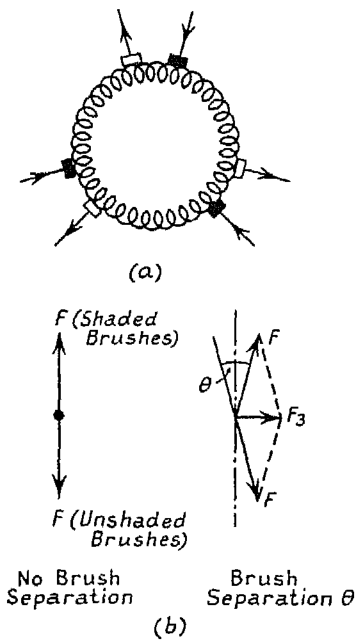


FIG. 9.7. M.M.F. OF TERTIARY WINDING
(a) Currents.
(b) M.m.f. vectors.

In zone 2–3 the currents in the top layer are seen to be $-J_A'$, $-J_B'(=-\lambda^2 J_A')$ and $-J_C'$ $(=-\lambda J_A')$. Adding these gives zero. Similar analyses can be made for each zone for both top and bottom conductors as shown in Table 9.1. The conditions when θ is between 60° and 120° and between 120° and 180° can also be similarly investigated and are shown in Tables 9.2 and 9.3. In all the tables the number of slots in each zone is also shown.

MAGNETOMOTIVE FORCE. From the instantaneous values of the conductor currents calculated from the above tables the m.m.f. per slot can easily be found.

Using these values the m.m.f. wave can be drawn by the step-by-step method in the usual way.

An alternative way of finding the m.m.f. wave is to consider the current distribution to be made up of two three-phase systems of currents as shown in Fig. 9.7(a). One system enters and leaves the winding by the brushes shaded in black and the other, with 180° phase displacement, by the unshaded brushes. Each of these three-phase systems of currents may be said to give rise to a rotating m.m.f. wave as described on page 9, the magnitude of the fundamental being

$$F = \{(\sqrt{6})/\pi\} k_{m120} I_b T = \{3(\sqrt{2})/2\pi^2\} I_b (T_a/a)$$

If there is no brush separation the two m.m.f.'s exactly neutralize

* $\lambda = 120°$ operator.

each other giving, as would be expected, no m.m.f. for the winding, but with a separation θ the conditions may be represented by the space vector diagram of Fig. 9.7(*b*). The fundamental m.m.f. of the winding when carrying brush currents I_b is thus

$$F_3 = 2F \sin(\theta/2)$$

It is sometimes convenient to express this in terms of the effective number of turns in series between brushes of one phase. If T_3 is the number of turns in series,*

$$T_3 = (T_a/a)(\theta/2\pi)$$

$$\therefore \quad F_3 = 2\{3(\sqrt{2})/2\pi^2\}I_b \,.\, T_3(2\pi/\theta)\sin(\theta/2)$$

$$= \frac{3\sqrt{2}}{\pi}\left\{\frac{\sin(\theta/2)}{\theta/2}\right\} I_b T_3$$

$$= \{3(\sqrt{2})/\pi\}k_{m\theta}I_b T_3 = \{3(\sqrt{2})/\pi\}I_b T_3' \quad . \quad . \quad (9.3)$$

By reference to page 9 it can be seen that when the current in a particular secondary phase is at its peak value this m.m.f. acts along an axis at 90° to the centre-line of the brush separation.

It may be noted that the m.m.f. expression for the tertiary winding (9.3) is similar in form to that for an ordinary three-phase winding, e.g. the primary and secondary windings (page 6).

Effective Resistance. The effective resistance can be found by determining the total loss due to the above currents. Let r_c be the resistance per conductor, z_s the conductors per slot and I_2 the total secondary current; the current per brush will thus be I_2/a. Considering the case when θ is less than 60°, the total number of conductors carrying the current I_2/a is $6S(z_s/2)(\theta/2\pi)$. Hence the total loss is

$$6(I_2^2/a^2)r_c \,.\, S(z_s/2)(\theta/2\pi)$$

The effective resistance may conveniently be compared with r_d, the resistance between diametrically-spaced brushes, as in the ordinary d.c. machine.

$$r_d = r_c \times Sz_s/(2a)^2$$

Hence the total loss may be written—

$$\text{Loss} = \frac{I_2^2}{a^2}\frac{4a^2}{Sz_s}r_d \times \frac{6Sz_s\theta}{2 \times 2\pi} = 3I_2^2 r_d \frac{2\theta}{\pi}$$

and the loss per phase $= I_2^2 r_d(2\theta/\pi) = I_2^2 r_3$.

The effective resistance per phase is thus

$$r_3 = r_d(2\theta/\pi)$$

* This assumes the usual lap winding with $a = p$; the effective turns per pole-pair setting up the m.m.f. wave will be T_3' divided by p for a simple wave winding or multiplied by a/p for a multiplex winding.

Similar calculations can be made for the cases where θ is greater than 60°, and greater than 120°. These give—

θ between 60° and 120°: $r_3 = r_d(2\theta/\pi)$
θ between 120° and 180°: $r_3 = r_d(4/3)$

It may be noted that when θ is greater than 120°, i.e. when the brushes begin to overlap, the resistance is constant. Curves of resistance to a base of brush separation are shown in Fig. 9.8 for a three-phase armature.

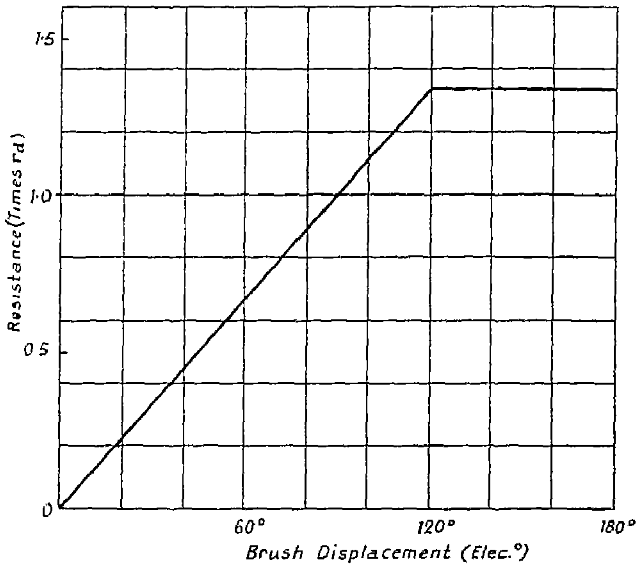


FIG. 9.8. VARIATION OF TERTIARY WINDING RESISTANCE WITH BRUSH DISPLACEMENT

The leakage reactance of the tertiary winding is closely associated with that of the primary winding, since it is in the same slots, and is discussed on page 145.

Resultant M.M.F. Due to All Windings

The resultant m.m.f. producing the main air-gap flux is due to the combined m.m.f.'s of the primary, secondary and tertiary windings. In what follows it is assumed that these are all sinusoidally distributed.

If the machine is running as an induction motor with no part of the tertiary winding in circuit then, neglecting magnetizing m.m.f., $F_1 = -F_2$, and the space vector diagram of these m.m.f.'s is as shown in Fig. 9.9(a).

At sub-synchronous speeds, with no brush shift, the primary

input current will, assuming the same torque, be less than when the machine is running as an induction motor, since the output is lower at the lower speed. The tertiary m.m.f. must therefore oppose

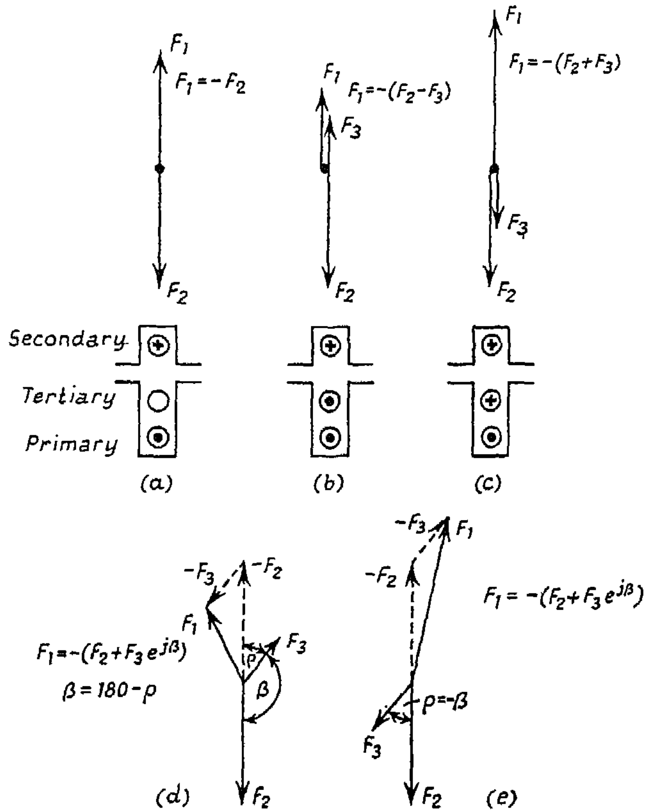


FIG. 9.9. SPACE VECTOR DIAGRAMS OF M.M.F.'s

(a) Induction motor.
(b) Sub-synchronous speed, $\rho = 0°$.
(c) Super-synchronous speed, $\rho = 0°$.
(d) Sub-synchronous speed, shift $= -\rho$.
(e) Super-synchronous speed, shift $= -\rho$.

the secondary m.m.f. to bring this about and conditions are as shown in Fig. 9.9(b) where $F_1 = -(F_2 - F_3)$. Similarly at super-synchronous speeds the primary current must increase for the same torque and conditions are as in Fig. 9.9(c) where $F_1 = -(F_2 + F_3)$.

If the brushes are given a shift ρ against the direction of rotation the space vector diagrams for sub-synchronous and super-

synchronous speeds become as shown in Fig. 9.9(*d*) and (*e*). The primary m.m.f. can thus be expressed by the general vector relation—

$$\mathbf{F}_1 = -(\mathbf{F}_2 + \mathbf{F}_3)$$
$$= -(F_2 + F_3 e^{j\beta})$$

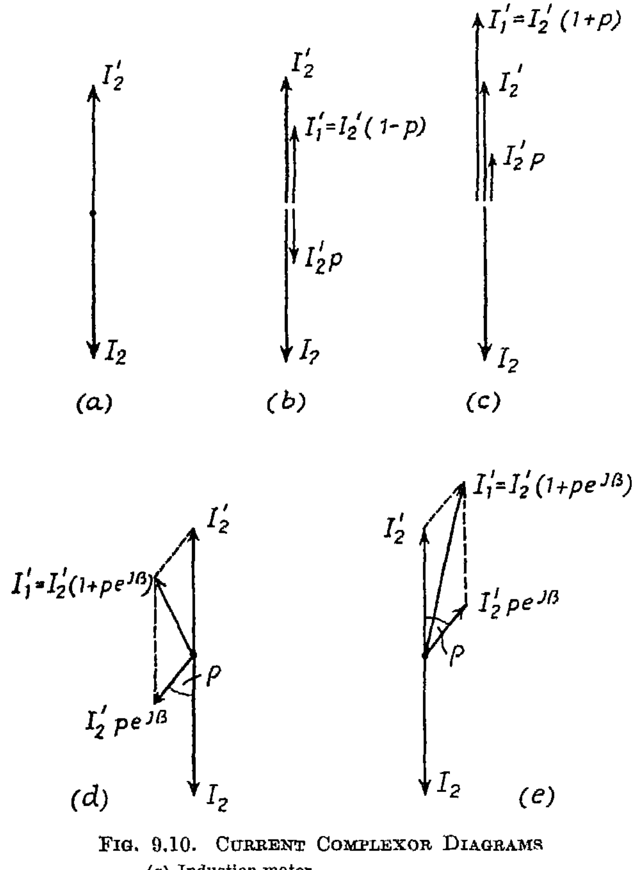


FIG. 9.10. CURRENT COMPLEXOR DIAGRAMS

(*a*) Induction motor.
(*b*) Sub-synchronous speed.
(*c*) Super-synchronous speed.
(*d*) Sub-synchronous speed, shift $= -\rho$.
(*e*) Super-synchronous speed, shift $= -\rho$.

where $\beta = 180 - \rho$ for sub-synchronous speeds and $\beta = -\rho$ for super-synchronous speeds.

CURRENT COMPLEXOR DIAGRAM. The expressions for the primary, secondary and tertiary m.m.f.'s in terms of the corresponding

currents can be substituted into the above general expression giving—

$$\frac{3\sqrt{2}}{\pi}\mathbf{I}_1'T_1' = -\left(\frac{3\sqrt{2}}{\pi}\mathbf{I}_2T_2' + \frac{3\sqrt{2}}{\pi}\mathbf{I}_2T_3'\,e^{j\beta}\right)$$

$$\therefore \quad \mathbf{I}_1' = -\{\mathbf{I}_2(T_2'/T_1') + \mathbf{I}_2(T_3'/T_1')\,e^{j\beta}\}$$
$$= -\{\mathbf{I}_2' + \mathbf{I}_2'(T_3'/T_2')\,e^{j\beta}\}$$
$$= -\mathbf{I}_2'(1 + \mathbf{p}) \qquad . \quad . \quad . \quad . \quad . \quad . \quad (9.4a)$$

where $\mathbf{p} = (T_3'/T_2')\,e^{j\beta}$ and is a complex quantity.

Where there is no brush shift this becomes

$$I_1' = -I_2'(1 \pm p) \qquad . \quad . \quad . \quad (9.4b)$$

where $p = T_3'/T_2'$; the negative sign refers to sub-synchronous speeds and the positive sign to super-synchronous speeds.

The current complexor diagrams for the cases of Fig. 9.9 are shown in Fig. 9.10. It can be seen that the effect of giving the brushes a shift ρ against the direction of rotation is to tend to improve the primary power factor at sub-synchronous speeds and to make it more lagging at super-synchronous speeds.

Leakage Fluxes and Reactance E.M.F.'s

As there are two windings in the rotor slots the disposition of the leakage fluxes and the determination of the leakage reactances is more complicated than in the ordinary induction motor.

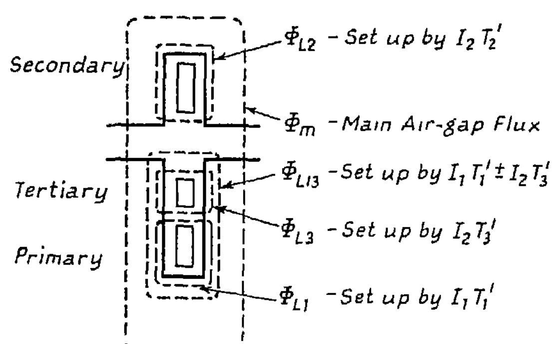


FIG. 9.11. LEAKAGE FLUXES

LEAKAGE FLUXES. The fluxes in the slot portion of the conductor may be represented as in Fig. 9.11. In the overhang, conditions are even more complicated but, to a first approximation, they may be treated similarly to those in the slot.

In practice Φ_{L3} and Φ_{L1} are small and may usually be neglected so that in the rotor the leakage flux Φ_{L13} linking both primary and tertiary windings is the only one that need be considered.

SECONDARY LEAKAGE REACTANCE. The secondary leakage reactance x_{L2} can be calculated in the usual way* from

$$\Phi_{L2} = (\sqrt{2})\Lambda_2 I_2 T_2' \text{ webers}$$

where Λ_2 is the permeance of the secondary leakage flux path in weber/ampere-turn.

Thus

$$x_{L2} = \omega\Phi_{L2}T_2'/I_2 = \omega(\sqrt{2})\Lambda_2(T_2')^2$$

PRIMARY LEAKAGE REACTANCE. If there were no secondary current the leakage flux associated with the primary winding would be produced by I_1T_1' and the primary leakage reactance would be

$$x_{L1} = \omega(\sqrt{2})\Lambda_1(T_1')^2$$

where Λ_1 is the permeance of the flux path for Φ_{L13}.

Since, however, the leakage flux associated with the primary winding is set up by the combined action of the primary and tertiary m.m.f.'s it may be written, neglecting magnetizing current (so that $I_1' = I_1$) and neglecting brush shift,

$$\Phi_{L13} = (\sqrt{2})\Lambda_1(I_1T_1' \pm I_2T_3')$$

Reference to Fig. 9.9 shows that the currents in the slot are in the same direction (positive sign) for sub-synchronous speeds and in opposite directions (negative sign) for super-synchronous speeds.

The e.m.f. induced in the primary winding by this flux is given by—

$$E_{L1} = \mathrm{j}\omega\Phi_{L13}T_1' = \mathrm{j}\omega(\sqrt{2})\Lambda_1T_1'(I_1T_1' \pm I_2T_3')$$

If brush shift is allowed for this becomes

$$\begin{aligned} E_{L1} &= \mathrm{j}\omega(\sqrt{2})\Lambda_1T_1'(I_1T_1' - I_2T_3'\mathrm{e}^{\mathrm{j}\beta}) \\ &= \mathrm{j}\omega(\sqrt{2})\Lambda_1(T_1')^2\{I_1 - I_2(T_3'/T_1')\mathrm{e}^{\mathrm{j}\beta}\} \\ &= \mathrm{j}x_{L1}\{I_1 - I_2'(T_1'/T_2')(T_3'/T_1')\mathrm{e}^{\mathrm{j}\beta}\} \\ &= \mathrm{j}x_{L1}(I_1 - I_2'\mathrm{p}) \qquad . \quad . \quad . \quad . \quad (9.5a) \end{aligned}$$

where $\mathrm{p} = (T_3'/T_2')\mathrm{e}^{\mathrm{j}\beta}$.

Remembering that $I_2'/I_1' = 1/(1 + \mathrm{p})$, this may be written

$$\begin{aligned} E_{L1} &= \mathrm{j}x_{L1}I_1\{1 - (I_2'/I_1)\mathrm{p}\} \\ &= \mathrm{j}x_{L1}I_1\{1 - \mathrm{p}/(1 + \mathrm{p})\} \\ &= \mathrm{j}x_{L1}I_1\{1/(1 + \mathrm{p})\} \qquad . \quad . \quad . \quad (9.5b) \end{aligned}$$

* Say, *The Performance and Design of A.C. Machines*, pp. 185–195 (Pitman).

The effect of the tertiary winding is thus to change the effective primary reactance from x_1 to $x_1/(1 + \mathbf{p})$. With no brush shift this becomes $x_1/(1 \pm p)$ where p is positive for super-synchronous and negative for sub-synchronous speeds.

TERTIARY LEAKAGE REACTANCE. The e.m.f. induced in the tertiary winding is

$$\begin{aligned} E_{L3} &= E_{L1} \times (T_3'/T_1') \\ &= \mathrm{j}x_1(I_1 - I_2'p)T_3'/T_1' \end{aligned}$$

Again neglecting magnetizing current,

$$\begin{aligned} E_{L3} &= \mathrm{j}x_1\{I_2'(1+p) - I_2'p\}T_3'/T_1' \\ &= \mathrm{j}x_1I_2'(T_3'/T_1') \\ &= \mathrm{j}x_1I_2(T_2'/T_1')(T_3'/T_1') \quad . \quad . \quad . \quad (9.6) \end{aligned}$$

The apparent reactance of the tertiary winding is thus

$$x_1(T_2'/T_1')^2(T_3'/T_2') = (x_1 \text{ referred to secondary})\ (T_3'/T_2')$$

The Complexor Diagram

The complexor diagram cannot be drawn in a logical sequence as with, say, the induction motor because the direction of the injected e.m.f. complexor, which governs the secondary current, and therefore also the primary current, is itself, to some extent, governed by the primary current.

MOTOR WITH NO BRUSH SHIFT ($\rho = 0$). Diagrams for the motor running above and below synchronous speed are given in Fig. 9.12. The main air-gap flux is represented by Φ as in the induction motor. The primary and secondary e.m.f.'s E_1 and E_2 lag the flux by 90° but are not shown on the diagram although $-E_1$ is given leading Φ by 90°. Below synchronous speed s is positive so that sE_2 lags Φ by 90° while above synchronous speed s is negative and sE_2 leads Φ by 90°.

The tertiary injected e.m.f. is made up of two parts, E_j produced by the main flux and another component E_{L3} produced by the leakage flux Φ_{L13} giving a total of E_{jt}. Below synchronous speed E_j leads Φ by 90° ($\beta = 180°$) and above synchronous speeds it lags Φ by 90° ($\beta = 0°$).

The resultant of E_{jt} and sE_2 overcomes the secondary-circuit impedance drop I_2z_{2s}. The impedance z_{2s} comprises the whole secondary-circuit resistance (secondary and tertiary winding and brush resistances) and the leakage reactance of the secondary winding only (sx_2), the effect of the tertiary winding reactance having already been allowed for in E_{L3}.

At *sub-synchronous speeds* the effect of E_{L3} is to cause I_2z_{2s} to lag E_2; as s is positive I_2 also lags I_2z_{2s} so that the power factor of the secondary current tends to be poor.

The current

$$I_1' = I_2' + I_2'\mathbf{p} = I_2' - I_2'p \text{ (since } \mathbf{p} = p\mathrm{e}^{-180} = -p)$$

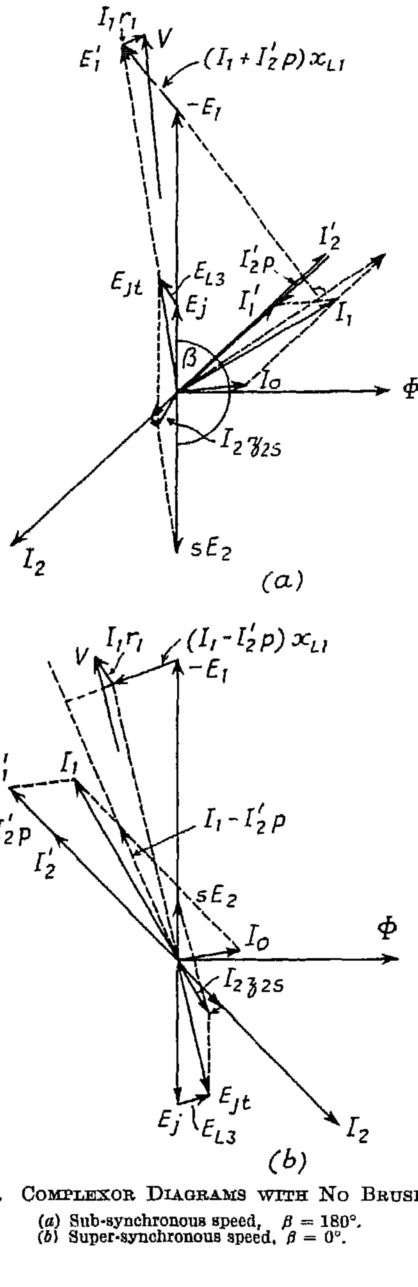


FIG. 9.12. COMPLEXOR DIAGRAMS WITH NO BRUSH SHIFT

(a) Sub-synchronous speed, $\beta = 180°$.
(b) Super-synchronous speed, $\beta = 0°$.

and it can be drawn as shown in Fig. 9.10(b). Adding the no-load current I_0 gives the primary current I_1.

The effective current producing the primary leakage flux is $I_1 - I_2'\mathrm{p}$ (eq. (9.5a), page 146). For sub-synchronous speeds and $\beta = 180°$ ($\mathrm{p} = -p$) this becomes $I_1 + I_2'p$ as shown dotted. The primary leakage e.m.f. is at 90° to this. Adding this to $-E_1$ gives E_1', the e.m.f. induced in the primary winding by the combined main rotor leakage fluxes. Since this same combined leakage flux is also and linked with the tertiary winding it also induces $E_{,t}$ so that $E_1'/E_{,t} = T_1'/T_3'$ and also $(I_1 + I_2'p)x_{L1}$ must be parallel to E_{L3}. Adding the primary resistance drop I_1r_1 to E_1' gives the terminal voltage V and makes the diagram complete.

At *super-synchronous* speeds $E_,$ is vertically downwards and sE_2 upwards (s negative) so that E_{L3} is similarly reversed as shown in Fig. 9.12(b), and I_2z_{2s} leads E_2 as shown in Fig. 9.12(b). The secondary reactance is negative, i.e. it acts as a capacitive reactance since s is negative, so that I_2 leads I_2z_{2s}; the secondary power factor is thus leading. The rest of the diagram is drawn similarly to that for sub-synchronous speeds.

The two diagrams illustrate the tendency of the motor to run at lagging power factors at sub-synchronous speeds and at high or leading power factors at super-synchronous speeds.

MOTOR WITH BRUSH SHIFT ρ. If the brushes are shifted against the direction of rotation by an angle ρ as described on page 130 the complexor diagrams for sub- and super-synchronous speeds are as shown in Fig. 9.13(a) and (b), the current complexors being as given in Fig. 9.10.

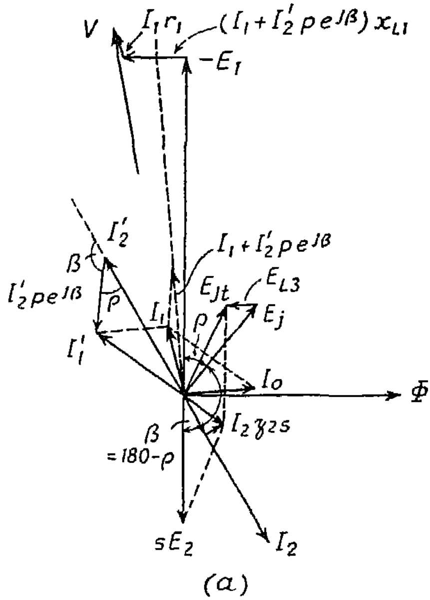
It can be seen that the effect of the brush shift is to make the power factor approximately unity at low speeds and lagging at high speeds. It is found, in practice, that if a satisfactory low-speed power factor is obtained by brush shifting the high-speed power factor may be excessively lagging; it is therefore desirable to make the brush shift variable between, say, 8° negative at low speeds and about zero or slightly positive at high speeds. This can be done by suitable gearing so that the rocker whose movement is against the direction of rotation moves more rapidly than the other.

Machines which may have to run in either direction must, of course, have their brushes in the neutral position.

Power Relations

From the complexor diagram the relations between the stator, rotor and mechanical powers can be obtained as was done for the doubly-fed motor on page 109. The relevant part of the complexor diagram is drawn to a larger scale in Fig. 9.14, the e.m.f. induced in the tertiary by the rotor leakage flux being neglected.

$$\text{Stator input } (P) = VI_1 \cos\phi_1$$
$$= E_1I_1 \cos\psi_1 + I_1^2r_1$$



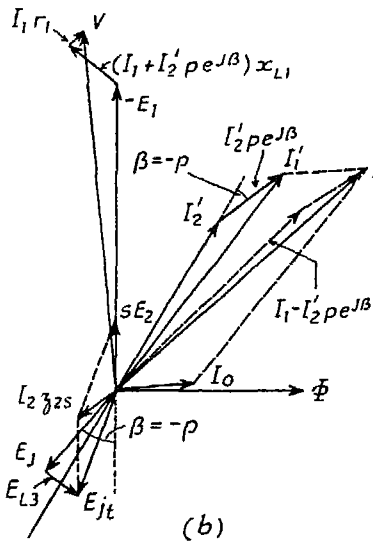


FIG. 9.13. COMPLEXOR DIAGRAMS WITH BRUSH SHIFT OF $-\rho$ (AGAINST ROTATION)

(a) Sub-synchronous speed.
(b) Super-synchronous speed.

Resolving the primary current complexors on to E_1—

$$I_1 \cos \psi_1 = I_0 \sin \alpha + I_2' \cos \psi_2 - I_2'p \cos (180 - \beta + \psi_2)$$
$$= I_0 \sin \alpha + I_2' \cos \psi_2 - I_2'p \cos \phi_2$$

Thus

$$P = E_1 I_0 \sin \alpha + E_1 I_2' \cos \psi_2 - E_1 I_2' p \cos \phi_2 + I_1^2 r_1 \quad . \qquad (9.7)$$

Considering the secondary circuit and resolving e.m.f.'s on to I_2—

$$sE_2 \cos \psi_2 - E_j \cos (180 - \beta + \psi_2) = I_2 r_2$$

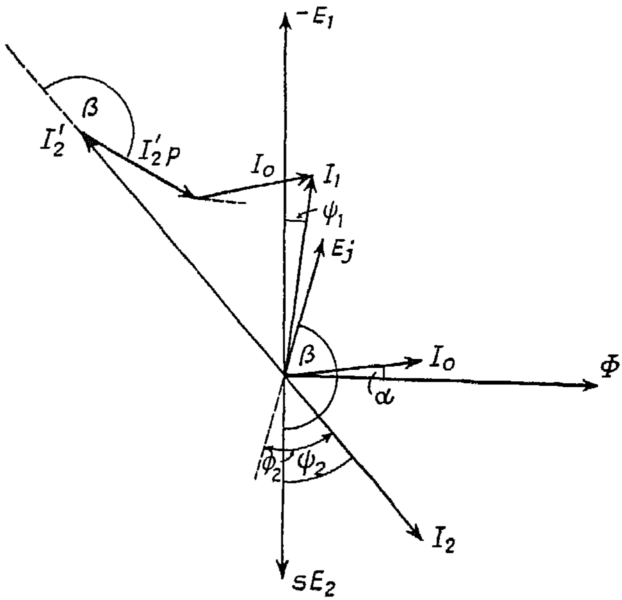


FIG. 9.14. COMPLEXOR DIAGRAM FOR POWER RELATIONS

Referring to primary,

$$sE_1 \cos \psi_2 - E_1 p \cos (180 - \beta + \psi_2) = I_2' r_2'$$

$$\therefore \qquad sE_1 I_2' \cos \psi_2 - E_1 p I_2' \cos \phi_2 = (I_2')^2 r_2' \quad . \qquad (9.8)$$

Adding expressions (9.7) and (9.8)—

$$P = E_1 I_0 \sin \alpha + (1 - s) E_1 I_2' \cos \psi_2 + (I_1)^2 r_1 + (I_2')^2 r_2'$$

i.e. stator input = iron loss + mechanical output + primary and secondary copper losses

Neglecting losses, the primary input power is thus equal only to the mechanical output. This may be compared to the doubly-fed

motor in which the stator has also to carry the power returned to the supply through the regulator when the motor is running at sub-synchronous speeds.

For a given stator input and a variable speed the torque thus decreases with an increase of speed—this corresponds to the constant-h.p. drive referred to on page 51.

Equivalent Circuit

From the complexor diagram it is possible to write down general equations regarding the behaviour of the motor similar to those of eq. (4.7), page 55. Solving these for the primary current I_1 in terms of the applied voltage V gives an expression for the impedance of the motor in terms of the various motor constants and the slip; from this expression an equivalent circuit can be developed.

Circuit Equations. Primary circuit voltages—

$$\mathbf{V} = \mathbf{I}_1 r_1 + \mathrm{j}x_{L1}(\mathbf{I}_1 - \mathbf{p}\mathbf{I}_2') + \mathbf{E}_1 \quad . \quad (9.9a)$$

where $\mathbf{p} = (T_3'/T_2')e^{j\beta}$.

Primary circuit currents—

$$\mathbf{I}_1 = \mathbf{I}_0 + \mathbf{I}_2'(1 + \mathbf{p}) \quad . \quad . \quad . \quad (9.9b)$$

Magnetizing circuit—

$$\mathbf{E}_1 = \mathbf{I}_0(r_0 + \mathrm{j}x_0) = \mathbf{I}_0\mathbf{z}_0 \quad . \quad . \quad . \quad (9.9c)$$

where r_0 and x_0 are the magnetizing resistance and reactance (equivalent-series circuit).

Secondary circuit—

$$s\mathbf{E}_2 = \mathbf{I}_2 r_2 + \mathrm{j}\mathbf{I}_2 s x_{L2} - \mathrm{j}x_{L1}\mathbf{p}(\mathbf{I}_1 - \mathbf{p}\mathbf{I}_2')(T_2'/T_1') - \mathbf{p}\mathbf{E}_2 \quad (9.9d)$$

Substituting for $\mathbf{I}_2'$ from (9.9b) in (9.9a),

$$\mathbf{V} = \mathbf{I}_1 r_1 + \mathrm{j}x_{L1}\mathbf{I}_1 - \mathrm{j}\mathbf{p}x_{L1}(\mathbf{I}_1 - \mathbf{I}_0)/(1 + \mathbf{p}) + \mathbf{E}_1$$

Since $\mathbf{I}_0 = \mathbf{E}_1/\mathbf{z}_0$ this becomes

$$\mathbf{V} = \mathbf{I}_1\left(r_1 + \mathrm{j}\,\frac{x_{L1}}{1 + \mathbf{p}}\right) + \mathrm{j}\,\frac{\mathbf{E}_1 x_{L1}}{\mathbf{z}_0} \cdot \frac{\mathbf{p}}{1 + \mathbf{p}} + \mathbf{E}_1$$

$$= \mathbf{I}_1\{r_1 + \mathrm{j}\,x_{L1}/(1 + \mathbf{p})\} + \mathbf{E}_1 \cdot \gamma \quad . \quad . \quad . \quad (9.10)$$

where $\gamma = 1 + \mathrm{j}\,\dfrac{x_{L1}}{\mathbf{z}_0}\,\dfrac{\mathbf{p}}{1 + \mathbf{p}}$.

Rewriting (9.9d) with $\mathbf{E}_2 = \mathbf{E}_1(T_2'/T_1')$ and $\mathbf{I}_2 = \mathbf{I}_2'(T_1'/T_2')$,

$$+ \mathbf{p})\mathbf{E}_1 \frac{T_2'}{T_1'} = \mathbf{I}_2' \frac{T_1'}{T_2'}(r_2 + \mathrm{j}sx_{L2}) - \mathrm{j}x_{L1}\mathbf{p}(\mathbf{I}_1 - \mathbf{p}\mathbf{I}_2')\frac{T_2'}{T_1'}$$

$$= \mathbf{I}_2' \frac{T_1'}{T_2'}(r_2 + \mathrm{j}sx_{L2}) - \mathrm{j}x_{L1}\mathbf{p}\{\mathbf{I}_0 + \mathbf{I}_2'(1 + \mathbf{p}) - \mathbf{p}\mathbf{I}_2'\}\frac{T_2'}{T_1'}$$

$$= \mathbf{I}_2' \frac{T_1'}{T_2'}(r_2 + \mathrm{j}sx_{L2}) - \mathrm{j}x_{L1}\mathbf{p}\,\frac{\mathbf{E}_1}{\mathbf{z}_0}\,\frac{T_2'}{T_1'} - \mathrm{j}x_{L1}\mathbf{p}\mathbf{I}_2'\,\frac{T_2'}{T_1'}$$

Referring r_2 and x_{L2} to the primary, by multiplying them by $(T_2'/T_1')^2$,

$$(s + \mathbf{p})\mathbf{E}_1 = \mathbf{I}_2'(r_2' + \mathrm{j}sx_{L2}') - \mathrm{j}x_{L1}\mathbf{p}(\mathbf{E}_1/\mathbf{z}_0) - \mathrm{j}x_{L1}\mathbf{p}\mathbf{I}_2'$$
$$\{s + \mathbf{p} + \mathrm{j}\mathbf{p}(x_{L1}/\mathbf{z}_0)\}\mathbf{E}_1 = \mathbf{I}_2'\{r_2' + \mathrm{j}(sx_{L2}' - \mathbf{p}x_{L1})\}$$

Substituting for $\mathbf{I}_2'$ in (9.9b) from this,

$$\mathbf{I}_1 = \frac{\mathbf{E}_1}{\mathbf{z}_0} + \frac{(s + \mathbf{p} + \mathrm{j}\mathbf{p}x_{L1}/\mathbf{z}_0)\mathbf{E}_1}{r_2' + \mathrm{j}(sx_{L2}' - \mathbf{p}x_{L1})}(1 + \mathbf{p})$$

Hence

$$\mathbf{E}_1 = \frac{\mathbf{I}_1}{\dfrac{1}{\mathbf{z}_0} + \dfrac{(1 + \mathbf{p})(s + \mathbf{p} + \mathrm{j}\mathbf{p}\,x_{L1}/\mathbf{z}_0)}{r_2' + \mathrm{j}(sx_{L2}' - \mathbf{p}x_{L1})}}$$

Substituting for $\mathbf{E}_1$ in (9.10),

$$\mathbf{V} = \mathbf{I}_1\left\{r_1 + \mathrm{j}\,\frac{x_{L1}}{1 + \mathbf{p}} + \frac{\boldsymbol{\gamma}}{\dfrac{1}{\mathbf{z}_0} + \dfrac{(1 + \mathbf{p})(s + \mathbf{p} + \mathrm{j}\mathbf{p}\,x_{L1}/\mathbf{z}_0)}{r_2' + \mathrm{j}(sx_{L2}' - \mathbf{p}x_{L1})}}\right\}$$

The expression in the curly brackets is the impedance of the motor.
Writing $\mathbf{p} = (T_3'/T_2')e^{\mathrm{j}\beta}$ in rectangular co-ordinates, i.e.

$$\mathbf{p} = m + \mathrm{j}n$$

where $m = (T_3'/T_2')\cos\beta$ and $n = \mathrm{j}(T_3'/T_2')\sin\beta$, gives

$$\mathbf{V} = \mathbf{I}_1\left[r_1 + \mathrm{j}\,\frac{x_{L1}}{1 + m + \mathrm{j}n} + \frac{\boldsymbol{\gamma}}{\dfrac{1}{\mathbf{z}_0} + \dfrac{(1 + m + \mathrm{j}n)\{s + m + \mathrm{j}n + \mathrm{j}(x_{L1}/\mathbf{z}_0)(m + \mathrm{j}n)\}}{r_2' + \mathrm{j}\{sx_{L2}' - x_{L1}(m + \mathrm{j}n)\}}}\right]$$

where $\boldsymbol{\gamma} = 1 + \mathrm{j}\,\dfrac{x_{L1}}{\mathbf{z}_0}\cdot\dfrac{m + \mathrm{j}n}{1 + m + \mathrm{j}n}$.

Multiplying some of the terms in the denominator and transferring $\boldsymbol{\gamma}$ to the denominator gives finally—

$$\mathbf{V} = \mathbf{I}_1\left[r_1 + \mathrm{j}\,\frac{x_{L1}}{1 + m + \mathrm{j}n} + \frac{1}{\dfrac{1}{\boldsymbol{\gamma}\mathbf{z}_0} + \dfrac{(1 + m + \mathrm{j}n)\{s + m(1 + \mathrm{j}x_{L1}/\mathbf{z}_0) + \mathrm{j}n(1 + \mathrm{j}x_{L1}/\mathbf{z}_0)\}}{\boldsymbol{\gamma}\{r_2' + nx_{L1} + \mathrm{j}(sx_{L2}' - mx_{L1})\}}}\right] \quad . \quad . \quad . \quad (9.11)$$

Rearranging the last term in the denominator gives

$$\mathbf{V} = \mathbf{I}_1 \left[r_1 + \mathrm{j}\,\frac{x_{L1}}{1 + m + \mathrm{j}n} + \frac{1}{\dfrac{1}{\gamma \mathbf{z}_0} + \dfrac{1}{\dfrac{\gamma(r_2' + n x_{L1})}{(1 + m + \mathrm{j}n)\{s + m(1 + \mathrm{j}x_{L1}/\mathbf{z}_0) + \mathrm{j}n(1 + \mathrm{j}x_{L1}/\mathbf{z}_0)\}} + \mathrm{j}\,\dfrac{\gamma(s x_{L2}' - m x_{L1})}{(1 + m + \mathrm{j}n)\{s + m(1 + \mathrm{j}x_{L1}/\mathbf{z}_0) + \mathrm{j}n(1 + \mathrm{j}x_{L1}/\mathbf{z}_0)\}}}} \right] \qquad (9.12)$$

It can be seen that the quantity in the square brackets represents an impedance $r_1 + \mathrm{j}x_{L1}/(1 + m + \mathrm{j}n)$ in series with two parallel circuits of impedances $\gamma \mathbf{z}_0$ and

$$\frac{\gamma(r_2' + n x_{L1})}{(1 + m + \mathrm{j}n)\{s + m(1 + \mathrm{j}x_{L1}/\mathbf{z}_0) + \mathrm{j}n(1 + \mathrm{j}x_{L1}/\mathbf{z}_0)\}} + \mathrm{j}\,\frac{\gamma(s x_{L2}' - m x_{L1})}{(1 + m + \mathrm{j}n)\{s + m(1 + \mathrm{j}x_{L1}/\mathbf{z}_0) + \mathrm{j}n(1 + \mathrm{j}x_{L1}/\mathbf{z}_0)\}}$$

The equivalent circuit for the motor can thus be drawn as shown in Fig. 9.15(*a*), the magnetizing impedance being represented by its more convenient parallel circuit. This enables the performance of the motor to be calculated for any value of slip s provided the various parameters for the motor are known.

The primary and referred secondary currents I_1 and I_2' can be found by ordinary circuit calculations. The total rotor power can be found from the $I_2^2 r$ loss in the right-hand branch, and the mechanical output by subtracting the secondary-circuit copper loss from this. The torque can then be obtained from the mechanical output and the speed.

A more accurate treatment can be made by using tensor analysis.*

SIMPLIFICATION OF EQUIVALENT CIRCUIT. With machines of normal design certain approximations can be made which simplify the diagram and still give reasonably accurate results.

The ratio $x_{L1}/\mathbf{z}_0$ is usually much less than 1 so that $\mathrm{j}x_{L1}/\mathbf{z}_0$ can be neglected relative to 1 and γ can be assumed equal to 1.

* W. J. Gibbs, "The Equations and Circle Diagram of the Schrage Motor," *J. Instn. Elect. Engrs.*, **93**, Pt. II, p. 621 (1946).

The expression thus becomes

$$\mathbf{V} = \mathbf{I}_1 \left[r_1 + \mathrm{j}\frac{x_{L1}}{1 + m + \mathrm{j}n} + \cfrac{1}{\cfrac{1}{\mathbf{z}_0} + \cfrac{1}{\cfrac{r_2' + nx_{L1}}{(1 + m + \mathrm{j}n)(s + m + \mathrm{j}n)} + \mathrm{j}\cfrac{(sx_{L2}' - mx_{L1})}{(1 + m + \mathrm{j}n)(s + m + \mathrm{j}n)}}} \right]$$

and the equivalent circuit is as shown in Fig. 9.15(b).

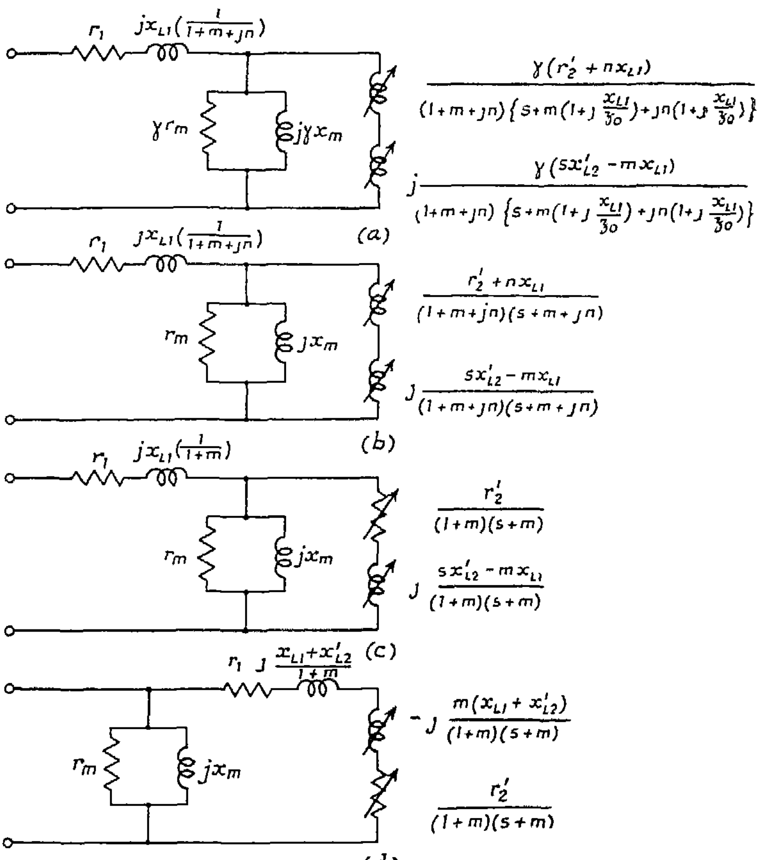


FIG. 9.15. EQUIVALENT CIRCUITS OF SCHRAGE MOTOR

(a) Complete equivalent circuit.
(b) Approximate equivalent circuit.
(c) Approximate equivalent circuit with no brush shift ($n = 0$).
(d) Further approximation to circuit (c).

Many motors are designed for operation with no brush shift ($\beta = 0°$ or $180°$); in such cases $n = (T_3'/T_2') \sin \beta = 0$ so that the expression becomes

$$V = I_1 \left[r_1 + \mathrm{j}\,\frac{x_{L1}}{1+m} + \cfrac{1}{\cfrac{1}{\mathbf{z}_0} + \cfrac{1}{\cfrac{r_2'}{(1+m)(s+m)} + \mathrm{j}\,\cfrac{(sx_{L2}' - mx_{L1})}{(1+m)(s+m)}}} \right]$$

The equivalent circuit can thus be drawn as in Fig. 9.15(c).
The last term in the denominator can be written

$$\mathrm{j}\,\frac{(sx_{L2}' - mx_{L1})}{(1+m)(s+m)} = \mathrm{j}\,\frac{x_{L2}'}{1+m} - \mathrm{j}\,\frac{m(x_{L1} + x_{L2}')}{(1+m)(s+m)}$$

Moving the magnetizing branch to the supply terminals, as is common with the induction-motor equivalent circuit, then gives the circuit of Fig. 9.15(d). The equivalent circuits give rise to circular current loci as shown in Fig. 9.16.

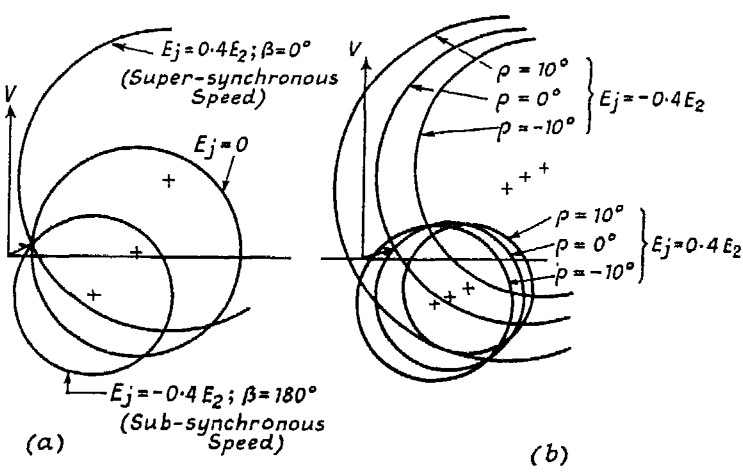


FIG. 9.16. EFFECT ON CIRCLE DIAGRAM OF VARYING E_j AND ρ

(a) Effect of varying E_j with no brush shift.
(b) Effect of varying ρ with $E_j = \pm\, 0{\cdot}4E_2$.

It can be seen from these circles that the power factor is likely to be high or leading at high speed and low at low speeds. By giving the brushes backward shift the power factor is made more lagging at high speeds, thus avoiding any tendency to excessive lead at top speed, and improved at low speeds, particularly at low loads. Power-factor curves plotted against torque are given in Fig. 9.19.

Approximate Current Loci

The approximate equivalent circuit of Fig. 9.15(*d*), although not suitable for accurate calculations, can be used to draw approximate circle diagrams which give a useful picture of the general behaviour.

The main branch of the circuit consists of a constant resistance and constant reactance in series with a variable resistance and reactance. The ratio of the variable resistance to the variable reactance is constant so that the expressions of Appendix 2 can be used to write down the following expressions for the co-ordinates of the centre—

$$P_x = \frac{V}{2\dfrac{x_{L1}+x_{L2}'}{1+m}\left\{1+\dfrac{r_1(1+m)}{x_{L1}+x_{L2}'}\cdot\dfrac{m(x_{L1}+x_{L2}')}{r_2'}\right\}}$$

$$= \frac{V}{2\dfrac{x_{L1}+x_{L2}'}{1+m}\left\{1+m(1+m)\dfrac{r_1}{r_2'}\right\}}$$

$$P_y = -P_x\cdot\frac{m(x_{L1}+x_{L2}')}{r_2'}$$

It must be remembered, however, that the circuit on which this is based assumes no brush shift.

Effect of Variable Parameters

It has been assumed in developing the circles of the preceding section that the various resistances and reactances are constant—this is not true in practice and considerable divergence from the theoretical circles may result.

As with other machines, the reactances may generally be regarded as constant. This is even more justifiable with commutator motors than with, say, a plain induction motor as the flux densities are generally low and saturation is therefore less likely to cause variations.

Resistances are complicated by the fact that brush contacts occur in both primary and secondary circuits, and the brush contact resistance decreases with increase of current. In the primary winding the brush resistance is generally low compared to the winding resistance and its variation can be neglected. In the secondary winding, however, the winding resistance is usually very low due to the low voltage and heavy current, and also there are two brushes in series in each phase. The brush resistance may therefore be as great as, or even greater than, the winding resistance, and variation of resistance with current may have an important effect on the characteristics.

The effect of the variable resistance on the current locus is illustrated in Fig. 9.17. Curve (*a*) shows the calculated locus for a super-synchronous speed using the light-load value of resistance and curve

(*b*) shows a similar calculation using the full-load value; the actual locus obtained on test differs from both of these.

It is thus evident that the circle diagram is not suitable for performance calculations although it is valuable for giving a general indication of the behaviour and the effect of making changes in various parameters. To calculate the performance of a motor it is thus necessary to use analytical methods and to make a preliminary

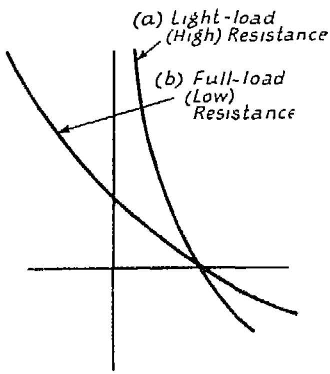


FIG. 9.17. EFFECT OF VARIABLE BRUSH RESISTANCE ON CALCULATED CURRENT LOCUS

estimate of the secondary current for each condition calculated in order to find the secondary resistance to be used in the calculation. With practice and experience, however, this can be done quite accurately and a good prediction of performance obtained.

Typical performance characteristics are shown in Fig. 9.19, page 164.

Operation at Synchronous Speed

The conditions when the motor is running at exactly synchronous speed are of interest. As the slip is now zero the flux is stationary relative to the stator phases, and the e.m.f. induced therein is also zero. To obtain synchronous speed, however, there must be a small injected e.m.f. from the tertiary winding to overcome the secondary-circuit impedance drop, and there must therefore be a small brush displacement θ_s. The conductors between the brushes which are producing this e.m.f. are running in a stationary field, and the e.m.f. is therefore of zero frequency, i.e. a direct e.m.f. and the stator currents are direct currents. The conditions are illustrated diagrammatically in Fig. 9.18 from which it is seen that the conductors supplying phase A are in the maximum flux while those supplying phases B and C are in a much smaller flux and are negative relative to those of phase A; the currents in the stator phases are therefore unequal. The actual position of the flux relative to the brushes at

the moment when the speed reaches synchronism is quite fortuitous and, of course, not necessarily that shown on the diagram.

The effect of this phenomenon on the torque can be found as follows.

The e.m.f. induced between brushes in a secondary phase due to a brush separation θ_s is, from eq. (2.9), page 24, given by

$$E_3 = 2(T_a/a)f_r\Phi \sin(\theta_s/2) \cos\beta \text{ volts}$$

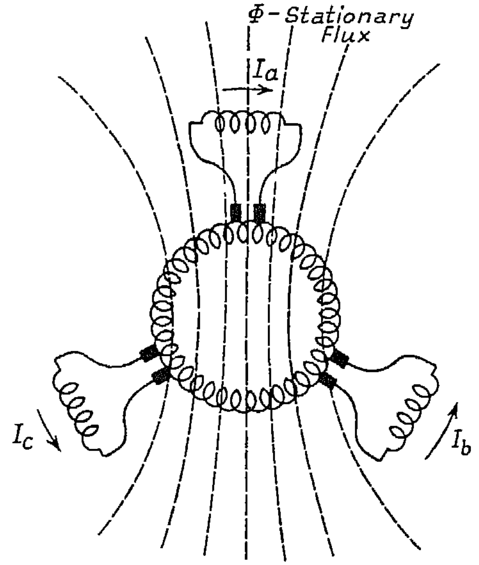


FIG. 9.18. CONDITIONS AT SYNCHRONISM

where β is the angle between the centre-line of the brush displacement and the flux axis and therefore also the angle between the flux axis and the stator-phase axis.

The current in a stator phase is

$$I_2 = (E_3/r_2) \text{ (since } x_{L2} = 0)$$

$$= \frac{2(T_a/a)f_r\Phi \sin(\theta_s/2) \cos\beta}{r_2}$$

$$= \frac{2(T_a/a)f_r\Phi(\theta_s/2) \cos\beta}{r_2} \text{ (since } \theta_s \text{ is small)}$$

The torque due to a phase is proportional to the current in the phase times the flux component which is coaxial with that phase, i.e.

$$TM_{ph} \propto I_2\Phi \cos\beta$$

Since $I_2 \propto \Phi\ \theta_s \cos\beta$ this becomes

$$TM_{ph} \propto \Phi^2\theta_s \cos^2\beta$$

If the above refers to phase A the corresponding angles for the other two phases will be $\beta + 2\pi/3$ and $\beta + 4\pi/3$, so that the total torque for the whole machine will be

$$TM = \Phi^2\theta_s\{\cos^2\beta + \cos^2(\beta + 2\pi/3) + \cos^2(\beta + 4\pi/3)\}$$
$$= \Phi^2 \,.\, 3\theta_s/2$$

Hence for a given brush separation the total torque is independent of β, i.e. it is independent of the relative position of the flux and the stator phases at the moment of attaining synchronism. There is thus no tendency for the machine to remain in synchronism and no *discontinuity in the torque curve* as the speed passes through synchronism.

EFFECT OF DISSYMMETRY. If, due to faulty brush settings, the angle θ_s is not the same at all three brushes but is θ_s for phase A, $\theta_s + \delta_B$ for phase B and $\theta_s + \delta_C$ for phase C, the expression for the torque becomes

$$TM \propto \Phi^2\{\theta_s \cos^2\beta + (\theta_s + \delta_B)\cos^2(\beta + 2\pi/3) + (\theta_s + \delta_C)\cos^2(\beta + 4\pi/3)\}$$
$$\propto \Phi^2\left\{\frac{3}{2}\theta_s + \frac{1}{4}(\delta_B + \delta_C)(1 + 2\sin^2\beta) + \frac{\sqrt{3}}{4}(\delta_B - \delta_C)\sin 2\beta\right\}$$

The second and third terms of this expression are dependent on the position of the flux relative to the stator phases, and variations in torque generated will thus occur in the neighbourhood of synchronous speed as the flux slowly rotates. These variations result in corresponding variations in speed and secondary current and are somewhat similar to the hunting of a synchronous machine. Operation over a small range of speed near synchronism may thus be impracticable.

Other causes of dissymmetry, such as bad brush contacts or faulty connexions, may cause similar effects.

Commutation

In addition to the reactance e.m.f. in the coil short-circuited by the brush there is a rotational e.m.f. due to its movement relative to the air-gap flux. The flux is always moving at synchronous speed relative to the conductors, so that the e.m.f. induced is independent of the motor speed; it is, however, moving past the brushes at slip speed so that the e.m.f. induced in the coil at a particular brush will depend on the magnitude of the flux being cut at the moment considered. When the flux axis is coincident with the coil axis, i.e. the flux is fully linked with the coil, the e.m.f. will be zero; when the flux is at 90° to this position the e.m.f. will be a maximum and in this position will have a value of

$$\hat{e}_{rb} = 2\pi f_1 T_c \Phi \text{ volts}$$

It will vary sinusoidally at slip frequency between this value and zero so that the r.m.s. value will be

$$E_{ib} = (\sqrt{2})\pi f_1 T_c \Phi \text{ volts}$$

It may be noted that this is different from the e.m.f. obtaining in the series and doubly-fed motors where the frequency is that of supply and the magnitude is proportional to slip.

As with other commutator motors the magnitude of this voltage must be limited to three or four volts so that it imposes a limit to the flux per pole that may be employed.

At synchronism the flux is stationary relative to the brushes as described in the last section, and the position of its axis is quite fortuitous. If the short-circuited coil happens to have its sides in a position of zero flux then commutating conditions will be similar to those of a d.c. machine, i.e. only the reactance e.m.f. will be present; if, however, it is in the position of maximum flux then conditions will be relatively bad. It is thus evident that conditions differ in the three phases and also differ at the two brushes of a phase. These differences tend to accentuate the unbalancing of the currents mentioned in the last section.

Control Gear

Equipment is required for starting and stopping and for variation of speed but the cost and complication is generally less than with other types of motor.

STARTING. A motor with the usual 3:1 speed range can be started by direct switching on to the supply provided the brushes are in the low-speed position; in this way about twice full-load torque is obtained with about $1\frac{1}{2}$ times full-load current. All that is required, therefore, is a main switch or contactor with an interlock to ensure that it cannot be closed unless the brushes are in the low-speed position.

For motors with a narrower speed range than 2:1 and for motors where a very gentle start is required, resistors may be added to the primary or secondary circuit and cut out in the usual way.

SPEED CONTROL. The following methods are available for moving the brushes to give speed variation—

1. By a simple handwheel mounted on the motor frame.
2. By a handwheel connected to the brush rockers through shafts, gears or chain drives, thus enabling a limited remoteness of control to be effected.
3. By a pilot motor. A f.h.p. induction motor is mounted on the main motor frame and connected to the rockers through gearing. Two push buttons are provided, one for forward motion (raise speed) and the other for reverse motion (lower speed), thus enabling completely remote control to be effected. Limit switches are necessary at each end of the brush travel to switch off the pilot motor and prevent its stalling. It may be noted that with this arrangement

the rate of brush travel, and therefore of speed change, is fixed and outside the control of the operator.

4. By a pilot motor with pre-set controller. In many cases it is desirable for the operator to have a controller giving a definite motor speed for each notch. This can be effected by having two drums, one coupled to the controller handle and the other reduction-geared or connected by a selsyn to the pilot motor. Moving the controller handle to a particular position starts the pilot motor, and as soon as the pilot-motor drum has rotated through the same angle the motor is stopped.

Where an occasional very low creeping speed is required, as in paper-making machinery, it is preferable to use resistors in the secondary circuit rather than design the motor for a very wide speed range.

The speed of operation of the brush gear should be such that about twice full-load current in the secondary is not exceeded. This gives a time for the change from minimum to maximum of approximately $2\sqrt{\text{h.p.}}$ seconds, e.g. 20 sec for a 100 h.p. motor.

General Design Features

As with the doubly-fed motor the basic design constants are similar to those of the induction motor. It must be remembered that the tertiary winding is in the same slots as the primary and adds 50 per cent or more to the total ampere-conductors around the rotor periphery; the values for the primary winding alone are therefore lower than in the induction motor.

Flux per Pole and Limiting Output. The e.m.f. induced in the short-circuited coils, which is constant at all speeds, should not exceed three to four volts even if a suitable discharge winding is fitted to assist commutation. As shown on page 34, this limits the flux per pole for 50 c/s machines to 0·02 weber, and the corresponding maximum output is about 25 h.p. per pole.

Primary Winding. Star or delta connexion can be used depending on the number and size of conductors required—it is generally found that for voltages up to 440, which is the usual upper limit for the Schrage motor due to the primary being supplied through slip rings, the delta connexion gives a more convenient size of conductor. A double-layer lap or wave winding, short-pitched by about 1/6th of a pole-pitch to minimize 5th and 7th harmonics as with induction motors, is usual.

Secondary Winding. It is desirable to keep the commutator voltage low so that the secondary currents are correspondingly high. A double-layer lap winding, which has several parallel paths and which can also be appropriately short-pitched to minimize harmonics, is therefore suitable for the secondary. The number of phases, which in the preceding theory has generally been supposed to be three, is open to the designer's choice, and 2, 3, 5, 7, or even more, phases are used with three-phase motors.

The advantages of a greater number of secondary phases are—

1. The change of current during commutation is less as explained on page 30.

2. The waveform of the secondary m.m.f. is improved, particularly when short-pitched coils are used.

The disadvantage is, of course, the greater number of brush arms required on the commutator, although this may enable a shorter commutator to be used.

TERTIARY WINDING. To assist commutation single-turn coils are generally used so that the number of commutator segments is necessarily large. To keep the number to a reasonable value the secondary voltage must be low and the current correspondingly high. A number of parallel circuits through the winding is therefore desirable, and a simplex lap ($a = p$) or a duplex lap ($a = 2p$) winding is usual on machines above about 5 h.p.

On smaller machines a wave winding is permissible and has the advantage that only one pair of brushes per phase, instead of one per phase per pole-pair, can be used, thus simplifying the mechanical arrangement of the brush gear in the limited space available.

To ensure satisfactory commutation, especially in the neighbourhood of synchronous speed, it is necessary for the various parallel circuits to share the current equally. Equalizing connexions between every third or fourth commutator segment are therefore desirable.

A further essential to secure good commutation is some form of discharge winding since compoles for neutralizing the reactance e.m.f. in the short-circuited coils are impracticable. Resistance connectors (page 37) are sometimes used but experience shows that the best results are usually obtained with the Robinson discharge winding (page 38) for machines up to about 50 h.p., where simplex tertiary windings with an e.m f. in the short-circuited coil of 2 to 2·5 V are used, while for larger machines the duplex arrangement described on page 38 is appropriate and permits e.m.f.'s in the short-circuited coil up to about 4·5 V.

Performance Characteristics

The curves of Fig. 9.19 show performance characteristics of typical motors.

SPEED-TORQUE CURVES. Curves (*a*) show the speed-torque curves over the normal working range of torque, these being similar for any size of motor and not appreciably affected by brush shift. The drop from no-load to full-load is somewhat greater than that for an induction motor of corresponding size on account of the resistance of the brushes and of the tertiary winding. A typical drop is about 100 r.p.m. at all speeds for a 100 h.p. 8-pole motor.

EFFICIENCY. The curves of (*b*) follow the usual shape, the efficiency being highest at synchronous speed when there is no tertiary winding loss and little stator iron loss.

POWER FACTOR. If there is no brush shift the curves of (*c*) are

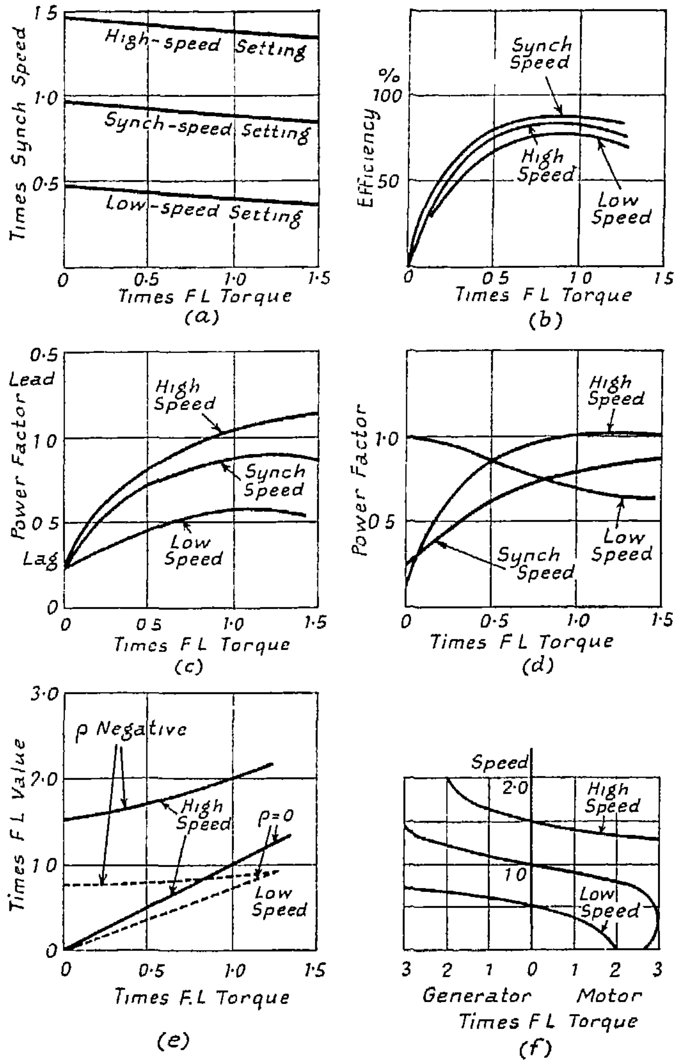


FIG. 9.19. TYPICAL PERFORMANCE CHARACTERISTICS

(a) Speed-torque.
(b) Efficiency.
(c) Power factor (symmetrical brush position).
(d) Power factor (unsymmetrical brush position).
(e) Secondary current.
(f) Speed-torque curves with generator action.

obtained showing high power factors at super-synchronous speeds and low values at sub-synchronous speeds. With a brush shift of a few degrees the curves of Fig. (*d*) are obtained showing an all-round improvement.

SECONDARY CURRENT. The curves of (*e*) show that with no brush shift the secondary current is approximately proportional to the torque but with large brush shift (8°–10°) the currents on no load are considerable; they lead the secondary e.m.f. E_2 at super-synchronous speeds and lag at sub-synchronous speeds.

GENERATOR OPERATION. If the machine is driven above no-load speed the injected e.m.f. E_j exceeds the secondary e.m.f. sE_2 and the secondary current is reversed—generator action results as with an induction motor driven at slightly above synchronous speed, the torque characteristics over the range of positive and negative torques up to about three times full-load value being as shown in Fig. 9.19(*f*). Operation is now on the lower part of the circle diagrams of Fig. 9.16, and it can be seen that the power factor is low at all speeds. To obtain a good power factor at low speeds the brush shift would have to be in the opposite direction to that for motor operation, i.e. *with* the rotation instead of *against*. As with the induction motor the machine can normally only generate when connected to a system capable of the supplying the magnetizing current.

Applications

The Schrage motor can be applied, almost without exception, to any industrial drive requiring a variable speed, and some cases where it has proved particularly successful are indicated below.

CRANES AND HOISTS. The Schrage motor (and also the doubly-fed motor) has the following advantages over the slip-ring induction motor.

1. External resistors are not necessary so that the overall efficiency is greater.

2. The speed at any controller notch is approximately independent of the load, making operation easier for the crane driver.

3. The characteristics are such that approximately full-load braking torque is available at speeds only slightly above the no-load speed corresponding to the brush setting in use. Safe control of descending loads at any speed is thus obtained without the use of mechanical brakes.

Disadvantages are the greater initial and maintenance costs and also the fact that the moment of inertia of the rotor is considerably higher.

The top speed of the motor is usually about 1,000 r.p.m., a somewhat higher figure than for other types of crane motor, and the speed range is about 20:1 giving a low speed of 50 r.p.m. corresponding to a hook speed of about 1 ft/min.

FANS AND CENTRIFUGAL PUMPS. Fans and centrifugal pumps

often require to be driven at variable speeds, typical examples being boiler-house fans. The torque required is proportional to the square of the speed, so that at low speeds the torque is very low, leading to high power factor and efficiency over the whole range.

PAPER-MAKING MACHINERY. The chief item of a paper mill is the machine on which the paper is actually formed. It is an example of a coincidental drive in that the relative speeds of each of the group of machines comprising the complete unit must remain the same or the paper will be torn, but these relative speeds must be adjustable to suit different qualities of paper. Furthermore, the speed of the whole unit must be adjustable over a wide range. For large machines, above about 100 in. wide and running at over

TABLE 9.4

Feature	Schrage Motor	Doubly-fed Motor
Supply Voltage	Limited to 660 V since it must be fed through slip rings.	Any value can be used for which the stator can be wound, e.g. 3·3 or 6·6 kV.
Construction	Self-contained	Requires a separate regulator of a size comparable with that of the motor. Also heavy cabling is needed between the regulator and motor.
Size of Motor	Larger than induction motor of same output.	About same size as induction motor.
Brushes and Commutator	Requires, except in very small motors, six brush sets per pole-pair and these have to be movable.	Brushes are fixed.
Enclosure	Total enclosure practicable on small motors *only* due to high rotor losses.	Total enclosure easier on account of fixed brushes and leads and lack of slip rings, and also relatively lower rotor losses.
Commutation	Commutator winding handles only slip power	Commutator winding handles total power making commutation more difficult, especially at starting.
Output	Limited to about 120 h.p. for 6-pole and 250 h.p. for 12-pole machines.	Up to 500 h.p.
Speed Characteristics	Shunt characteristics	Slightly steeper characteristics, especially at low speeds, due to regulator impedance.

600 ft/min, it is desirable to drive each of the component machines by a separate motor with suitable electric control to fulfil the above requirements. The Schrage motor has been used for this service for many years with entire satisfaction.

RING SPINNING FRAMES. Most ring spinning frames in this country are driven by squirrel-cage induction motors at an approximately constant speed, this speed being adjustable when desired by changing gear or pulley ratios. Increased production can, however, be obtained by varying the speed as the layers of yarn on the bobbins are built up. The Schrage motor with an automatic speed regulating device has been used for this purpose and, although more expensive, has resulted in 10–12 per cent greater production and about 50 per cent less breakages of yarn. The size of motor required is usually 5–10 h.p., and it is totally enclosed.

Comparison between Schrage and Doubly-fed Motors

These two motors have been seen to have similar characteristics so that they are often in competition when a motor is being selected for a particular variable-speed drive. Table 9.4 gives a comparison of some of the salient features.

EXERCISES 9

1. A 4-pole armature has 104 coils each having 4 turns and is running at 1,500 r.p.m. in a stationary field of 0·02 Wb per pole. The e.m.f. between two brushes, symmetrically spaced relative to the flux axis, is 200 V. Find the angle (mech. degrees) of brush separation, assuming the flux to be sinusoidally distributed over the pole.

If the armature is rotating at 3,000 r.p.m. and the field at 1,500 r.p.m. in the same direction, find the r.m.s. value and the frequency of the e.m.f. between the brushes.

2. A 3-ph., 400-V, 6-pole, 50-c/s Schrage motor has a star-connected primary winding with 230 effective turns per phase, a 3-phase secondary with 40 effective turns per phase, and a lap-connected tertiary having 198 total turns. Find the approximate no-load speed of the motor for a brush-separation of 45° (elec.).

3. A 6-pole, 3-ph., 50-c/s Schrage motor has a lap-connected tertiary winding and a 3-ph. secondary winding. The primary winding has 100 effective turns in series per phase each carrying 10 A; the secondary winding has 40 effective turns in series, each carrying 30 A. The motor is running at a sub-synchronous speed with no brush shift and a brush displacement of 20 mech. degrees. Determine the number of turns on the tertiary winding.

4. A 6-pole Schrage motor has the following double-layer lap windings—

	Primary	Secondary (4 parallel circuits)	Tertiary (duplex)
Number of phases	3	3	—
Number of turns (total)	180	240	180
Coil span (per cent)	83·3	80	100

If the secondary winding carries 70 A and the brush separation is 80° (elec.) in a direction to give sub-synchronous running, determine the primary current. Neglect magnetizing current and assume no brush displacement.

If the primary leakage reactance when running as an induction motor is 0·6 Ω, estimate the leakage e.m.f. in the primary winding when running as above.

5. The lap-connected tertiary winding of a 4-pole Schrage motor has 72 slots with 6 conductors per slot and the 3 sets of brushes per pole-pair have a displacement of 90°. The secondary phase current is 100 A. Draw the current distribution diagram and the m.m.f. wave for a pole-pair when the current in one phase is at its peak value. Sketch the fundamental of the wave and compare with the calculated value.

6. A 10-pole, 50-c/s Schrage motor has the following particulars when running at approximately 400 V with a secondary current of 100 A lagging the secondary e.m.f. by 30°—

Effective primary turns/ph. (star-connected)	= 50
Effective secondary turns/ph. (3-phase)	= 30
Primary resistance drop	= 1 per cent
Primary reactance drop	= 10 per cent
Secondary circuit resistance	= 0·2 Ω
Magnetizing and iron-loss current	= 15 A at 80°

The brushes are adjusted so that the tertiary e.m.f. set up by the main flux is 75 V at 180° to the secondary e.m.f. Assuming a primary e.m.f. due to the main flux of 370 V, draw the complexor diagram and find the secondary reactance, the primary power factor and the speed.

7. The following particulars refer to a 400-V, 50-c/s, 8-pole, delta-connected, 120/40-h.p., 1,050/350-r.p.m., Schrage motor with a brush shift of − 5° (elec.). The ratio of diametric commutator e.m.f. to the secondary e.m.f. per phase is $E/E_2 = 0{\cdot}52$. All parameters are referred to one phase of the primary winding.

$$r_1 = 0{\cdot}11\ \Omega \qquad r_2' + r_3{}^1 = 0{\cdot}41\ \Omega$$
$$x_{L1} = 0{\cdot}75\ \Omega \qquad x_{L2}' = 0{\cdot}5\ \Omega$$
$$x_m = 15{\cdot}7\ \Omega$$

Determine, using the equivalent circuits of Fig. 9.15(*a*), (*b*), (*c*) and (*d*), the input current, power factor and power, the output power and torque and the secondary current when running with a brush displacement of 132° (high speed) and a slip of − 0 4.

CHAPTER 10

THE OSNOS (NO-LAG) MOTOR

Principle and Construction

The Osnos motor may be regarded as a special case of the Schrage motor, although, as mentioned on page 100, it was developed earlier. The construction is similar to that of the Schrage motor but the tertiary e.m.f. collected from the commutator is not variable and is usually injected into the secondary circuit at an angle leading the secondary e.m.f. by about 90°, so that power-factor improvement but

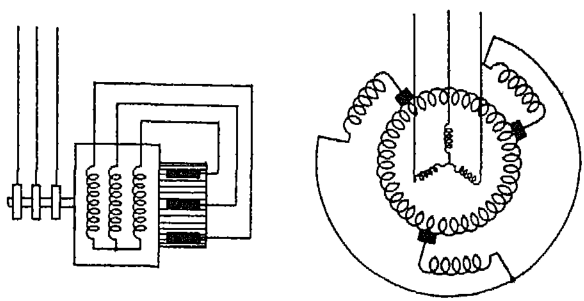


Fig. 10.1. Connexions of Osnos Motor

no speed control is obtained. The motor, as normally designed, thus runs at approximately synchronous speed but with unity or leading power factor.

As a fixed e.m.f. is to be collected from the commutator, only three sets of brushes per pole-pair, spaced at 120°, are necessary as shown diagrammatically in Fig. 10.1, and the position of these is normally fixed by the manufacturer.

Since the motor runs at nearly synchronous speed, the value of the secondary e.m.f. sE_2 is small. The tertiary injected e.m.f. will be less than this and in practice is usually between about 2 per cent of the standstill secondary e.m.f. E_2 for large machines and 10 per cent for small machines. The number of turns required on the tertiary winding is thus very small.

Tertiary E.M.F.

From eq. (2.11), the e.m.f. between brushes is, for a winding with full-pitch coils,

$$E = (\sqrt{2})(T_a/a)f_1\Phi \sin (\theta/2)$$

In this case $\theta = 120°$ so that $\sin (\theta/2) = \sqrt{3}/2$ and

$$E = (\sqrt{3}/\sqrt{2})(T_a/a)f_1\Phi \text{ volts}$$

The equivalent-star value is thus

$$E = (1/\sqrt{2})(T_a/a)f_1\Phi \text{ volts per phase} \quad . \quad (10.1)$$

Tertiary M.M.F.

If I_2 is the secondary current per phase, the fundamental of the tertiary m.m.f. is, from eq. (1.5),

$$F_3 = (3/\sqrt{2}\pi^2)(I_2/a)(T_a/p) \text{ ampere-turns} \quad . \quad (10.2)$$

As the number of turns on the tertiary winding is small compared to the number on the secondary winding, the effect of this m.m.f. on the air-gap flux is negligible.

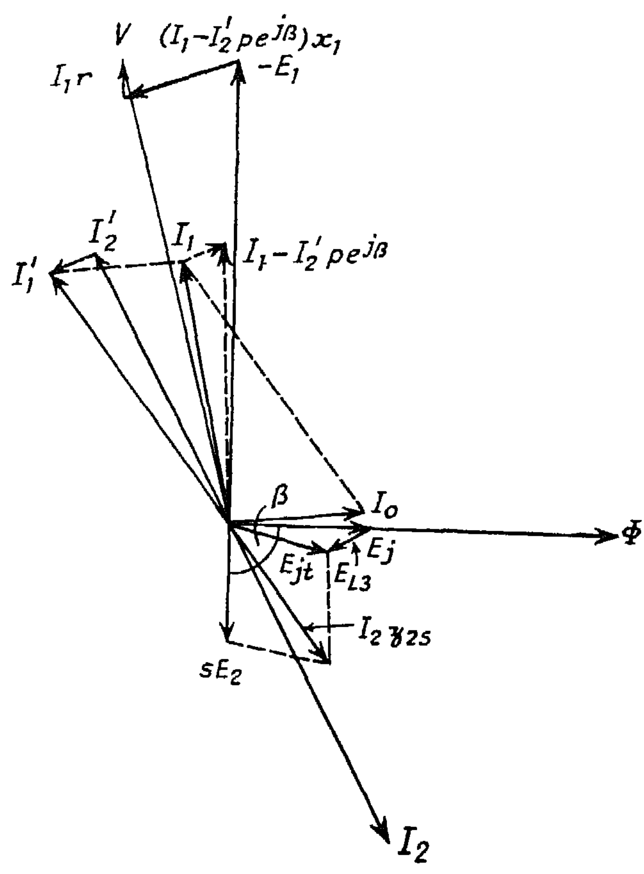


FIG. 10.2. COMPLEXOR DIAGRAM

Complexor Diagram

The complexor diagram is shown in Fig. 10.2 and is similar to that for the Schrage motor (page 147), except that the injected e.m.f. due to the main flux is leading E_2 by 90°, and the primary current component to counteract the tertiary m.m.f. is very small.

The current I_2 thus leads sE_2 by a considerable angle. The small m.m.f. due to the tertiary winding demands a primary current of $I_2'(T_3'/T_2') = I_2'p$ leading I_2' by 90°, so that the total primary current, including the magnetizing component I_0, is almost at unity power factor. The reactance e.m.f. in the primary winding is set up by the total effective current in the primary slots $(I_1 - I_2'p)$ and leads it by 90°. The reactance e.m.f. set up in the tertiary winding by the effective current in the primary slots is $(I_1 - I_2'p)x_{L1}T_3'/T_1' = E_{L3}$, giving the total injected e.m.f. E_{jt} as shown.

Equivalent Circuit

The behaviour of the motor can be calculated from the general expression for the Schrage motor given on page 154. This expression can, however, again be simplified by making approximations resulting chiefly from the fact that the tertiary winding has only a very small number of turns relative to the primary and secondary windings.

The term (x_{L1}/z_0) is small relative to 1 so that γ can be assumed equal to 1. Also jn can be neglected relative to $(1 + m)$ as it is usually less than 0·1. Expression (9.11) can thus be written

$$\mathbf{V} = \mathbf{I}_1 \left\{ r_1 + \mathrm{j}\,\frac{x_{L1}}{1+m} + \frac{1}{\dfrac{1}{\mathbf{z}_0} + \dfrac{(1+m)(s+m+\mathrm{j}n)}{r_2' + nx_{L1} + \mathrm{j}(sx_{L2}' - mx_{L1})}} \right\} \quad (10.3)$$

Re-arranging the last term in the denominator, this becomes

$$\mathbf{V} = \mathbf{I}_1 \Bigg\{ r_1 + \mathrm{j}\,\frac{x_{L1}}{1+m}$$
$$+ \frac{1}{\dfrac{1}{\mathbf{z}_0} + \dfrac{\mathrm{j}n(1+m)}{r_2' + nx_{L1} + \mathrm{j}(sx_{L2}' - mx_{L1})} + \dfrac{(1+m)(s+m)}{r_2' + nx_{L1} + \mathrm{j}(sx_{L2}' - mx_{L1})}} \Bigg\}$$

Since m is small relative to 1 and the reactance terms are small relative to r_2' the term $\mathrm{j}n(1+m)/\{r_2' + nx_{L1} + \mathrm{j}(sx_{L2}' - mx_{L1})\}$ is approximately equal to $\mathrm{j}(n/r_2')$ which is independent of slip. x_{L1} may also be neglected relative to r_2' in all except very small machines.

The expression can thus be written

$$\mathbf{V} = \mathbf{I}_1 \left\{ r_1 + \mathrm{j}\,\frac{x_{L1}}{1+m} + \frac{1}{\dfrac{1}{\mathbf{Z}_0} - \dfrac{1}{\mathrm{j}r_2'/n} + \dfrac{1}{\dfrac{r_2'}{(1+m)(s+m)} + \mathrm{j}\,\dfrac{sx_{L2}' - mx_{L1}}{(1+m)(s+m)}}} \right\}$$

The last term in the denominator can be written

$$\mathrm{j}\,\frac{(sx_{L2}' - mx_{L1})}{(1+m)(s+m)} = \mathrm{j}\,\frac{x_{L2}'}{1+m} - \mathrm{j}\,\frac{m(x_{L1}+x_{L2}')}{(1+m)(s+m)}$$

so that the expression finally becomes

$$\mathbf{V} = \mathbf{I}_1 \left\{ r_1 + \mathrm{j}\,\frac{x_{L1}}{1+m} + \frac{1}{\dfrac{1}{\mathbf{Z}_0} - \dfrac{1}{\mathrm{j}\,\dfrac{r_2'}{n}} + \dfrac{1}{\dfrac{r_2'}{(1+m)(s+m)} + \mathrm{j}\,\dfrac{x_{L2}'}{1+m} - \mathrm{j}\,\dfrac{m(x_{L1}+x_{L2}')}{(1+m)(s+m)}}} \right\} \quad . \quad . \quad . \quad (10.4)$$

The quantity in the curly brackets represents the impedance of the motor and can be represented by the circuit shown in Fig. 10.3(a) —this is therefore an approximate equivalent circuit for the motor. As with the ordinary induction motor the circuit can be further simplified by moving the shunt branch to the terminals, giving the circuit of Fig. 10.3(b).

The value of m is always small and will be zero if $\beta = 90°$ in which case the circuit is similar to that of an induction motor except for the term r_2'/n which acts as though a capacitor were connected at the motor terminals, i.e. it causes the usual induction-motor circle diagram to be shifted to the left.

Approximate Current Loci

The circuit of Fig. 10.3(b) has a shunt branch and a main branch, the latter comprising a variable resistance and reactance in series with a constant resistance and reactance. The ratio of the variable resistance to the variable reactance is $r_2'/\{m(x_{L1} + x_{L2}')\}$ and is a

constant. It is shown in Appendix 2 that the locus of the current complexor in such a circuit, when supplied at a constant voltage, is a circle, the co-ordinates of whose centre are given by—

$$P_x = \frac{V}{2 \times \dfrac{x_{L1} + x_{L2}'}{1+m}\left\{1 + \dfrac{r_1(1+m)m}{r_2'}\right\}} \qquad . \quad (10.5a)$$

$$P_y = P_x \cdot \frac{m(x_{L1} + x_{L2}')}{r_2'} \qquad . \quad . \quad . \quad . \quad (10.5b)$$

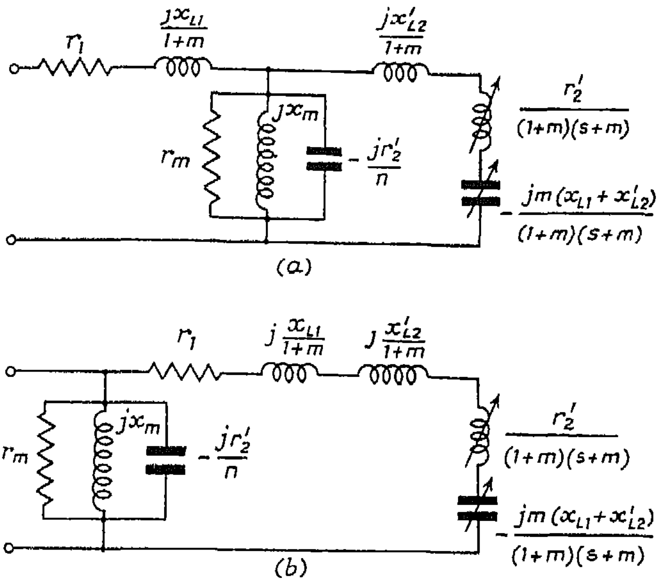


FIG. 10.3. APPROXIMATE EQUIVALENT CIRCUITS

(a) Circuit from eq. (10.4).
(b) Final approximation.

If $m = 0$, i.e. if $\beta = 90°$, these become $P_x = V/\{2(x_{L1} + x_{L2}')\}$ and $P_y = 0$ as for a plain induction motor; the only effect of the tertiary winding is thus to alter the shunt branch causing the circle to be shifted towards the left. Conditions are similar to those with the Scherbius phase advancer, and the current locus can be drawn as described on page 91. The following example illustrates the procedure.

Example 10.1. Draw the current loci for a 20-h.p., 420-V, 4-pole,

50-c/s, Osnos motor having the following particulars and with values of β equal to 0, $\pi/4$, $\pi/2$, $3\pi/4$, π and $3\pi/2$.

Primary winding, delta connected	
Exciting circuit reactance	$x_m = 77\ \Omega$/ph.
Exciting circuit resistance	$r_m = 550\ \Omega$/ph.
Primary winding resistance	$r_1 = 1{\cdot}8\ \Omega$/ph.
Primary winding reactance	$x_{L1} = 4{\cdot}1\ \Omega$/ph.
Secondary circuit resistance	$r_2' = 2{\cdot}6\ \Omega$/ph.
Secondary circuit reactance	$x_{L2}' = 3{\cdot}9\ \Omega$/ph.
$E_3/E_2 = T_3'/T_2' = p$	$= 0{\cdot}07$

From eqs. (10.5a) and (10.5b) the co-ordinates of the centre are—

$$P_x = \frac{420}{2 \times \dfrac{4{\cdot}1 + 3{\cdot}9}{1 + m}\left\{1 + \dfrac{1{\cdot}8(1 + m)m}{2{\cdot}6}\right\}}$$

$$= \frac{26{\cdot}2(1 + m)}{1 + m(1 + m)/1{\cdot}45}\ \text{A/ph.}$$

$$P_y = P_x \times m(4{\cdot}1 + 3{\cdot}9)/2{\cdot}6$$
$$= P_x \times 3{\cdot}08m\ \text{A/ph.}$$

Magnetizing and core-loss current

$$= 420/550 - \mathrm{j}(420/77) = 0{\cdot}76 - \mathrm{j}5{\cdot}45\ \text{A/ph.}$$

Current in r_2'/n branch

$$= I_c = 420n/2{\cdot}6 = 161n\ \text{A/ph.}$$

The values of the co-ordinates of the centres of the circles are calculated as shown in Table 10.1 for the given values of β.

TABLE 10.1

β	$m = p\cos\beta$	$n = p\sin\beta$	I_c	$1 + m$	$(1 + m)m$	A*	P_x (amperes)	P_y (amperes)
0	0·07	0	0	1·07	0·075	1·051	26·6	5·75
$\pi/4$	0·049	0·049	7·9	1·049	0·051	1·035	26·5	3·98
$\pi/2$	0	0·07	11·3	1·0	0	1·0	26·2	0
$3\pi/4$	− 0·049	0·049	7·9	0·951	− 0·047	0·967	25·9	− 3·90
π	− 0·07	0	0	0·93	− 0·065	0·955	25·5	− 5·5
$3\pi/2$	0	− 0·07	− 11·3	1·0	0	1·0	26·2	0

* $A = 1 + (1 + m)m/1{\cdot}45$

The circles for the six values of β are shown in Fig. 10.4. It can be seen that, although an angle $\beta = \pi/2$ gives the maximum leading reactive current at no-load, $\beta = \pi/4$ gives a better power factor

over a range of load. The angle normally employed is therefore generally between 40° and 60°. Angles greater than $\pi/2$ are, of course, impracticable as they make the power factor worse than that of the plain induction motor as shown by the circle for $3\pi/2$.

Analytical Treatment

The equivalent circuits of Fig. 10.3(*a*) or 10.3(*b*) can be used for the analytical calculation of the behaviour as illustrated in Example 10.2, which refers to the same machine as Example 10.1.

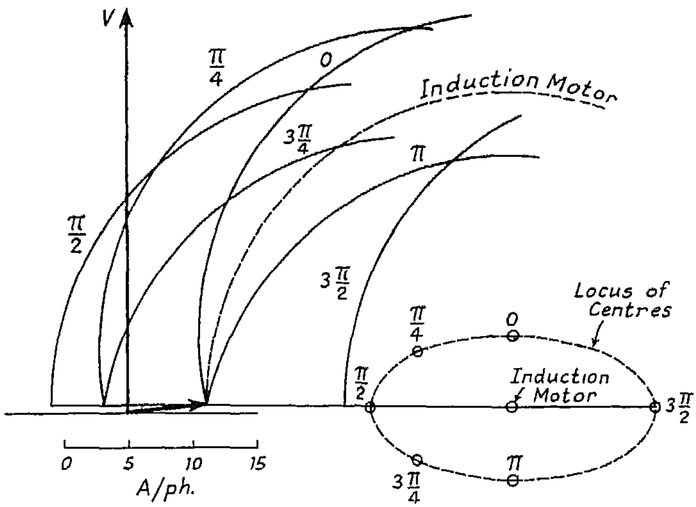


FIG. 10.4. CURRENT LOCI FOR OSNOS MOTOR WITH VARIOUS VALUES OF β

Example 10.2. Calculate for the 20-h.p. Osnos motor of Example 10.1 the primary current and power factor, the torque and the h.p. output when running at a slip of 0·03, the brushes being adjusted to give $\beta = 50°$. Find also the standstill current and power factor.

For an injection angle $\beta = 50°$—

$$m = 0{\cdot}07 \cos\beta = 0{\cdot}045$$

$$n = 0{\cdot}07 \sin\beta = 0{\cdot}0535$$

For the shunt branch—

$$1/z_0 = 1/r_m + 1/jx_m = 1/550 + 1/j77 = 0{\cdot}00182 - j0{\cdot}013 \text{ mho}$$
$$= 0{\cdot}0132\underline{/-82{\cdot}1} \text{ mho}$$

Hence

$$x_{L1}/z_0 = 4{\cdot}1 \times 0{\cdot}0132 = 0{\cdot}054$$

This is small relative to 1 so that $\gamma = (1 + x_{L1}/z_0)$ can be assumed equal to 1. Also $jn = j0{\cdot}0535$ is small relative to 1 so that expression (10.3) is appropriate for a reasonably accurate calculation. The effect of the other approximations is discussed later.

The calculation of primary current according to expression (10.3) is shown in Table 10.2.

TABLE 10.2

	$s = 0{\cdot}03$ (On load)	$s = 1$ (Standstill)
$1 + m$	$1{\cdot}045$	$1{\cdot}045$
$s + m + jn$	$0{\cdot}075 + j0{\cdot}0535 = 0{\cdot}0923/\underline{35{\cdot}6^\circ}$	$1{\cdot}045 + j0{\cdot}0535 = 1{\cdot}045/\underline{2{\cdot}95^\circ}$
$r_2' + nx_{L1}$	$2{\cdot}6 + 0{\cdot}22 = 2{\cdot}82$	$2{\cdot}6 + 0{\cdot}22 = 2{\cdot}82$
$j(sx_{L2}' - mx_{L1})$	$j(0{\cdot}117 - 0{\cdot}184) = -j0{\cdot}067$	$j(3{\cdot}9 - 0{\cdot}184) = j3{\cdot}716$
$r_2' + nx_{L1} + j(sx_{L2}' - mx_{L1})$	$2{\cdot}82 - j0{\cdot}067 = 2{\cdot}82/\underline{-1{\cdot}35^\circ}$	$2{\cdot}82 + j3{\cdot}716 = 4{\cdot}68/\underline{53{\cdot}9^\circ}$
$\dfrac{(1+m)(s+m+jn)}{r_2' + nx_{L1} + j(sx_{L2}' - mx_{L1})} = a$	$\dfrac{1{\cdot}045 \times 0{\cdot}0923/\underline{35{\cdot}6^\circ}}{2{\cdot}82/\underline{-1{\cdot}35^\circ}} = 0{\cdot}0343/\underline{36{\cdot}95^\circ} = 0{\cdot}0274 + j0{\cdot}0206$	$\dfrac{1{\cdot}045 \times 1{\cdot}045/\underline{2{\cdot}95^\circ}}{4{\cdot}68/\underline{53{\cdot}9^\circ}} = 0{\cdot}234/\underline{-50{\cdot}95^\circ} = 0{\cdot}15 - j0{\cdot}189$
$1/z_0$	$0{\cdot}00182 - j0{\cdot}0130$	$0{\cdot}00182 - j0{\cdot}0130$
$1/z_0 + a$	$0{\cdot}0292 + j0{\cdot}0076 = 0{\cdot}30/\underline{14{\cdot}6^\circ}$	$0{\cdot}152 - j0{\cdot}202 = 0{\cdot}255/\underline{-53{\cdot}5^\circ}$
$1/(1/z_0 + a)$	$33{\cdot}2/\underline{-14{\cdot}6^\circ} = 32{\cdot}2 - j8{\cdot}4$	$3{\cdot}93/\underline{53{\cdot}5^\circ} = 2{\cdot}34 + j3{\cdot}15$
$r_1 + jx_{L1}/(1 + m)$	$1{\cdot}8 + j3{\cdot}92$	$1{\cdot}8 + j3{\cdot}92$
Total impedance/phase (ohms)	$34{\cdot}0 - j4{\cdot}48 = 34{\cdot}3/\underline{-7{\cdot}5^\circ}$	$4{\cdot}14 + j7{\cdot}07 = 8{\cdot}2/\underline{59{\cdot}8^\circ}$
Current/phase (amperes)	$12{\cdot}3/\underline{7{\cdot}5^\circ}$	$51{\cdot}2/\underline{-59{\cdot}8^\circ}$

The line currents are thus 20·6 A at 0·99 power factor leading when the motor is on load at $s = 0{\cdot}03$, and 89 A at 0·5 power factor lagging when at standstill.

$$\text{Power input} = 3 \times 420 \times 12{\cdot}3 \times 0{\cdot}99 = 15{,}300 \text{ W}$$

The torque in synchronous watts is equal to the rotor input, and this is the above total input minus stator copper loss and loss in the shunt branch (core, friction and windage loss).

$$\text{Stator copper loss} = 3I^2r_1 = 3 \times 12{\cdot}3^2 \times 1{\cdot}8$$
$$= 810 \text{ W}$$

$$\text{Voltage across shunt branch} = V - I\{r_1 + jx_{L1}/(1+m)\}$$
$$= 420 - 12{\cdot}3\underline{/7{\cdot}5^\circ}(1{\cdot}8 + j3{\cdot}92)$$
$$= 420 - 53{\cdot}0\underline{/72{\cdot}9^\circ}$$
$$= 420 - 15{\cdot}6 - j50{\cdot}6$$
$$= 407 \text{ V}$$

$$\text{Shunt circuit loss} = 3E^2/r_m = 3 \times 407^2/550 = 950 \text{ W}$$

$$\text{Torque} = 15{,}300 - 810 - 950 = 13{,}540 \text{ W}$$
$$= 13{,}540 \times 0{\cdot}117/25$$
$$= 63{\cdot}2 \text{ Lb-ft}$$

$$\text{Output} = \text{rotor input } (1 - s)$$
$$= 13{,}540 \times 0{\cdot}97$$
$$= 13{,}150 \text{ W}$$
$$= 17{\cdot}6 \text{ h.p.}$$

$$\text{Efficiency} = (13{,}150/15{,}300) \times 100 = 86{\cdot}0 \text{ per cent}$$

If a similar calculation is made according to the equivalent circuit of Fig. 10.3(*a*) the phase current becomes $13{\cdot}0\underline{/7{\cdot}1^\circ}$ for $s = 0{\cdot}03$ and $49{\cdot}3\underline{/-59{\cdot}4^\circ}$ for $s = 1$, while for the circuit of Fig. 10.3(*b*) the currents are $12{\cdot}8\underline{/9{\cdot}5^\circ}$ and $47{\cdot}5\underline{/-57{\cdot}6^\circ}$ respectively. It can thus be seen that the approximations are justified.

Design Features

The design is similar to that of a Schrage motor except for the tertiary winding which may require special consideration on account of the very low voltage which it has to generate.

TERTIARY WINDING. Whereas in the Schrage motor the design of the tertiary winding is governed by the maximum voltage between segments which will permit satisfactory commutation, in the Osnos motor the difficulty is usually to devise a winding which will give a sufficiently low voltage and at the same time permit the use of a large flux per pole without excessive voltage between segments. Practical windings are illustrated in Fig. 10.5.

With the *simple lap winding* shown in Fig. 10.5(*a*) the equivalent-star e.m.f. is, from eq. (10.1),

$$E_j = (1/\sqrt{2})(T_a/a)f_1\Phi k_e \text{ volts per phase} \qquad (10.6)$$

where k_e is the coil-span factor if the winding has short-pitched coils. The value of E_j can thus be made small by the use of short-pitched coils, the limit being a coil having a span of one slot pitch. The e.m.f. induced in the coil short-circuited by the brush, i.e. the segment e.m.f., is given by

$$E_{rb} = (\sqrt{2})\pi f_1 T_c \Phi k_e \text{ volts} \qquad (10.7)$$

and is similarly reduced by short pitching. With a permissible segment voltage of 2, the maximum flux per pole for a 50-c/s machine having single-turn coils of varying pitches is shown below.

Per-cent Pitch	100	66·7	50	33·3	20
Flux/Pole (weber)	0·009	0·0104	0·0127	0·018	0·029

The above values of flux per pole can be doubled, for the same values of e.m.f., by using a duplex winding (Fig. 10.5(*b*)).

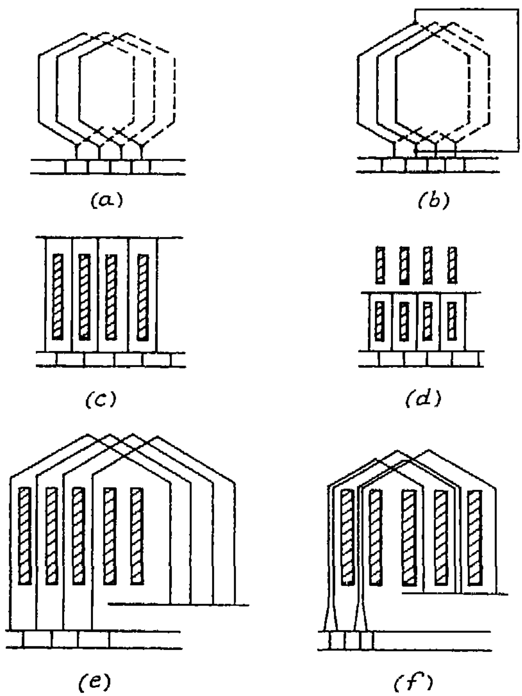


FIG. 10.5. TERTIARY WINDINGS

(*a*) Simple lap winding.
(*b*) Duplex lap winding.
(*c*) Simple bar winding.
(*d*) Fractional-length bar winding.
(*e*) Full-turn bar winding.
(*f*) Full-turn bar winding with 2 bars/slot.

A simpler arrangement consists of single conductors from each commutator segment joined to a common ring at the far end of the machine as shown in Fig. 10.5(*c*). The e.m.f. between brushes is

now that due to the two conductors in contact with the brushes, i.e. due to a single coil having a span equal to the brush pitch. For a brush pitch of 120° $k_e = \sqrt{3}/2$ and $T = 1$ so that

$$E_j = (\sqrt{2})\pi f\Phi \times (\sqrt{3}/2) = 3{\cdot}84 f\Phi \text{ volts} \quad . \quad (10.8)$$

The voltage between segments is due to two adjacent conductors, i.e. to a coil with a span of one slot pitch. Thus

$$E_{rb} = (\sqrt{2})\pi f_1 \Phi k_e = (\sqrt{2})\pi f_1 \Phi \sin(\psi/2) \quad . \quad (10.9)$$

where ψ is the slot-pitch angle.

If the e.m.f. obtained by this is too great, only a portion of each conductor can be used by making the common connexion part of the way along the core, in one of the ventilating ducts, as shown in Fig. 10.5(*d*).

If the brush e.m.f. from a single turn is not great enough it can be increased by using a full turn, of any desired span, instead of a single conductor, the arrangement then being as shown in Fig. 10.5(*e*). If k_e is the coil-span factor of the turn the e.m.f. will be that of Fig. 10.5(*b*) multiplied by $2k_e$. The brush e.m.f. will be similarly increased.

A decrease of the segment e.m.f. without appreciable change in the e.m.f. between brushes E_j can be effected by the arrangement of Fig. 10.5(*f*). The e.m.f. between brushes is the same as for case (*e*) but the segment e.m.f. is halved due to two conductors being in the same slot.

The limiting voltage between segments is usually about 2 V, although a lower value is generally used with bar-type windings. The maximum flux per pole and the injected e.m.f. can thus be found as in Example 10.3.

TABLE 10.3

Type of Winding	Coil Span	k_e	Φ	E_j
(*a*) Simple Lap	1 slot	0·075	0·0600	6·7
	2 slots	0·149	0·0304	6·7
	3 slots	0·221	0·025	6·7
(*b*) Duplex Lap	1 slot	0·075	0·120	6·7
	2 slots	0·149	0·061	6·7
	3 slots	0·221	0·041	6·7
(*c*) Bar Winding	120°	0·075	0·060	11·6
(*d*) Bar Winding (mid-core conn.)	120°	0·075	0·120	11·6
(*e*) Bar Winding (joined at comm. end)	10 slots	0·679	0·044	11·6
	14 slots	0·866	0·035	11·6
	18 slots	0·974	0·031	11·6
(*f*) Bar Winding (joined at comm. end with 2 conds. in same slot)	10 slots	0·679	0·088	23·2
	14 slots	0·866	0·70	23·2
	18 slots	0·974	0·62	23·2

Example 10.3. A 50-c/s Osnos motor has 21 slots per pole on its rotor and the voltage between segments is limited to 1 V. Determine the maximum allowable flux per pole and the brush e.m.f. (equivalent-star value) for the various types of winding illustrated in Fig. 10.5.

Single-turn coils are assumed for the lap windings so that $T_a/a = 42$. The slot-pitch angle is $\psi = 180/21 = 8{\cdot}56°$ (electrical degrees).

Various possibilities are illustrated in Table 10.3.

Applications and Performance

Typical performance curves are shown in Fig. 10.6 from which it is seen that the power factor is nearly unity or slightly leading over

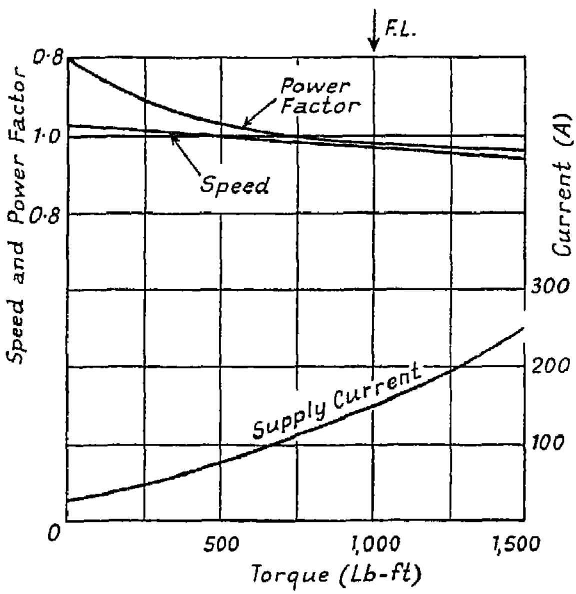


FIG. 10.6. TYPICAL CHARACTERISTICS OF 250 H.P. OSNOS MOTOR

the whole range of load. Due to the component of E_j in phase with E_2, the speed is slightly higher than that of an induction motor and may be slightly above synchronous speed at no load.

The motor is started as an ordinary induction motor, with resistors in the rotor circuit, giving a starting torque of 2 to $2\frac{1}{2}$ times full-load torque with 1 to $1\frac{1}{2}$ times full-load current. The pull-out torque is somewhat higher than that of a corresponding induction motor, e.g. about three times full-load torque.

The fact that the motor operates at about unity power factor results in a lower stator current than would obtain in an induction motor of similar rating—for low- and medium-speed motors the size

of the stator winding may therefore be less and the stator copper losses lower, thus counterbalancing, to some extent, the extra cost of the commutator.

It can be seen that the performance of the Osnos motor is in many respects similar to that of a synchronous-induction motor but it has the advantage of not requiring a separate exciter. Owing to the supply being fed to the rotor, the supply voltage is limited to about 650 V and design difficulties limit the output to about 500 h.p. at 1,500 r.p.m. or slightly larger at lower speeds.

EXERCISES 10

1. Draw curves of power factor to a base of primary current up to 40 A for the following motor, neglecting all leakage reactances, tertiary m.m.f. and primary resistance, (*a*) with no injected e.m.f. and (*b*) with an injected e.m.f. between brushes of 29·4 V leading the secondary e.m.f. by 90°.

Motor data: 30-h.p., 420-V, 3-ph., delta-connected primary.
No-load current $= 0{\cdot}7 - j6{\cdot}0$ A/ph.
Secondary circuit resistance $= 2{\cdot}6\ \Omega$.

2. The data refer to a 100-h.p., 420-V, 4-pole, 3-ph., 50-c/s, delta-connected Osnos motor having an e.m.f. of 18 V between brushes leading the secondary e.m.f. by 60° injected into the stator circuit.

Exciting circuit resistance	$= 100\ \Omega$/ph.
Exciting circuit reactance	$= 15\ \Omega$/ph.
Stator resistance	$= 0{\cdot}1\ \Omega$/ph.
Stator leakage reactance	$= 0{\cdot}8\ \Omega$/ph.
Secondary circuit resistance	$= 0{\cdot}3\ \Omega$/ph. (referred to primary)
Secondary circuit reactance	$= 0{\cdot}7\ \Omega$/ph. (referred to primary)

Determine the primary current and power factor and the h.p. output when running at synchronous speed, and the speed when running at no-load.

3. A 150-h.p., 50-c/s Osnos motor has a flux per pole of 0·02 Wb, 15 slots per pole on the rotor and 15 commutator segments per pole. Determine the voltage between brushes spaced at 120° (elec.), and the voltage between adjacent segments for the following arrangements of tertiary winding—

(*a*) Simple lap winding with single-turn coils and a coil span of 2 slot pitches.
(*b*) Duplex lap winding with single-turn coils and a coil span of 2 slot pitches.
(*c*) Bar winding joined at remote end.
(*d*) Bar winding joined at commutator end and having coil span of 12 slot pitches.

4. A 6-pole, 50-c/s Osnos motor has a flux per pole of 0·02 Wb and a tertiary winding with 84 lap-connected single-turn coils. The required injected e.m.f. (equivalent-star value) is 5 V. Determine the percentage coil span the tertiary-winding coils should have.

5. A 50-c/s Osnos motor has a tertiary winding consisting of bars connected to the commutator at one end and to an end ring at the other. If the injected e.m.f. between the brushes is to be 7 V, what must be the flux per pole?

CHAPTER 11

THE THREE-PHASE SERIES MOTOR

If a three-phase motor having a series characteristic, i.e. the speed falling with increasing load, is required the three-phase series motor is available. The stator of such a machine is similar to that of an induction motor, and the rotor carries an ordinary commutator winding with three sets of brushes per pole-pair spaced at 120° (elec.). The stator and rotor are connected in series as shown in Fig. 11.1 from which it is evident that the flux will be dependent

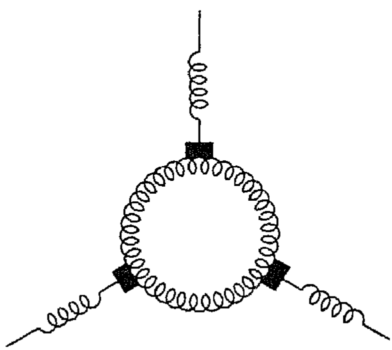


Fig. 11.1. Diagrammatic Representation of Three-phase Series Motor

on the current resulting in the "series" speed-torque characteristic.

Speed control is effected by simultaneous movement of all the brushes round the commutator.

To ease commutation difficulties and for other reasons explained later (page 201) there is usually a transformer connected between stator and rotor in order to reduce the voltage applied to the commutator; if this is a phase-shifting transformer it can give the same effect as movement of the brushes and so can be used for speed control.

Although not widely used in industrial practice the series motor is useful for drives in which the load increases with speed as with fans, centrifugal pumps and compressors or where a large starting torque is required as with transporters or turntables.

Magnetomotive Forces

The motor can conveniently be represented diagrammatically as in Fig. 11.2, the rotor being represented by an equivalent-star winding as described on page 14.

The currents in both stator and rotor windings set up rotating m.m.f.'s moving at synchronous speed as described in Chapter 1; the connexions are so arranged that both m.m.f.'s move in the same direction so that the resultant m.m.f., which sets up the air-gap flux, also moves at synchronous speed. The relative positions of the m.m.f.'s depend, however, on the relative positions of the stator-winding axes and the brushes. For most purposes it may be assumed

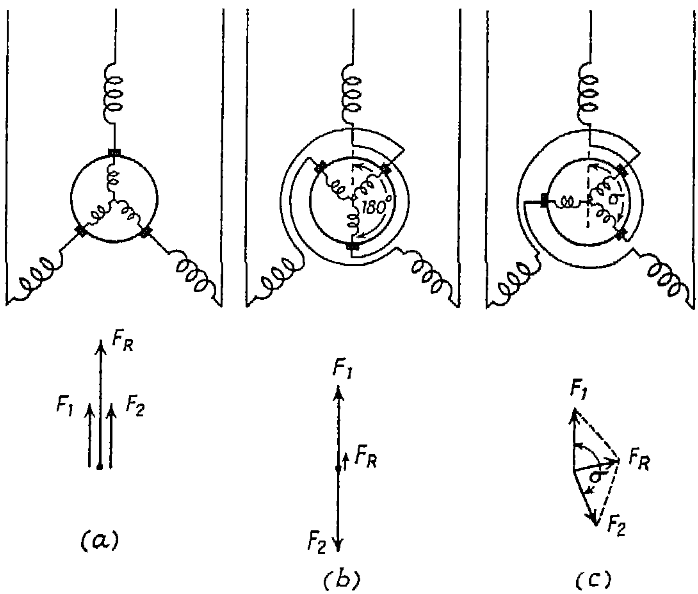


FIG. 11.2. SPACE VECTOR DIAGRAMS OF M.M.F. FOR DIFFERENT BRUSH POSITIONS

(*a*) Neutral position.
(*b*) Short-circuit position.
(*c*) Running position.

that the distribution of both stator and rotor m.m.f.'s over the gap surface is sinusoidal so that they may be represented by space vectors having magnitudes as calculated from eqs. (1.3) and (1.5).

EFFECT OF BRUSH POSITION. If the position of the brushes is adjusted so that the two m.m.f.'s act along the same axis and assist each other as shown in Fig. 11.2(*a*), the resultant m.m.f. acting in the air gap will be $F_R = F_1 + F_2$ as shown by the space vector diagram. A flux acting along this axis will thus be set up but, since it is in the same axis as the rotor m.m.f., there will be no torque produced on the rotor and the machine will not rotate. The magnitude of the flux will be sufficient to generate e.m.f.'s in the windings equal and opposite to the applied voltage, and the current which flows will be just large enough to set up this flux, i.e. between 10

and 20 per cent of full-load current—the conditions are thus somewhat similar to those obtaining in a transformer on open-circuit. This brush position is thus called the no-load or *neutral* position.

If the brushes are moved through 180° to the position shown in Fig. 11.2(*b*) the stator and rotor m.m.f.'s still act along the same axis but now oppose each other—the resultant m.m.f. is now $F_1 - F_2$ and will set up a relatively small flux (zero if $F_1 = F_2$); the effective reactance is thus low and the current high. Conditions are therefore similar to those of a transformer on short-circuit and this is known as the *short-circuit* position.

If the brushes are displaced by an angle σ from the neutral position as shown in Fig. 11.2(*c*) the m.m.f.'s act along different axes and produce a resultant m.m.f. as shown in the vector diagram. This resultant sets up a flux which has a component at 90° to the rotor m.m.f.; a torque is therefore produced and the motor rotates. The value of σ for normal running is between 140° and 170°.

Although the motor is normally started by movement of the brushes from the neutral position it is more convenient, when discussing the theory of the machine, to measure the brush displacement as the angle θ ($= 180° - \sigma$) from the short-circuit position, this angle being normally between 10° and 40°.

Direction of Rotation. If the motor is stationary in the neutral position conditions are as though there were a N pole on the stator opposite a S pole on the rotor; these will attract each other and there will be no tendency to rotate. If, however, the brushes are moved, say, anticlockwise the S pole on the rotor will be moved in this direction; the attraction between the two poles will therefore tend to pull the S pole back towards its original position and the motor will rotate in the opposite direction to that in which the brushes were displaced from the neutral position. Similarly, if the brushes had been moved clockwise, rotation would have been anticlockwise. Reversal of rotation of the motor can thus easily be carried out by moving the brushes, but in one case the motor would be running in the same direction as the rotating field and in the other case in the opposite direction to it. If the motor is allowed to run in the opposite direction to that of the field, rotor iron losses are excessive and the commutation is poor so that, in practice, the brushes are always displaced from the neutral position in a direction against that of the rotating field, and reversal, if required, is effected by interchanging two of the stator supply leads as in the induction motor.

Vector and Complexor Diagrams

To avoid confusion in drawing the complete diagrams for the motor it is desirable to separate the space vector diagrams for the m.m.f.'s from the complexor diagrams for the currents and voltages.

Motor Running below Synchronous Speed. If the brushes are shifted by an angle σ from the neutral (high-impedance) position,

i.e. θ from the short-circuit position, the space vector diagram of the m.m.f.'s will be as shown in Fig. 11.3(*a*); the horizontal axis is chosen to bisect the angle between the stator and rotor m.m.f. vectors so that the resultant m.m.f. F_R is at an angle ψ to the horizontal. It is assumed also that the ratio of rotor effective turns to stator effective turns (T_2'/T_1') is K which, in the diagram, is slightly less than 1. For a given current all these m.m.f.'s are of

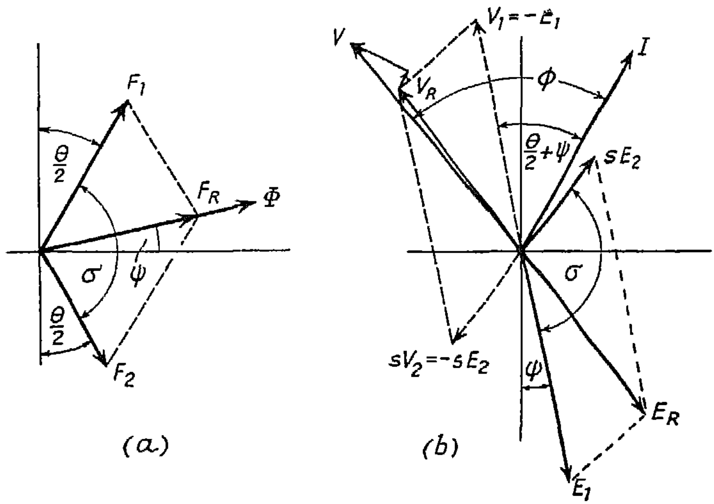


FIG. 11.3. VECTOR AND COMPLEXOR DIAGRAMS FOR SUB-SYNCHRONOUS SPEED

(*a*) Space diagram of m.m.f.'s.
(*b*) Complexor diagram of currents and voltages.

constant magnitude and rotating at synchronous speed, and the resultant sets up the flux Φ.

This rotating flux Φ sets up in the stator conductors an e.m.f. $E_1 = (\sqrt{2})\pi f T_1'\Phi$ and, as in the induction-motor complexor diagram, this may be drawn at 90° to Φ as shown. Since the brushes have been moved an angle σ against the direction of the rotation of the flux, the flux will cut the corresponding rotor phase before cutting the stator phase by a time depending on this angle, i.e. the rotor e.m.f. will lead E_1 by the angle σ. At standstill the magnitude of the rotor e.m.f. will be given by $E_2 = (\sqrt{2})\pi f T_2'\Phi = KE_1$. At synchronous speed the rotor conductors will be moving at the same speed as the flux and so the rotor e.m.f. will be zero; at a slip s it will be $sE_2 = sKE_1$.

When the current in a particular phase of the stator is at a peak value it can be seen from m.m.f. diagrams that the direction of the m.m.f. F_1 produced by it is 90° displaced from the centre-line of the phase in the same direction as the flux rotation; the resultant m.m.f.

lags this spatially by $\{90 - (\theta/2 + \psi)\}$. The e.m.f. E_1 thus lags the current by $\{180 - (\theta/2 + \psi)\}$, so that the current complexor can be drawn on the complexor diagram as shown.

The resultant generated e.m.f. is the sum of E_1 and $sE_2 = E_R$. The applied voltage must overcome this and also the resistance and leakage reactance drops of both windings, these being in phase and in quadrature respectively with the current as indicated. It is often

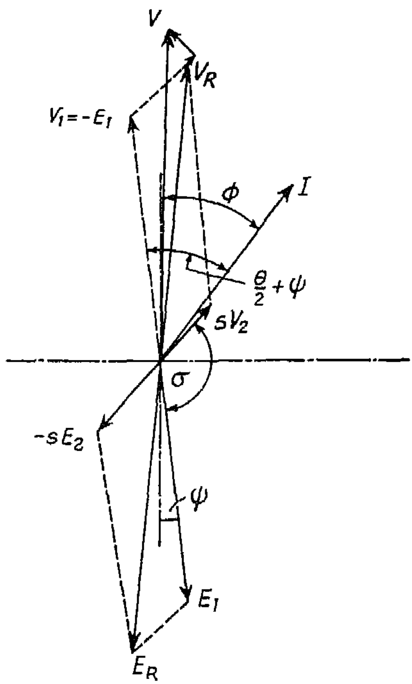


FIG. 11.4. COMPLEXOR DIAGRAM FOR SUPER-SYNCHRONOUS SPEED

convenient to utilize, not the generated e.m.f.'s E_1 and sE_2, but the voltage components that have to be applied to overcome them, i.e. V_1 and sV_2, and these are also drawn on the diagram. It may be noted that the current I always lags the voltage component V_1 by the angle $(\theta/2 + \psi)$ whatever the slip.

It can be seen also that the current I has a component in phase opposition to E_1, showing that power is being delivered *to* the stator, but that it has a component in phase with sE_2 showing that power is being delivered *from* the rotor back to the supply; this is in accordance with the state of affairs obtaining in an induction motor running below synchronous speed where the slip power also appears in the rotor circuit.

MOTOR RUNNING ABOVE SYNCHRONOUS SPEED. If the motor is allowed to run above synchronous speed, the brush shift being the same, the space diagram will be similar to that of Fig. 11.3(*a*) but all the magnitudes will be reduced in proportion to the reduction of current. The complexor diagram will, however, be different as shown in Fig. 11.4 since the slip is now negative. It can be seen that the current has a component in phase opposition to both E_1 and sE_2 showing that power is now being delivered to both rotor and stator. An improvement in the power factor can also be noted.

Circle Diagram

A simplified locus diagram based on the above, but with resistance and leakage reactance neglected, can be drawn and enables some simple relations regarding the behaviour of the motor to be observed.

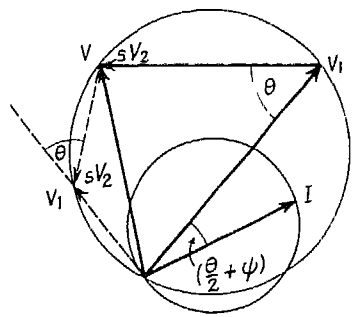


FIG. 11.5. LOCUS DIAGRAM OF VOLTAGES AND CURRENT

The terminal voltage V is constant and this is made up of the stator and rotor components V_1 and sV_2 which, for a given brush setting, are inclined to each other at the angle θ. As the slip s is varied the apex of the triangle formed by V_1 and sV_2 based on V therefore moves around a circle subtending the angle θ as shown in Fig. 11.5.

It has been shown that the current I lags V_1 by a constant angle $\theta/2 + \psi$. If I is assumed proportional to the flux Φ then I will be proportional to V_1 so that the end of the current complexor also moves on a circular locus.

The part of the circle corresponding to the normal working range of speed (between $s = 1$ and s = approximately -1) depends on the ratio T_2'/T_1' and three cases may usefully be considered.

RATIO $T_2'/T_1' = 1$. If $s = 1$ (standstill) then $V_1 = V_2$ so that the working point lies on the horizontal diameter as shown in Fig. 11.6(*a*). With $T_2'/T_1' = 1$, $\psi = 0$ so that I lags V_1 by $\theta/2$ and is therefore horizontal, i.e. it lags V by 90° as would be expected since, neglecting losses, no power is absorbed when the motor is stationary. As s decreases V_1 increases until it reaches a maximum when lying

along the diameter shown dotted. Since the current I is proportional to V_1 the current reaches a maximum when lagging this diameter by $\theta/2$ and this value of current is the diameter of the current

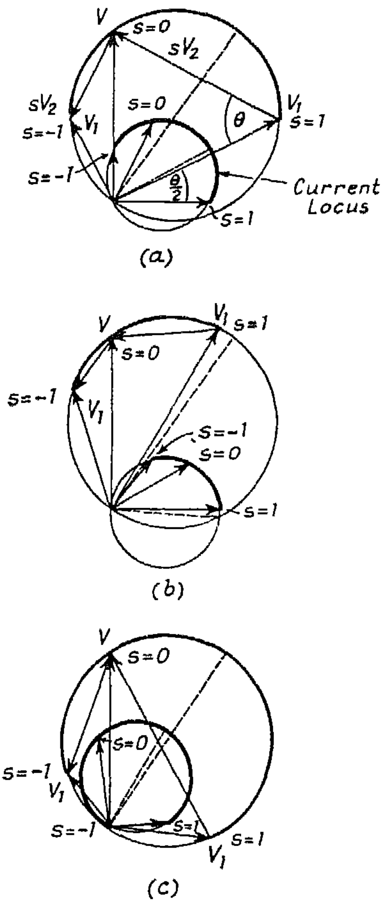


FIG. 11.6. LOCUS DIAGRAMS WITH DIFFERENT VALUES OF T_2'/T_1'

(a) Ratio $T_2'/T_1' = K = 1$.
(b) Ratio $T_2'/T_1' = 0.5$.
(c) Ratio $T_2'/T_1' = 2$.

circle. At $s = 0$, sV_2 is zero and V_1 coincides with V; I then lags V, the applied voltage, by $\theta/2$, this being the power factor at synchronous speed. At $s = -1$ the end of V_1 again lies on the horizontal diameter and I is in phase with the voltage V, i.e. the power factor is unity. At speeds above that corresponding to

$s = -1$, I will lead V by a small angle. The working range between $s = 1$ and $s = -1$ is shown by a heavy line on the locus diagrams.

RATIO $T_2'/T_1' = 0{\cdot}5$. When $s = 1$, $sV_2 = 0{\cdot}5V_1$ and the operating point is as shown in Fig. 11.6(*b*). I again lags V by 90°. At $s = 0$, I lags V by $\theta/2 + \psi$, i.e. by a greater angle than when $T_2'/T_1' = 1$.

RATIO $T_2'/T_1' = 2$. Conditions are now as shown in Fig. 11.6(*c*) and it can be seen that power-factor conditions are improved.

The diagrams of Fig. 11.6 thus show that a ratio for T_2'/T_1' greater than 1 is desirable in order to obtain a good power factor;

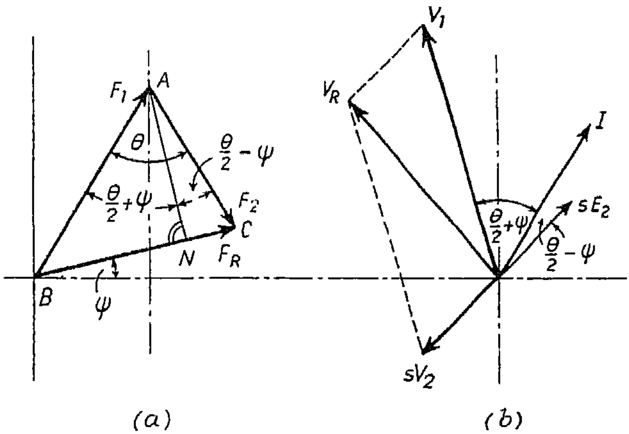


FIG. 11.7. DIAGRAMS FOR DEVELOPMENT OF POWER EQUATION

(*a*) Space vectors.
(*b*) Complexors.

it is shown later, however, that such a ratio may lead to instability. That part of the circle below the $s = 1$ point corresponds to rotation in the opposite direction to the field which, as already explained, is not employed in normal practice.

Power

The power handled by the stator and rotor can be determined by considering the current in relation to the stator and rotor voltages.

The relevant parts of the diagrams of Fig. 11.3 may be redrawn as shown in Fig. 11.7. AN in the vector diagram is parallel to V_1 and therefore at 90° to BC. Also the voltage V_1 leads I by $(\theta/2 + \psi)$.

The power delivered to the stator is thus

$$V_1 I \cos(\theta/2 + \psi) = V_1 I \times AN/AB$$

Since
$$\sin\theta = (2/F_1F_2) \times \text{area of } ABC$$
$$= (2/F_1F_2)(1/2)AN \,.\, F_R$$
$$AN = (F_1F_2/F_R)\sin\theta = (KF_1{}^2/F_R)\sin\theta$$

Hence stator power $= V_1I(KF_1{}^2/F_1F_R)\sin\theta = V_1IK(F_1/F_R)\sin\theta$.

In the rotor $sV_2 = sKV_1$ and it leads the current by $180 - (\theta/2 - \psi) = 180° - \angle NAC$. The rotor power is thus $sV_2I\cos(180 - \angle NAC)$. But

$$\cos(180 - \angle NAC) = -\cos NAC = -AN/F_2$$
$$= -(F_1F_2/F_RF_2)\sin\theta = -(F_1/F_R)\sin\theta.$$

Hence $\qquad$ rotor power $= -sV_2I(F_1/F_R)\sin\theta$
$$= -sKV_1I(F_1/F_R)\sin\theta$$

The total power supplied is the sum of the stator and rotor powers and is, neglecting losses, equal to the mechanical output. Thus—

$$\text{Mech. output} = V_1IK(F_1/F_R)\sin\theta - sKV_1I(F_1/F_R)\sin\theta$$
$$= V_1IK(F_1/F_R)\sin\theta(1 - s) \quad . \quad . \quad . \quad (11.1)$$

The relations between stator power, rotor power and mechanical output are thus in the ratio $1:s:(1 - s)$ exactly as in the induction motor. The slip power in the rotor is, however, returned to or taken from the supply as a result of the change of frequency effected at the commutator.

Torque

If a flux Φ is linking a coil of T turns to produce an e.m.f. E, the e.m.f. may be written $E = Ix$ where x is the effective reactance of the coil and I the current setting up the flux. Thus

$$\Phi = E/\{(\sqrt{2})\pi fT\} = Ix/\{(\sqrt{2})\pi fT\} = IT \,.\, x/\{(\sqrt{2})\pi fT^2\}$$
$$= Fx/\{(\sqrt{2})\pi fT^2\} = kFx$$

where $k = 1/\{(\sqrt{2})\pi fT^2\}$.

Flux may thus be related to the m.m.f. producing it and to the coil reactance.

The stator e.m.f. E_1 set up by the flux Φ can thus be written

$$E_1 = (\sqrt{2})\pi fT_1'kF_Rx = F_Rx/T_1'$$

The torque is proportional to the flux and the component of the rotor m.m.f. in quadrature with it. Referring to Fig. 11.7, AN represents the quadrature component of the rotor m.m.f. and has been shown (page 189) to be given by $AN = (KF_1^2/F_R)\sin\theta$.

The flux has been shown to be given by $\Phi = kF_Rx$ so that

$$TM = kF_Rx \times (KF_1^2/F_R)\sin\theta = kKF_1^2x\sin\theta \qquad (11.2)$$

CONDITION FOR MAXIMUM TORQUE. For a given value of K the expression for the torque is a maximum when $\angle ACN = 90°$, i.e. when $F_1\cos\theta = F_2 = KF_1$ or when $K = \cos\theta$.

Putting this value in the torque expression (eq. (11.2)) gives

$$TM = k_1F_1^2x\sin\theta\cos\theta$$

which has a maximum value when $\theta = 45°$ and $\cos\theta = 0{\cdot}707$.

For a given stator, therefore, the number of rotor conductors to give the maximum torque would be $T_2' = 0{\cdot}707T_1'$, and the brush shift from the short-circuit position should be 45°. In practice such a low ratio and such a large angle would give a poor power factor as seen from the locus diagrams of Fig. 11.6, and a ratio near to 1 and angles between 10° and 30° are usual.

TORQUE-SLIP CURVES. By substituting for F_1 in terms of V and s in eq. (11.2) it is possible to obtain the torque-slip or torque-speed curve, resistance and leakage reactance being neglected for simplicity.

From the voltage and m.m.f. diagrams of Fig. 11.7, assuming $V = V_R$, the following relations can be obtained—

$$V^2 = V_1^2 + s^2V_2^2 - 2V_1sV_2\cos\theta$$
$$= V_1^2(1 + s^2K^2 - 2Ks\cos\theta) \quad . \quad . \quad (11.3)$$

and
$$F_R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\theta$$
$$= F_1^2(1 + K^2 - 2K\cos\theta)$$

But
$$V_1 = F_Rx/T_1' = (F_1x/T_1')\sqrt{(1 + K^2 - 2K\cos\theta)}$$

Substituting this in eq. (11.3) gives

$$V^2 = \{F_1^2x^2/(T_1')^2\}(1 + K^2 - 2K\cos\theta)(1 + s^2K^2 - 2Ks\cos\theta)$$

The value of F_1 obtained from this may now be substituted in the torque expression (eq. (11.2)).

$$TM = kKx\sin\theta\,\frac{V^2(T_1')^2}{x^2(1 + K^2 - 2K\cos\theta)(1 + s^2K^2 - 2Ks\cos\theta)}$$
$$= \frac{kKV^2(T_1')^2\sin\theta}{x(1 + K^2 - 2K\cos\theta)(1 + s^2K^2 - 2Ks\cos\theta)}$$

By putting $s = 1 - f_r/f_1$, the above equation can be expressed in terms of speed as follows—

$$\frac{n_r}{n_1} = \frac{f_r}{f_1} = 1 - \frac{\cos\theta}{K} \pm \sqrt{\left\{\frac{kV^2(T_1')^2\sin\theta}{Kx(TM)(1 + K^2 - 2K\cos\theta)} - \frac{\sin^2\theta}{K^2}\right\}} \quad . \quad . \quad . \quad (11.4)$$

A set of typical torque-speed curves is shown in Fig. 11.8.

Stability

It can be seen from the curves, and also from the fact that eq. (11.4) has an optional sign before the root, that there are two values of speed for a given torque; for speeds below that corresponding to maximum torque the motor is unstable in that a decrease in speed is accompanied by a decrease in torque or, conversely, that an increase in torque is accompanied by an increase in speed. The motor can thus not normally be run at speeds below the critical

maximum-torque value, and this limit is indicated by the dotted curve in Fig. 11.8.

The unstable range can also be observed on the circle diagrams of Fig. 11.6. As the complexors of current and voltage move round the circle from the standstill position they increase until they become diameters; until these positions are reached the torque, which is proportional to I^2, is increasing with an increase of speed and the motor is therefore unstable. If, however, the standstill position is on or beyond the diameter the motor is stable at all speeds.

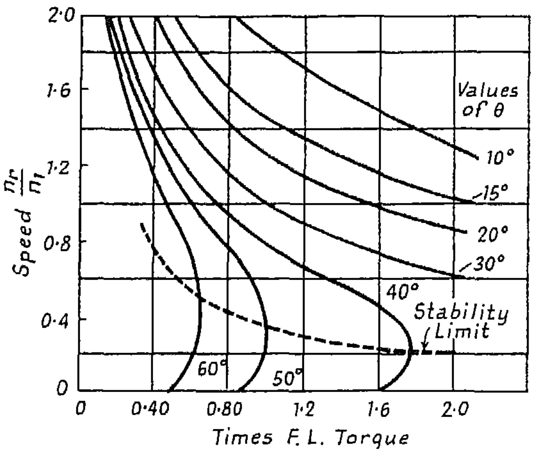


FIG. 11.8. SPEED-TORQUE CURVES

Using eq. (11.4) it can be seen that the critical speed, n_c, occurs when the quantity under the root vanishes, the speed then becoming univalued and given by

$$n_c/n_1 = 1 - (\cos\theta)/K \quad . \quad . \quad . \quad (11.5)$$

It is evident that the value of K has an important effect on stability and three cases may usefully be considered.

CASE 1: $K = \cos\theta$. This relation has been shown (page 190) to give maximum torque, and inserting it into the speed equation (11.4) gives—

$$\frac{n_r}{n_1} = \pm\sqrt{\left(\frac{kV^2(T_1')^2}{(TM)\,.\,x\,.\,\sin\theta\cos\theta} - \tan^2\theta\right)}$$

The limiting stable speed, i.e. the speed at which the above becomes univalued, is zero. The motor is thus stable over the whole range of speed, a typical curve being shown in Fig. 11.9 for a motor with $\theta = 30°$.

The same conclusion can be reached by considering the circle diagram (Fig. 11.5) since if V_1 lies along a diameter $K = sV_2/V_1 = \cos\theta$.

CASE 2: $K = 1$. The speed equation becomes—

$$\frac{n_r}{n_1} = 1 - \cos\theta \pm \sqrt{\left\{\frac{kV^2(T_1')^2\sin\theta}{2x(TM)(1-\cos\theta)} - \sin^2\theta\right\}}$$

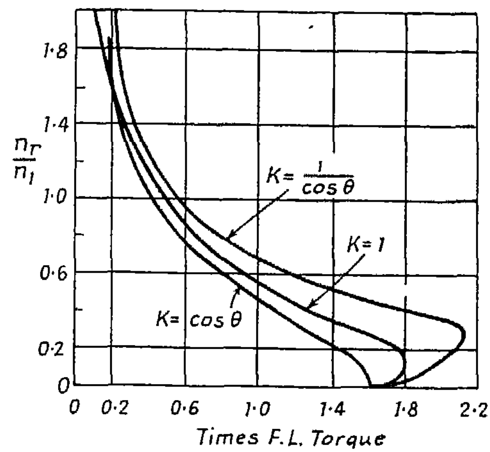


FIG. 11.9. SPEED-TORQUE CURVES WITH DIFFERENT VALUES OF $K(= T_2'/T_1')(\theta = 30°)$

The limiting stable speed is thus given by $n_r/n_1 = 1 - \cos\theta$, and the speed-torque curve follows the shape shown in Fig. 11.9, the curve being drawn to give the same starting torque as in the previous case.

CASE 3: $K = 1/\cos\theta$. With normal values of θ, K will be greater than 1 and the speed-torque curve will have a more pronounced peak value of torque as shown in Fig. 11.9.

Power Factor

It has already been mentioned (page 191) that the most desirable brush shift with regard to maximum torque may be inadmissible on account of poor power factor and also that the ratio K has an important influence.

A general expression for power factor can be developed by finding the voltage components in phase and in quadrature with the current.

The in-phase component $V\cos\phi$ can be found by dividing the power equation (11.1) by I—

$$\begin{aligned} V\cos\phi &= V_1K(F_1/F_R)\sin\theta\,.\,(1-s) \\ &= K(F_1x/T_1')\,.\,\sin\theta\,.\,(1-s) \end{aligned}$$

The reactive component $V_1 \sin\left(\frac{\theta}{2} + \psi\right)$ in the stator winding is produced by the flux set up by the component of m.m.f. in the stator axis, i.e. by

$$F_1 - F_2 \cos\theta = F_1(1 - K\cos\theta)$$

Hence $$V_1 \sin\left(\frac{\theta}{2} + \psi\right) = (F_1 x/T_1')(1 - K\cos\theta)$$

The reactive component $sV_2 \sin\left(\frac{\theta}{2} - \psi\right)$ induced in the rotor winding is set up by the m.m.f. acting along the rotor axis, i.e. by

$$F_2 - F_1\cos\theta = F_1(K - \cos\theta)$$

If running with a slip s the e.m.f. induced will have to be multiplied by s so that

$$sV_2 \sin\left(\frac{\theta}{2} - \psi\right) = (sF_1 x/T_2')(K - \cos\theta)$$

The total reactive e.m.f. is the sum of these components so that

$$\begin{aligned} V\sin\phi &= (F_1 x/T_1')(1 - K\cos\theta) + (sF_1 x/T_2')(K - \cos\theta) \\ &= (F_1 x/T_1')\{1 + s - \cos\theta(K + s/K)\} \end{aligned} \qquad (11.6)$$

Hence $$\tan\phi = \frac{V\sin\phi}{V\cos\phi} = \frac{K(1+s) - (K^2+s)\cos\theta}{K^2(1-s)\sin\theta} \qquad (11.7)$$

and $$\cos\phi = \frac{1}{\sqrt{(1+\tan^2\phi)}}$$

The power factor is thus seen to be dependent on the turns ratio K as well as on the slip and the brush shift. The expressions with the values of K considered when dealing with stability (page 192) may usefully be investigated.

CASE 1: $K = \cos\theta$. The above expression for $\tan\phi$ becomes

$$\tan\phi = \tan\theta/(1 - s)$$

and $$\cos\phi = \frac{1}{\sqrt{\{1 + \tan^2\theta/(1-s)^2\}}}$$

At synchronous speed, when $s = 0$, this becomes $\cos\theta$, and therefore the power factor at this speed can never be unity. The advantage of working with a small brush shift from the short-circuit position is thus evident since the synchronous-speed power factor is equal to the cosine of this angle. In Fig. 11.10 is shown a power-factor curve for this case on a machine having $\theta = 30°$.

CASE 2: $K = 1$. The expression (11.7) for $\tan \phi$ becomes

$$\tan \phi = \frac{(1+s)(1-\cos \theta)}{(1-s)\sin \theta}$$

and

$$\cos \phi = \frac{1}{\sqrt{\left[1 + \left\{\frac{(1+s)(1-\cos \theta)}{(1-s)\sin \theta}\right\}^2\right]}}$$

$$= \frac{(1-s)\cos(\theta/2)}{\sqrt{(1+s^2-2s\cos\theta)}}$$

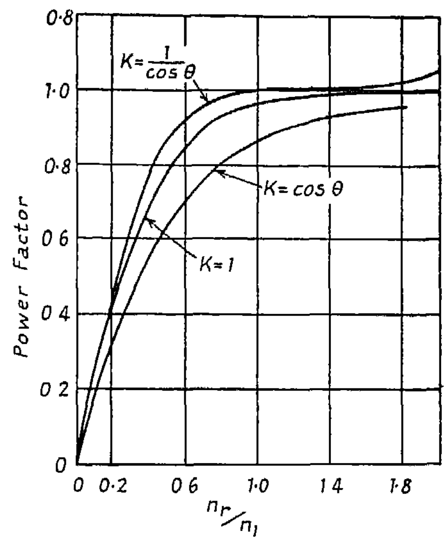


FIG. 11.10. POWER FACTOR AT VARIOUS SPEEDS AND VALUES OF $K(\theta = 30°)$

At synchronous speed the power factor becomes $\cos(\theta/2)$, at $s = -1$ it becomes unity and at speeds above this it leads, the curve for a machine having $\theta = 30°$ being shown in Fig. 11.10.

CASE 3: $K = 1/\cos \theta$. The expression for $\tan \phi$ becomes

$$\tan \phi = s \sin \theta \cos \theta/(1-s)$$

and

$$\cos \phi = \frac{1}{\sqrt{\left\{1 + \left(\frac{s \sin \theta \cos \theta}{1-s}\right)^2\right\}}}$$

At synchronous speed this becomes unity and above synchronous speed the power factor is leading.

CONDITIONS FOR HIGH POWER FACTOR. It is thus seen that for high power factors at low speeds the value of K must be high, i.e.

the rotor must be magnetically stronger than the stator; it has, however, already been seen (page 193) that such a condition leads to instability so that the two requirements are incompatible. If the motor is to be stable down to zero speed, the value of K must be less than $\cos\theta$ and unity-power-factor operation is not possible. The range of stable running and high power factor can be increased by using a small brush shift θ but the available torque falls off rapidly when θ is less than 30°.

The speed at which the power factor becomes unity can be found by equating the reactive e.m.f. given in eq. (11.6) to zero. Thus

$$1 + s - \cos\theta(K + s/K) = 0$$

whence $$n_r/n_1 = 1 - s = 1 - \frac{1 - K\cos\theta}{(\cos\theta)/K - 1}$$

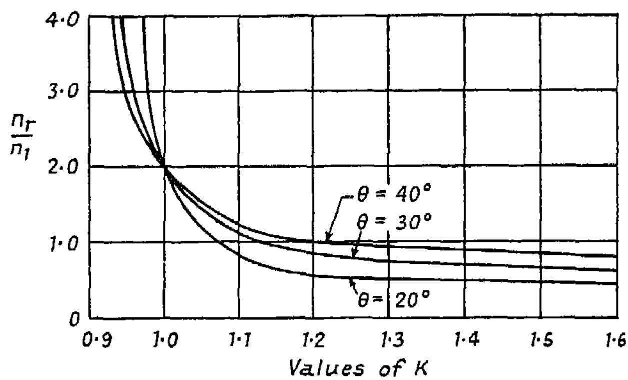


FIG. 11.11. SPEEDS AT WHICH POWER FACTOR BECOMES UNITY

At speeds below this the current will lag and at higher speeds it will lead. Curves showing the speed for unity power factor for motors having brush shifts of 20°, 30° and 40° are given in Fig. 11.11.

The difficulties which these stability and power-factor relations introduce into the design of the simple motor are illustrated by the following example.

Example 11.1. A three-phase series motor is to be designed to have its lowest speed characteristic with $\theta = 60°$ and to be stable down to zero speed. Find the required value of K and determine the power factor when running at a high speed with $\theta = 20°$.

What should be the ratio K to give unity power factor at synchronous speed with $\theta = 20°$ and what would be the limiting speed for stable running with this ratio?

From eq. (11.5), putting $n_r = 0$, gives $K = \cos\theta = \cos 60° = 0{\cdot}5$.

With this ratio and $\theta = 20°$ ($\cos\theta = 0{\cdot}94$; $\sin\theta = 0{\cdot}342$) the power factor at synchronous speed is given by eq. (11.7)—

$$\tan\phi = (0{\cdot}5 - 0{\cdot}25 \times 0{\cdot}94)/(0{\cdot}25 \times 0{\cdot}342) = 3{\cdot}1$$

Hence $\cos \phi = 0{\cdot}306$, which is an impracticably low value.

To give unity power factor at synchronous speed, with $\theta = 20°$, the value of K would have to be

$$K = 1/\cos \theta = 1/0{\cdot}94 = 1{\cdot}06$$

The limiting speed for stable running is given by

$$n_r/n_1 = 1 - (\cos \theta)/K = 1 - 0{\cdot}94/1{\cdot}06 = 0{\cdot}11$$

i.e. 11 per cent of synchronous speed.

It is thus desirable to be able to change the effective ratio K as the angle θ is varied; this can be done by a variable-ratio transformer between stator and rotor or by a motor with fixed and movable brushes as described on pages 201 and 206.

Effect of Voltage Drops and Saturation on Characteristics

In the previous analysis the resistance and leakage reactance drops and saturation have been neglected. In general the voltage drops tend to increase stability while saturation, which causes a reduction of the effective reactance x at high currents, reduces the torque for a given current but increases it for a given speed. The current locus ceases to be a circle and becomes an ellipse with the longer axis passing through the origin. The effect of these items is shown in the following section.

Predetermination of Behaviour

To use the previously-described complexor diagrams for predetermining the characteristics, the magnetization curve, i.e. the relation between current and e.m.f. E_1 induced in the stator winding, must be known. Assuming current to pass through the stator winding only, this can easily be calculated from design data or obtained by test, giving a curve as shown by curve A in Fig. 11.13(b). When the motor is operating the flux is produced, not by F_1, but by the resultant m.m.f. F_R. From the known brush shift and the vector diagram of Fig. 11.3(a) the ratio F_1/F_R can be found. For a given voltage on curve A, and therefore a given degree of saturation, the current necessary to produce the same voltage when both stator and rotor windings are excited can be found by multiplying the original current ordinate by F_1/F_R, giving curve B on Fig. 11.13(b).

To draw a diagram for predetermining the behaviour, the direction of the stator voltage complexor V_1 can be drawn vertically along OA, as shown in Fig. 11.12, with the line AB (as shown dotted) at an angle of θ to OA and thus representing the direction of the rotor voltage complexor sV_2. A circle about O of radius V represents the locus of the end of the applied voltage complexor, and a line OC lagging OA by $(\theta/2 + \psi)$ represents the direction of the current. On the right is drawn the magnetization curve with both windings excited (curve B of Fig. 11.13(b)).

Neglecting resistance and leakage reactance drops, consider a stator voltage V_1 equal to OA'—the current necessary to induce this is obtained from the magnetization curve as DF. If $A'B'$ is drawn parallel to AB to cut the applied voltage circle at B', then $OA'B'$ is the voltage triangle; OB' is the position of the applied voltage complexor and $A'B'$ the rotor voltage sV_2.

The slip is obtained from $A'B'/OA' = sV_2/V_1$, the phase angle ϕ by the angle between $V(OB')$ and the direction of the current complexor, and the torque by

$$V_1 I \cos(\theta/2 + \psi) = OA' \,.\, DF \cos(\theta/2 + \psi)$$

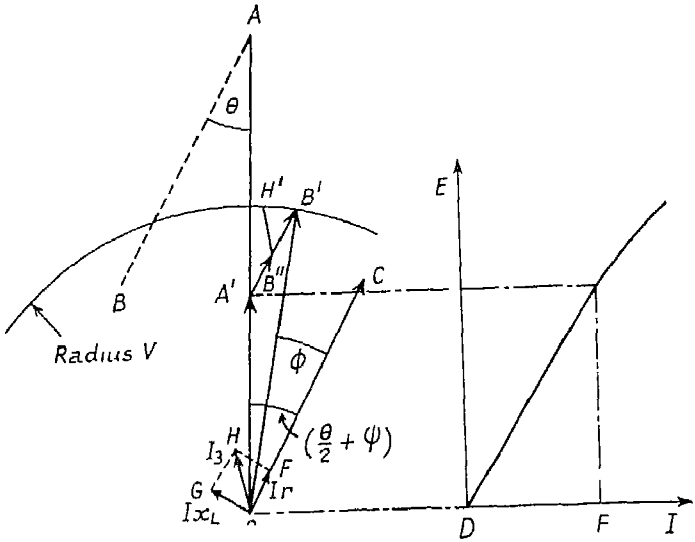


FIG 11.12. COMPLEXOR DIAGRAM FOR PREDETERMINATION OF CHARACTERISTICS

If resistance and leakage reactance drops are to be taken into account, the two drops can be drawn in phase and in quadrature with the current as shown by OF and OG giving the impedance drop OH. This can be inserted, by trial and error, between the end of the sV_2 complexor $A'B''$ and the applied voltage circle as shown by $B''H'$, giving a new position for the applied voltage and a new value of sV_2 given by $A'B''$. Core loss can, for a given flux, be treated as an additional resistance and included with the resistance drop.

The procedure and the effect of the drops and saturation are illustrated, for a small machine in which the effects are important, by Example 11.2.

Example 11.2. A 3-phase, 5-h.p., 50-c/s, 4-pole series motor is fed from the mains voltage through a transformer which steps the voltage down to 60 V. The effective turns on the stator and rotor

are equal, the brush angle θ is 30°, total resistance (including effect of core loss) $= 0{\cdot}12\ \Omega$, total leakage reactance $= 0{\cdot}07\ \Omega$. The magnetization curve with current passed through stator winding alone is given by curve A of Fig. 11.13(b).

From a vector diagram similar to that of Fig. 11.3, for an angle $\theta = 30°$ the ratio $F_1/F_R = 1/(2 \sin 15°) = 1{\cdot}93$.

(a) Neglecting saturation and impedance drop. The air line of curve A (Fig. 11.13(b)) may be regarded as the magnetization curve; multiplying the current abscissae by 1·93 gives curve B, the stator e.m.f. for various currents when both windings are excited.

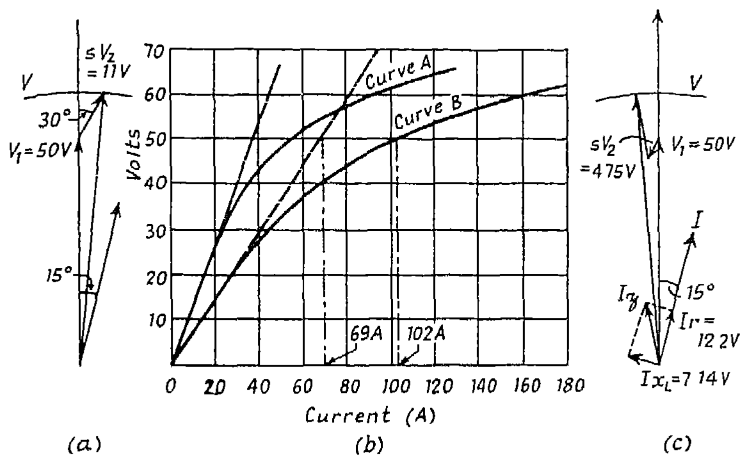


FIG. 11.13. PREDETERMINATION OF CHARACTERISTICS

(a) Neglecting impedance drop.
(b) Magnetization curves.
(c) Allowing for impedance drop

Since $F_1 = F_2$, the angle ψ is zero so that the current complexor lags V by $\theta/2 = 15°$.

Selecting a stator voltage of $V_1 = 50$ V the complexor diagram may be drawn as in Fig. 11.13(a); projecting from this complexor on to the air line of curve B gives the current as 69 A. Drawing the sV_2 complexor from the end of V_1 and at 30° to it gives the position of V and the length of sV_2 as 11 V; as V is to the right of V_1 this gives a speed above synchronism, i.e. the slip is negative and equal to $-11/50 = -0{\cdot}22$. The speed is therefore

$$1{,}500(1 + 0{\cdot}22) = 1{,}830 \text{ r.p.m.}$$

This point is plotted on the lower speed/current curve of Fig. 11.14; other points can be similarly obtained by taking other values of V_1 and the rest of the curve drawn.

(b) Allowing for saturation. At $V_1 = 50$ V the complexor diagram remains the same, but to find the current the projection

must now be carried across to the actual magnetization curve giving a current of 102 A. This point and the rest of the characteristic is shown by the upper dotted curve of Fig. 11.14.

(*c*) Allowance for impedance drop. The complexor diagram with this allowance is shown in Fig. 11.13(*c*)—the current is again 102 A, corresponding to $V_1 = 50$ V, so that $Ir = 102 \times 0{\cdot}12 = 12{\cdot}2$ V and $Ix_L = 102 \times 0{\cdot}07 = 7{\cdot}14$ V. These are drawn respectively in phase with and in quadrature to I giving Iz as shown; fitting this

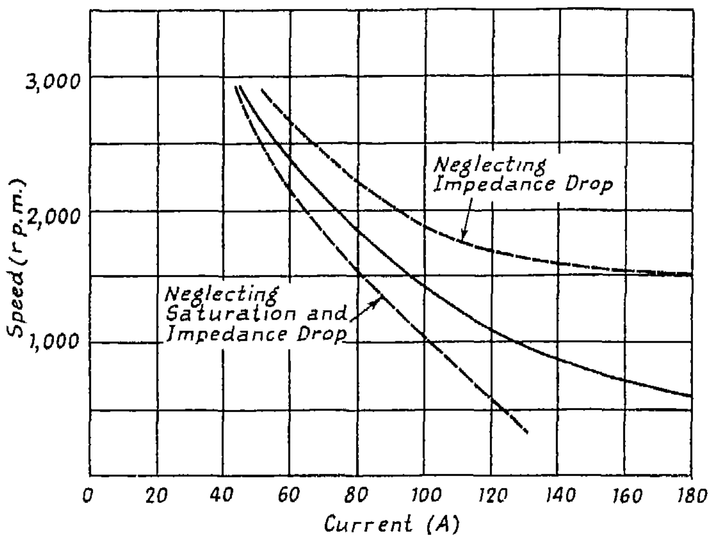


FIG. 11.14. SPEED-CURRENT CHARACTERISTICS

between the applied voltage circle and the sV_2 vector completes the diagram, and sV_2 may be measured as $+\,4{\cdot}75$ V so that $s = 4{\cdot}75/50 = 0{\cdot}095$ and the speed is $1{,}500(1 - 0{\cdot}095) = 1{,}360$ r.p.m. This and similar points for other values of V_1 give the full curve of Fig. 11.14.

Commutation

As the coils short-circuited by the brushes are moving in the rotating field at a speed of $n_r - n_1$ a rotational e.m.f. E_{rb} is set up in them as described on page 32 and given by

$$E_{rb} = (\sqrt{2})\pi(f_r - f_1)T\Phi \text{ volts}$$

This is zero at synchronous speed when the coils are moving at the same speed as the field and, for a given flux (i.e. a given current), varies proportionately at other speeds as shown in Fig. 11.15. Reference to a complexor diagram of coil e.m.f.'s shows that E_{rb} is

in quadrature with the equivalent-star e.m.f. associated with the same brush.

The reactance e.m.f. is also present as in all commutator machines and is proportional to the speed as indicated in Fig. 11.15. The reactance e.m.f. is in phase with the current so that the total e.m.f. in the short-circuited coil shown in Fig. 11.15 is seen to reach a minimum at about 75 per cent of synchronous speed. Commutation

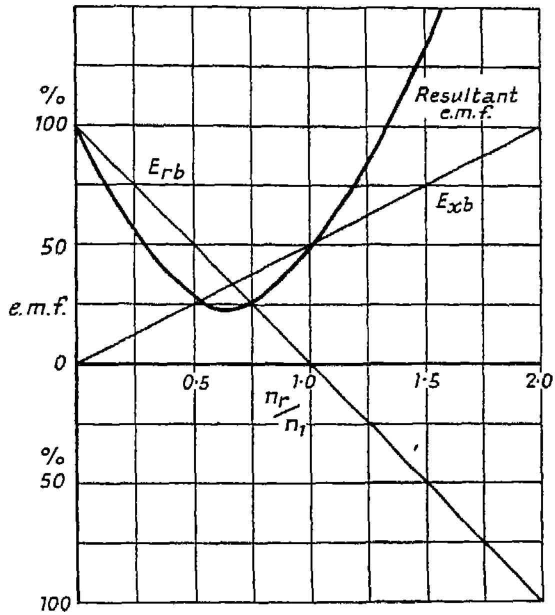


FIG. 11.15. E.M.F. IN SHORT-CIRCUITED COILS

is thus likely to be difficult if the speed exceeds about twice synchronism, and it can also be seen that bad commutation will also result if the motor is allowed to run in the opposite direction to that of the field.

In order to keep down the value of E_{rb} the flux per pole must be limited, and the total rotor voltage cannot, in practice, exceed about 150 V. In almost all cases, therefore, a transformer is necessary between the supply and the stator or, as described in the next section, between the stator and rotor.

Rotor Transformer

It has been seen that for stable running at low speeds the value of K should be low, while to obtain a good power factor at high speeds it should be high. To reconcile these conflicting requirements a variable value of K is desirable and this can be done by connecting

a variable-ratio series transformer between the stator and rotor as shown in Fig. 11.16. This may not necessitate additional expense since, as mentioned above, a main transformer would otherwise be

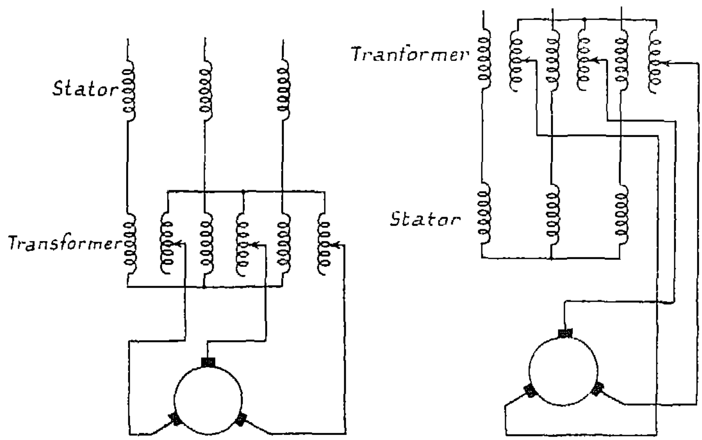


FIG. 11.16. SERIES MOTOR WITH ROTOR TRANSFORMER

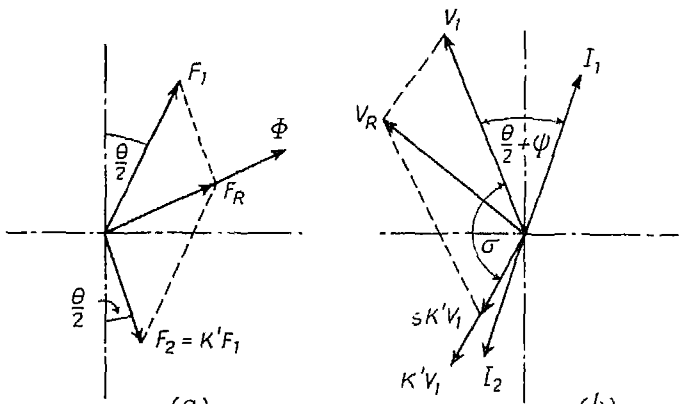


FIG. 11.17. DIAGRAMS WITH ROTOR TRANSFORMER (NEGLECTING TRANSFORMER MAGNETIZING CURRENT)

(a) Vector diagram.
(b) Complexor diagram.

necessary to step the supply voltage down to a value suitable for commutation. Moreover, the rotor transformer has to handle only the slip power and may therefore be smaller than a main transformer, usually about half the size.

COMPLEXOR DIAGRAM. The complexor diagram is similar to that of the simple motor—the stator current I_1 sets up the m.m.f. F_1

as shown in the space vector diagram of Fig. 11.17(*a*). The current in the transformer secondary, and therefore in the rotor, is

$$I_2 = I_1(T_{T1}/T_{T2})$$

so that
$$F_2 = KF_1(T_{T1}/T_{T2}) = K'F_1$$

where K' is the effective rotor/stator turns ratio, including the transformer, i.e.

$$K' = (T_2'/T_1')(T_{T1}/T_{T2})$$

The voltage V_1 is drawn to lead the flux by 90° and I_1 is drawn parallel to F_1 as in the diagram for the simple motor. The magnitude of V_2 at standstill is KV_1 and is equal to the transformer secondary voltage V_{T2}; the primary transformer voltage is therefore

$$V_{T1} = V_{T2}(T_{T1}/T_{T2}) = KV_1(T_{T1}/T_{T2}) = K'V_1$$

At slip s this becomes $sK'V_1$. The total voltage is the sum of these two giving V_R as in the simple motor.

Effect of Transformer Magnetizing Current. Near synchronous speed the voltage across the transformer is small, but at speeds differing widely from synchronism it will be large and the saturation of the transformer may result in an appreciable magnetizing current.

At *sub-synchronous* speeds the rotor is returning power to the supply and therefore supplies the magnetizing current—this current therefore lags V_2 by 90°. The actual rotor current is therefore $I_2 = I_2' + I_m$ where I_2' is the current to counterbalance I_1 and I_m is the magnetizing current. This is shown in Fig. 11.18(*a*), and has the effect of retarding the position of the secondary m.m.f. by the angle δ to the position F_2. The conditions without the magnetizing current are shown dotted, and it can be seen that the resultant m.m.f. F_R, and therefore the flux, are reduced; this increases the current for a given torque but improves the power factor.

At *super-synchronous* speeds the power is being delivered to the transformer from the supply, and the magnetizing current is therefore supplied from the stator current. Conditions are thus as shown in Fig. 11.18(*b*) from which it is seen that, for a given current, the flux and torque are increased and the power factor reduced.

General Effect of Rotor Transformer. If the transformer is designed for high saturation it can have desirable effects on the behaviour of the motor—at low speeds the flux is reduced and the current increased; the former effect outweighs the latter and results in improved commutation, particularly at starting. At high speeds the saturation limits the rise of rotor voltage and hence limits the speed at low loads, so that the motor can be arranged to run at a safe speed on no-load. The saturation leads to high iron losses and a lower efficiency—the same speed-limiting effect can, however, be

obtained by designing a transformer with an air gap which takes a high magnetizing current without unduly high flux densities.*

Advantage may be taken of the use of the rotor transformer to have six instead of three secondary phases, thus improving the

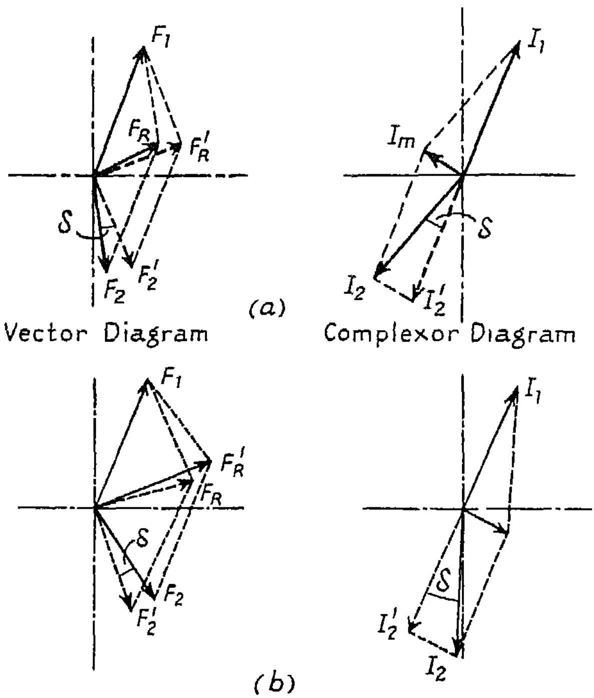


FIG. 11.18. EFFECT OF MAGNETIZING CURRENT ON DIAGRAMS WITH ROTOR TRANSFORMER

(a) Sub-synchronous speed.
(b) Super-synchronous speed

commutation on account of the smaller current change (page 30) and giving a slightly greater output as a result of the higher distribution factor for the rotor winding (0·955 instead of 0·827).

On account of the above advantages the use of a rotor transformer is general whenever a three-phase series motor is employed.

Equivalent Circuits

The following expressions can be written down, as with the motors considered previously, to express the behaviour.

* J. W. Hibbert, "Notes on Characteristics of Polyphase Series Motors," *Beama. J.*, **58**, p. 306 (Nov. 1951).

E.m.f.'s in main circuit—

$$V = Iz_L + E_1 + sE_2 \quad . \quad . \quad . \quad (11.8)$$

where z_L is the total leakage impedance of the machine.

E.m.f.'s induced by air-gap flux—

$$E_2 = E_1(T_2/T_1)e^{j\sigma} \quad . \quad . \quad . \quad (11.9)$$

Magnetization—

$$E_1 = I_m z_m \quad . \quad . \quad . \quad . \quad (11.10)$$

From (11.8) and (11.9)

$$V = Iz_L + E_1\{1 + s(T_2/T_1)e^{j\sigma}\}$$
$$= Iz_L + E_1\alpha_1 \quad . \quad . \quad . \quad . \quad (11.11)$$

M.m.f.'s—

$$F_R = F_1 + F_2$$
$$I_0 T_1 = IT_1 + IT_2$$
$$I_m = I\{1 - (T_2/T_1)e^{-j\sigma}\} = I\alpha_2 \quad . \quad . \quad (11.12)$$

From (11.10), (11.11) and (11.12)

$$V = I(z_L + z_m\alpha_1\alpha_2)$$

The quantity in the brackets thus represents the effective impedance of the machine so that the equivalent circuit is as shown in Fig. 11.19. This circuit enables the current to be found for a

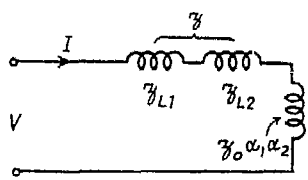


FIG. 11.19. EQUIVALENT CIRCUIT

given slip; if stator iron loss is neglected the rotor input, and hence the torque in synchronous watts, are given by the I^2r loss in the above circuit less the stator I^2r loss.

EQUIVALENT CIRCUIT WITH ROTOR TRANSFORMER. The equivalent circuit of Fig. 11.19 also represents the machine with a rotor transformer if suitable modification is made to α_1 and α_2 and if the impedance z_L is made to include that of the transformer, all secondary impedances being referred to the primary. The values of α_1 and α_2 must now include the turns ratio of the transformer so that they become—

$$\alpha_1 = 1 + s(T_{r1}/T_{r2})(T_2/T_1)e^{j\sigma}$$

and

$$\alpha_2 = 1 - (T_{r1}/T_{r2})(T_2/T_1)e^{-j\sigma}$$

Use of Fixed and Movable Brushes

Instead of having a variable-ratio transformer the same effect can be obtained with a fixed-ratio transformer and an additional set of

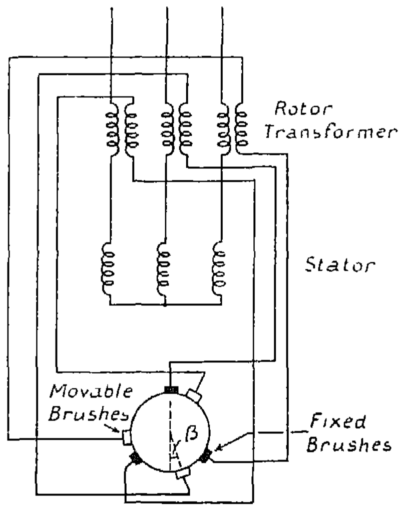


FIG. 11.20. SERIES MOTOR WITH FIXED AND MOVABLE BRUSHES

brushes, one set then being fixed and the other movable as shown in Fig. 11.20.

EFFECT OF BRUSH MOVEMENT. Considering the pair of brushes associated with one phase, if they are placed at AA' in Fig. 11.21(a)

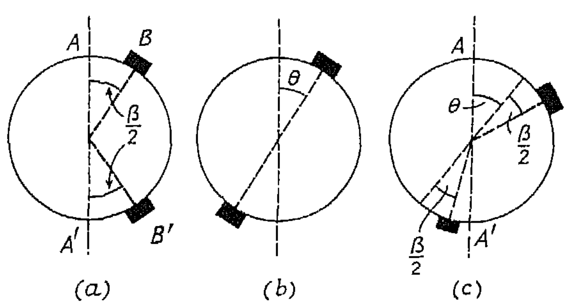


FIG. 11.21. BRUSH MOVEMENT

the e.m.f. generated between them and the reactance of the circuit will be a maximum, and the m.m.f. will be acting along the axis AA'. If each brush be given a shift of $\beta/2$ as shown in the figure the direction of the m.m.f. remains unaltered but the magnitudes

of the m.m.f., e.m.f. and the reactance, and therefore of the effective turns ratio, are reduced in the ratio of the chord BB' to the diameter AA', i.e. by the factor $\cos(\beta/2)$.

In Fig. 11.21(*b*) both brushes are moved simultaneously through the angle θ, and this changes the direction of the m.m.f. but leaves its magnitude unaltered.

If now both movements are combined as in Fig. 11.21(*c*) the top brush moves $\theta + \beta/2$ from A and the bottom brush $\theta - \beta/2$ from A' so that the axis of the m.m.f. has been moved through the angle θ and its magnitude has been reduced by the factor $\cos(\beta/2)$.

MOVEMENT TO GIVE CONSTANT LIMITING STABLE SPEED. The limiting stable speed for a simple motor is given by $n_c/n_1 = 1 - (\cos\theta)/K$; if it is desired to make this speed constant for all values of θ then K must vary proportionately to $\cos\theta$.

Suppose K_0 is the effective turns ratio with the brushes diametrically spaced; this ratio must be reduced by adding the shift to each brush in such a way that $K = K_0 \cos\theta$. It has, however, been shown above that $K = K_0 \cos(\beta/2)$ so that $\beta/2$ must be made equal to θ. The movement of the top brush must thus be $\theta + \beta/2 = \beta$ and of the bottom brush $\theta - \beta/2 = 0$. The required effect of making the stable speed constant for all values of θ is thus obtained if the top brush is moved an angle $\beta = \theta$ and the bottom brush is held stationary. The value of this limiting stable speed is given by

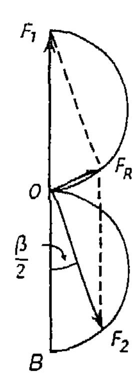


FIG. 11.22. M.M.F. VECTOR DIAGRAM

$$n_c/n_1 = 1 - \cos\theta/(K_0 \cos\theta) = 1 - 1/K_0$$

For stability over the whole speed range K must equal unity. The space vector diagram for this condition is shown in Fig. 11.22. The stator m.m.f. is drawn vertically and the rotor m.m.f. is inclined at an angle $\beta/2 = \theta$ as shown. If OB represents the maximum rotor m.m.f. (with diametrical brushes), which is equal to F_1 since $K = 1$, then $F_2 = OB \cos(\beta/2)$ and moves round a semicircle as shown. Adding the stator m.m.f. to F_2 gives the resultant m.m.f. F_R which clearly must move round the semicircle having OF_1 as diameter. F_R is thus always at 90° to the rotor m.m.f. so that all the exciting m.m.f. is provided by the stator and operation at unity power factor is impossible.

COMMUTATION. Although the magnitudes of the rotational e.m.f. E_{rb} and the reactance e.m.f. E_{xb} at the fixed and movable brushes are the same, they are not at the same phase angle to each other in the two cases so that the resultant e.m.f. in the short-circuited coils at the two brush sets, and therefore the circulating currents, are not the same. If the two brush sets are placed side by side they bear on different parts of the commutator and unequal wear may occur. It is preferable, therefore, to arrange for each brush arm to

cover the whole axial length of the commutator, and if the rotor is wave-wound this can easily be arranged by having all the fixed brushes on the bottom half of the commutator and all the movable brushes on the top half. Compoles are of course impracticable with movable brushes.

ADVANTAGE OF FIXED- AND MOVABLE-BRUSH MOTOR. The motor with fixed and movable brushes does not provide stability at the same time as an improved power factor, and it would appear to

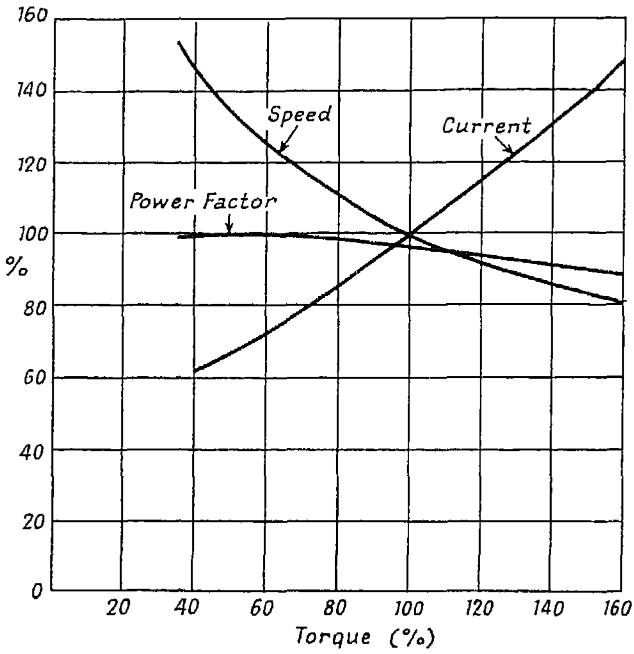


FIG. 11.23. CHARACTERISTICS OF THREE-PHASE SERIES MOTOR

show no advantage over the ordinary motor. Its merit lies, however, in the fact that for any brush shift the motor is in a state of being just stable over its whole speed range so that the highest power factor compatible with stability is always obtained.

Performance

The various characteristics which have been referred to in the preceding pages are summarized below.

RUNNING CHARACTERISTICS. Typical speed-torque characteristics are given in Fig. 11.8, while current, speed and power factor plotted against torque for a typical motor are given in Fig. 11.23. It may be noted that the speed-torque curves of the motor with the brushes

set for high speeds are considerably flatter than those of the d.c. series motor so that, with suitable design, the motor could be used where a d.c. compound motor would otherwise be required.

STARTING. No special starting equipment is required as starting is effected with the brushes in the minimum speed position, i.e. with $\theta = 50°$ or $60°$. The starting torque and starting current in this position are each about 50 per cent of the full-load values. Moving the brushes towards the short-circuit position, i.e. reducing θ, enables starting torques up to 200 per cent of full-load torque to be obtained with about 150 per cent of full-load current.

SPEED CONTROL. Although speed control by supply-voltage variation is possible, the simplicity of control by brush shifting makes this method almost universal. Commutation difficulties usually limit the range available to about 3 to 1, i.e. between about 40 and 140 per cent of synchronous speed.

EXERCISES 11

1. A 420-V, 50-c/s, 6-pole, 3-ph. series motor has a rotor/stator turns ratio of 1·43 and a brush shift of 140° from the neutral position. If the current, in amperes, is proportional to the stator voltage and numerically equal to half of it, draw the circle diagrams of voltage and current; plot a curve of power factor against speed.

2. A 3-ph., 420-V series motor with a brush shift of 35° from the short-circuit position has a rotor/stator effective turns ratio of (*a*) 1·0, (*b*) 0·5 and (*c*) 2·0. Draw the space vector and complexor diagrams and find the stator and rotor voltages and the power factor for slips of $s = 1·0, 0·5, 0$ and $-1·0$ in each case. Resistance and leakage reactance may be neglected.

3. A 220-V, 3-ph. series motor with three brush sets per pole-pair has 100 full-pitch stator turns per phase and a total of 500 full-pitch turns per pole-pair on the rotor. Estimate the power factor when the brushes are shifted 40° (elec.) from the short-circuit position and the motor is running at one-half synchronous speed.

PART IV: SINGLE-PHASE MOTORS

CHAPTER 12

GENERAL RELATIONS

Development of the Single-phase Motor

As stated in the introduction, the first recorded mention of a single-phase motor was made in 1884 when it was stated that a d.c. series motor would work on alternating current. In 1887 Elihu Thomson described his repulsion experiments which form the basis of the modern repulsion motor and the single-phase induction motor. At about the same time Wightman discussed the principle whereby a single-phase machine can be excited by passing a current through the rotor winding instead of through the stator winding. Deri and Blathy of the Ganz works, Budapest, were also experimenting between 1889 and 1891 with single-phase series motors, while in this country Atkinson, in 1898, showed how a single-phase commutator motor could be made to give a shunt characteristic. It is thus seen that interest was world wide and a very large number of patents were taken out for different types of motor.

Large motors suitable for railway work were developed about 1900 and gave an impetus to single-phase traction—this is still an important application of the single-phase commutator motor. The other sphere of application for the single-phase motor is for machines of low output operating in circumstances where a three-phase supply is not economically available.

Types of Motor

A large number of different types of single-phase motor have been devised but only a few have become commercially satisfactory. All these motors are made up of two or more of the four essential parts or circuits shown in Fig. 12.1. By different methods of connecting these circuits different types of motor are obtained, the more important being illustrated in Fig. 12.2 and discussed briefly in the following notes.

PLAIN SERIES MOTOR (No. 1). This is exactly similar to the d.c. series motor and operates satisfactorily on a.c. or d.c. It is widely used as a universal motor in fractional-horse-power sizes for domestic and similar applications. When run on a.c., the power factor tends to be low but this is unimportant for the small sizes normally obtaining.

COMPENSATED SERIES MOTORS (NOS. 2 AND 3). The compensating winding results in a better power factor and enables larger motors to be built. Motors of this type are now used almost exclusively on

single-phase traction systems in sizes up to several hundred horse-power.

ROTOR-EXCITED COMPENSATED SERIES MOTORS (NOS. 4 AND 5). The characteristics are similar to those of 2 and 3, but the motors require more brushes and are therefore less practicable and not used commercially.

PLAIN REPULSION MOTORS (NOS. 6 AND 7). This type is used in sizes up to 1 h.p. Speed control over a limited range can be obtained by shifting the brushes if desired. The single-winding type is used

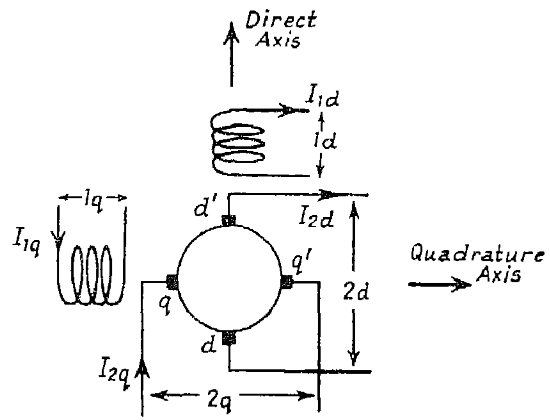


FIG. 12.1. ESSENTIAL CIRCUITS OF SINGLE-PHASE MOTOR

1*d*. Stator winding in direct axis.
2*d*. Rotor winding in direct axis.
1*q*. Stator winding in quadrature axis.
2*q*. Rotor winding in quadrature axis.

in all cases except where reversing is necessary, this being effected by reversing the connexions of one of the windings of the double-winding type.

INVERTED REPULSION MOTORS (NOS. 8 AND 9). These give similar characteristics to those of 6 and 7 but have little merit as it is generally more difficult to design a rotor winding for connexion to supply voltage than a stator winding.

COMPENSATED REPULSION MOTOR (NO. 10). This gives a better power factor than the plain repulsion motor, and also better commutation at low speeds. It is the type originally used on the old L.B. and S.C. Railway although it was later replaced by the compensated series motor. (With the forming of the Southern Railway in 1923 the use of a.c. traction was abandoned and replaced by d.c.)

REPULSION MOTOR WITH SECONDARY EXCITATION (NO. 11). The exciting winding is connected in the rotor instead of in the stator circuit, the characteristics being approximately the same as for No. 6.

SERIES REPULSION MOTORS (NOS. 12 AND 13). The series and repulsion arrangements are combined in order to obtain the best commutating conditions of each.

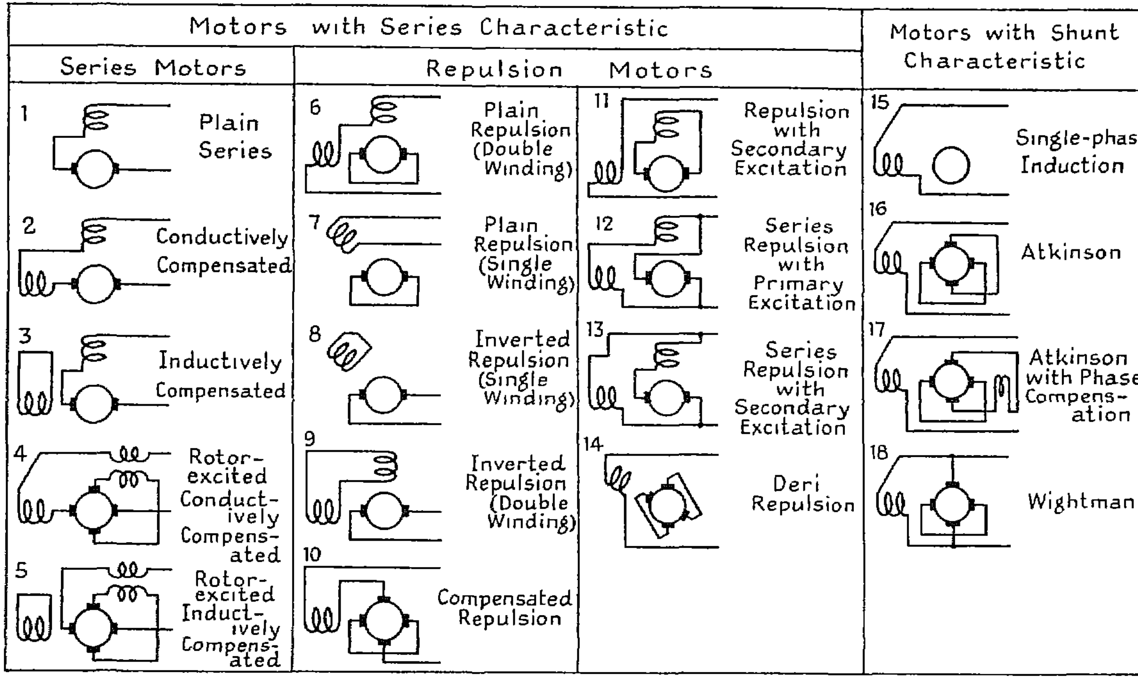


FIG. 12.2. TYPES OF SINGLE-PHASE MOTOR

THE DERI BRUSH-SHIFTING REPULSION MOTOR (No. 14). By having a set of movable and a set of fixed brushes, a better speed control is obtained than with the plain repulsion motor.

SINGLE-PHASE INDUCTION MOTOR (No. 15). The motor is not inherently self-starting, and a special starting device has to be employed: this type is the most common of all the single-phase motors on account of its simplicity and robustness, no commutator being required.

ATKINSON SHUNT MOTOR (Nos. 16 AND 17). The motor as illustrated has no advantage over the single-phase induction motor and is, on account of its commutator, more complicated. Arrangements for speed control can, however, be made while the power factor can be improved by the phase-compensation arrangement of No. 17.

WIGHTMAN MOTOR (No. 18). This gives similar characteristics to the Atkinson motor but is more difficult to design as the commutator winding is connected across the supply voltage.

Torque

Interaction between the current in the conductors and the flux density in the air gap gives rise to the torque. Reference to Fig. 12.3

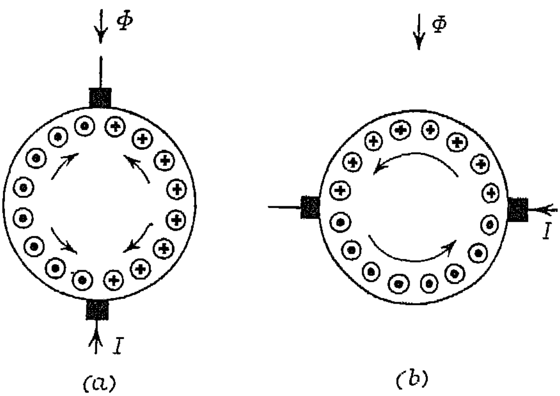


FIG. 12.3. TORQUE PRODUCTION

(*a*) Brush axis in line with flux axis: no torque.
(*b*) Brush axis at 90° to flux axis: maximum torque.

and the use of Fleming's left-hand rule to find the direction of the force on a conductor shows that when the brush axis is coincident with the flux axis there is no resultant torque, since that of corresponding conductors on each side of the armature are in opposition whereas with the brush axis at 90° to the flux axis there is a maximum resultant torque since the torques of all individual conductors act in the same direction. It may therefore be stated that torque is only produced by the *component of flux which is at* 90° *to the brush axis.* Any flux produced by the armature current itself can thus produce no torque.

CALCULATION OF TORQUE. The flux and current may be assumed to vary sinusoidally with time, i.e.

$$\Phi = \hat{\Phi} \sin \omega t$$

and

$$i_{t_1} = \hat{i}_{t_1} \sin (\omega t + \psi)$$

where ψ is the time phase angle between flux and current.

The flux density in the air gap may also be assumed to vary sinusoidally in space around the periphery so that at any moment in the cycle

$$B_\theta = \hat{B}_\theta \cos \theta$$

where θ is the angle measured from the centre-line of the flux axis.

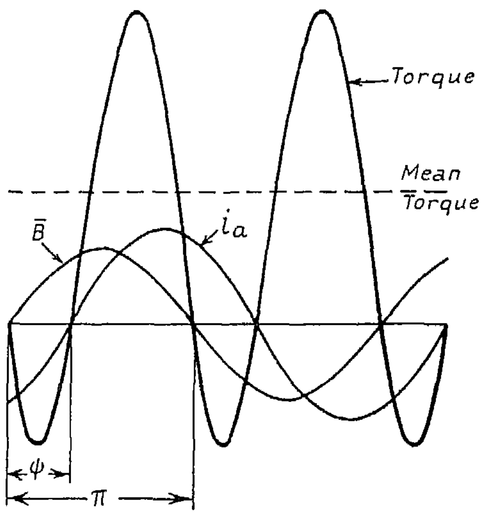


FIG. 12.4. VARIATION OF TOTAL TORQUE WITH TIME

The tangential force on any conductor is thus—

$$F_\theta = B_\theta i_t L \text{ newtons}$$
$$= B_\theta (i_a/2a) L \text{ newtons}$$

where i_a is the total current entering the armature.

The total tangential force at any moment will thus be

$$F = \Sigma F_\theta \,.\, z_\theta = (i_a/2a) L \Sigma B_\theta z_\theta = (i_a/2a) L \bar{B} z$$

where z_θ is the number of conductors at position θ,

$\bar{B}$ is the mean flux density over the pole-pitch at a particular moment in the cycle.

Thus—

$$F = (i_a/2a)\sin(\omega t + \psi)\bar{B}_m \sin\omega t \, . \, L \, . \, z$$
$$= (Lz/4a)i_a\bar{B}_m\{\cos\psi - \cos(2\omega t + \psi)\} \text{ newtons}$$

where $\bar{B}_m$ is the peak value (time) of the mean density.

This expression, plotted in Fig. 12.4, shows that the force, and therefore the torque, pulsate at twice the supply frequency. The inertia of the armature is usually sufficient to render these pulsations innocuous but there is always a tendency, with single-phase motors, towards noisy operation and vibration troubles, particularly if ψ is large resulting in big negative pulses of torque.

The mean force is due to the first term in the above expression, i.e.

$$\bar{F} = (I_a/2\sqrt{2}a)Lz\bar{B}_m \cos\psi \text{ newtons}$$

where I_a is the r.m.s. value of the current entering the armature brushes.

The mean torque is $\bar{F}D/2$ so that, using $\pi D = 2p \, . \, Y$ and $\bar{B}_m \pi DL = 2p\bar{\Phi}$, the torque becomes

$$TM = (1/2\pi\sqrt{2})(p/a)I_a\Phi z \cos\psi \text{ newton-metres}$$
$$= (0{\cdot}117/\sqrt{2})(p/a)I_a\Phi z \, . \cos\psi \text{ Lb-ft} \quad . \quad . \quad (12.1)$$

It may be noted that this is $1/\sqrt{2}$ times the torque of a similar d.c. machine having the same current and maximum flux values.

Mechanical Power

The power (in watts) corresponding to the above torque is $2\pi n$ times the torque in newton-metres, i.e.

$$P = TM \times 2\pi n$$
$$= (1/\sqrt{2})(p/a)I_a\Phi zn \cos\psi$$

Substituting E_r, the e.m.f. of rotation due to the conductors moving in the flux Φ, (eq. (2.12)), this becomes—

$$P = E_r I_a \cos\psi \text{ watts} \quad . \quad . \quad . \quad (12.2)$$

General Complexor Diagrams

As with other types of machine the single-phase motor can be investigated by complexor diagrams or by analytical methods. In both cases the flux in the air gap is produced by the resultant m.m.f. due to the currents in the various windings; it is, however, most convenient to resolve this main flux into two components acting respectively along the direct and quadrature axes of Fig. 12.1.

The complexor diagram will therefore comprise complexors representing these main flux components, the currents in the various windings, the transformer and rotational e.m.f.'s due to the windings linking with or rotating in the main flux components, transformer and rotational e.m.f.'s in the leakage fluxes and the resistance drops.

In Fig. 12.5(a) are the complexors relating to the main direct-axis flux; there is a transformer e.m.f. in the stator winding, E_{tsd}* and a transformer e.m.f. in the rotor winding, E_{tad}, between brushes dd'

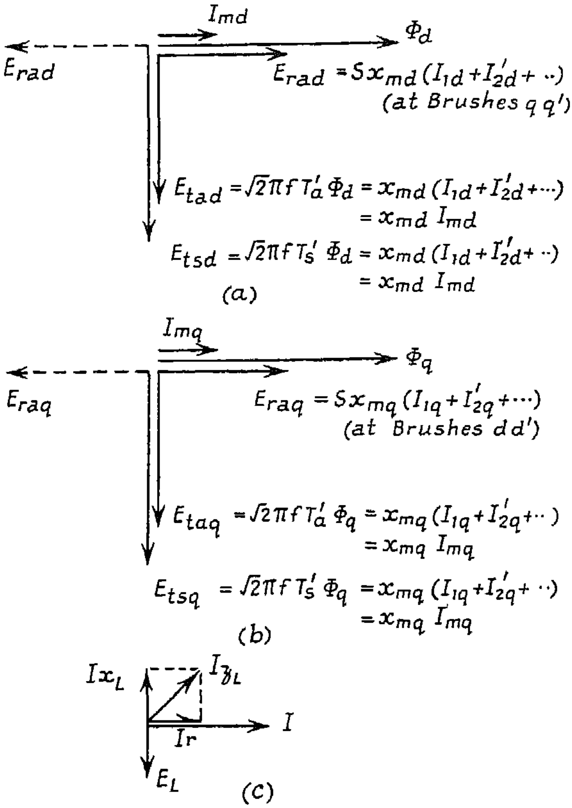


FIG. 12.5. ELEMENTS OF COMPLEXOR DIAGRAMS

(a) Complexors related to direct-axis flux.
(b) Complexors related to quadrature-axis flux.
(c) Complexors related to leakage flux.

due to these windings being linked with the direct-axis flux component Φ_d; these lag the flux by 90°. There is also, between brushes qq' a rotational e.m.f., E_{rad}, due to rotation of the rotor in the quadrature flux component Φ_q; this is in phase with or phase opposition to the flux Φ_q depending on the direction of rotation.

* Suffixes t or r signify that the e.m.f. is set up by transformer action or by rotation, suffixes s or a that it is induced in the stator or rotor (armature) and suffixes d or q that it is produced by the direct- or quadrature-axis component of the flux.

The magnitude of these e.m.f.'s can be calculated from the expressions of Chapter 2. The flux is produced by magnetizing ampere-turns which are the resultant of the ampere-turns of all the windings on the direct axis; these are represented by the magnetizing current I_{md} in phase with the flux, this being given by $\mathbf{I}_{1d} + \mathbf{I}_{2d}' + \mathbf{I}_{3d}' + \ldots$, all currents being referred to the stator winding.

In Fig. 12.5(*b*) are shown the corresponding complexors for the quadrature axis.

Transformer e.m.f. E_L due to the leakage flux lags the flux, and therefore the current producing the flux, by 90°; it is usual, however, to represent this e.m.f. as a leakage reactance drop Ix_L leading the current by 90° as in Fig. 12.5(*c*). The resistance drop is, of course, in phase with the current.

By selecting the appropriate complexors from the above a complete complexor diagram for any type of motor can be built up.

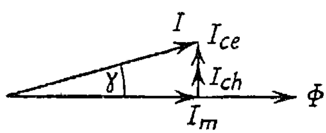


FIG. 12.6. IRON LOSS

IRON LOSS. It has been assumed above that the main flux components Φ_d and Φ_q are in time phase with the resultant m.m.f. producing them; actually the flux lags the magnetizing m.m.f. by a small angle γ due to iron loss. The hysteresis effect results in a distortion of the magnetizing current and a small phase shift ahead of the flux it is producing; this can be represented approximately by a small component of current I_{ch} leading the flux by 90° as shown in Fig. 12.6. Eddy currents set up in the laminations are approximately in phase with the e.m.f. producing them since the resistance of the eddy-current path is relatively high; the eddy currents may thus be drawn lagging the flux by 90° and a corresponding current I_{ce} leading the flux by about 90° is drawn from the primary. The total iron loss can thus be represented by a component of current leading the flux by 90°.

Analytical Treatment and General Circuit Equations

As any motor consists of two or more of the circuits shown in Fig. 12.1 it is possible to write down, from Kirchhoff's second law, an equation for each circuit relating the e.m.f.'s and voltage drops; the e.m.f.'s are most conveniently expressed in terms of the mutual and leakage reactances, as described on page 18. Inserting the particular boundary conditions relating to the connexions of the machine under consideration enables the equations to be solved and the behaviour of the motor determined. To obtain greater accuracy two additional circuits corresponding to the paths of the currents

in the coils short-circuited by the brushes may be added as in Fig. 12.7 so that there are six possible circuits.

In developing the equations all currents, voltages and impedances are most conveniently referred to a common reference winding, e.g. the direct-axis stator winding. The six equations can thus be obtained as follows.

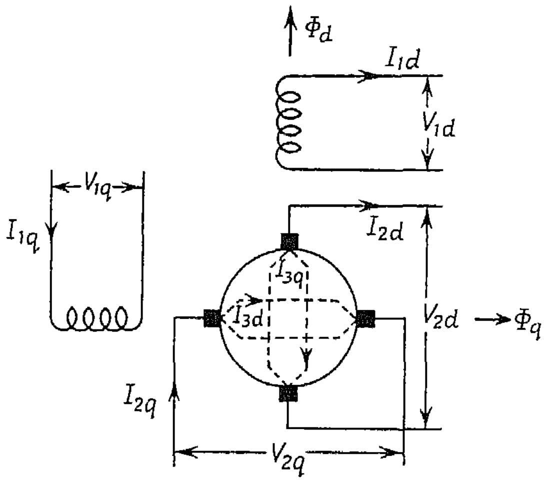


FIG. 12.7. CIRCUITS FOR GENERAL EQUATIONS

STATOR WINDING IN DIRECT AXIS. The mutual direct-axis flux Φ_d links the stator winding in this axis as well as the rotor and brush-circuit windings so that the e.m.f.'s set up in the stator winding are—

— $\mathrm{j}x_{md}I_{1d}$ due to its self inductance

— $\mathrm{j}x_{md}I_{2d}'$ due to mutual induction from circuit $2d$ (rotor circuit)

— $\mathrm{j}x_{md}I_{3d}'$ due to mutual induction from circuit $3d$ (brush circuit)

In addition there is an e.m.f. of self inductance caused by the leakage flux Φ_{L1d} of the stator winding which is $-\mathrm{j}x_{L1d}$ and a resistance voltage drop $I_{1d}r_{1d}$. Treating all the e.m.f.'s as voltage drops the voltage to be applied to overcome them is

$$V_{1d} = I_{1d}r_{1d} + \mathrm{j}x_{L1d}I_{1d} + \mathrm{j}x_{md}(I_{1d} + I_{2d}' + I_{3d}') \quad . \ (12.3a)$$

ROTOR WINDING IN DIRECT AXIS. Similar conditions obtain in the rotor winding except that in addition to the transformer e.m.f. resulting from the flux there is a rotational e.m.f. due to rotation in Φ_q. The mutual reactance x_{mq} relating to this flux differs from that of the direct-axis flux if the machine is not mechanically symmetrical

in the two axes. The sign of the rotational term is conventionally taken as positive for e.m.f.'s induced in the direct axis by quadrature-axis flux, movement from the quadrature to the direct axis being regarded as positive on the normal complexor diagram. The equation is thus—

$$V_{2d}' = I_{2d}'r_{2d}' + jx_{L2d}'I_{2d}' + jx_{md}(I_{1d} + I_{2d}' + I_{3d}') + Sx_{mq}(I_{1q} + I_{2q}' + I_{3q}') \quad . \quad (12.3b)$$

STATOR WINDING IN QUADRATURE AXIS. The corresponding expression for the quadrature axis is similar in form to (12.3*a*)—

$$V_{1q}' = I_{1q}'r_{1q}' + jx_{L1q}'I_{1q}' + jx_{mq}(I_{1q}' + I_{2q}' + I_{3q}') \quad (12.3c)$$

ROTOR WINDING IN QUADRATURE AXIS. The sign of the rotational term is here negative since the e.m.f. is induced in the quadrature axis by the direct-axis flux, i.e. backwards as compared to (12.3*b*).

$$V_{2q}' = I_{2q}'r_{2q}' + jx_{L2q}'I_{2q}' + jx_{mq}(I_{1q} + I_{2q}' + I_{3q}') - Sx_{md}(I_{1d} + I_{2d}' + I_{3d}') \quad . \quad (12.3d)$$

BRUSH CIRCUIT IN DIRECT AXIS. There is no applied voltage in this circuit so that the sum of the drops is zero.

$$0 = I_{3d}'r_{3d}' + jx_{L3d}'I_{3d}' + jx_{md}(I_{1d} + I_{2d}' + I_{3d}') + Sx_{mq}(I_{1q} + I_{2q}' + I_{3q}') \quad . \quad (12.3e)$$

BRUSH CIRCUIT IN QUADRATURE AXIS

$$0 = I_{3q}'r_{3q}' + jx_{L3q}'I_{3q}' + jx_{mq}(I_{1q} + I_{2q}' + I_{3q}') - Sx_{md}(I_{1d} + I_{2d}' + I_{3d}') \quad . \quad (12.3f)$$

These six equations contain ten variables; if four of these are fixed by the boundary conditions relating to the particular type of motor under consideration, the remainder can be found by solving the equations, and the behaviour of the motor determined.

A further refinement to the above is to add terms for the rotational e.m.f. set up by the rotor leakage flux, i.e. $jSx_{L2}I_{2d}$ in the direct axis and $-jSx_{L2}I_{2q}$ in the quadrature axis.

It must also be remembered that the actual values of voltage, current and impedance are related to the above referred values by the relations—

$$V = V'/k; \; I = I'k; \text{ and } z = z'/k^2$$

where k = (effective turns in reference winding)/(effective turns in winding considered).

Iron loss cannot usually be introduced into the above very easily, and is generally included with friction and windage loss and subtracted from the gross output.

CHAPTER 13

THE PLAIN SERIES MOTOR (FRACTIONAL H.P. UNIVERSAL TYPE)

General Construction

Since reversing the polarity of the supply to an ordinary d.c. series motor does not affect the direction of the torque, it might be expected that such a motor could operate from an a.c. supply; this is, in fact, the case, although operation would be unsatisfactory on account of the high iron loss caused by the alternating flux in the solid poles and yoke. For sizes of less than about $\frac{1}{2}$ h.p., however, satisfactory operation can be obtained if the poles and yoke are laminated; with larger sizes certain other modifications to the design, as compared to that of a d.c. motor, are necessary and such motors are discussed in the next chapter.

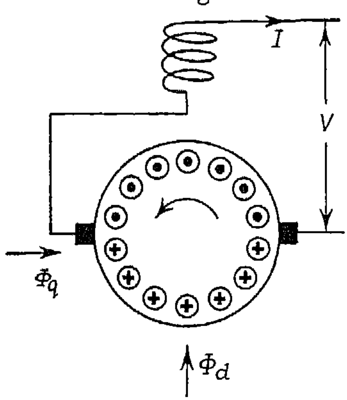


FIG. 13.1. SIMPLE SERIES MOTOR

The fractional-horse-power a.c. series motor is therefore similar in construction to a d.c. motor of the same size except that its magnet frame is laminated; such motors have the special merit that they can run satisfactorily on either a.c. or d.c. supplies and are called *universal* motors and used in large numbers for driving small domestic and similar apparatus. Typical sizes range from 1/100 to 1/4 h.p.

The connexions are shown in Fig. 13.1 from which it can be seen that only two of the circuits illustrated in Fig. 12.1 (page 211) are employed, i.e. the stator winding in the direct or exciting axis which produces the exciting flux Φ_d and the rotor winding in the quadrature axis which produces a quadrature flux Φ_q.

Due to the salient-pole construction the space distribution of the flux may deviate considerably from a sine wave (page 1), but as rotor torque and e.m.f. are dependent on the total flux and not on its distribution this has little effect on the behaviour.

The rotational e.m.f. is proportional to the main flux and the transformer e.m.f.'s to $d\Phi/dt$; as these make up the major part of the sinusoidal supply voltage, the time variation of the flux is approximately sinusoidal. To produce this the current must be non-sinusoidal and, with normal values of saturation, may contain up to

30 per cent of third harmonic. This gives rise to a negative torque but this can usually be neglected and in what follows sinusoidal quantities are assumed.

Complexor Diagram

Assuming the air-gap flux to be resolved into its two components Φ_d and Φ_q, the former may be used as the starting point for the diagram as shown in Fig. 13.2. The main current sets up this flux

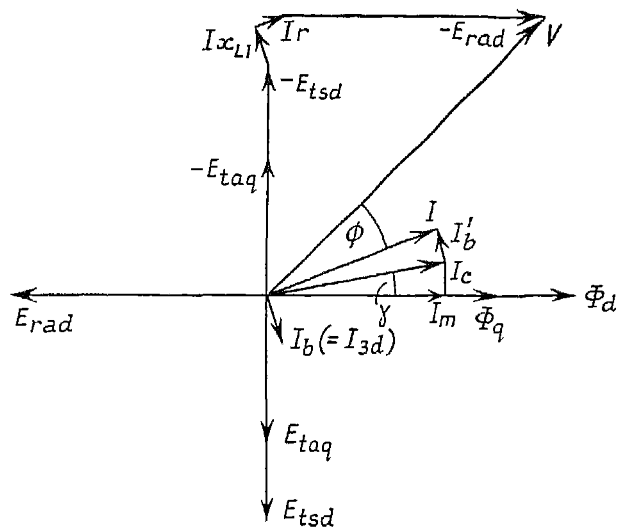


FIG. 13.2. COMPLEXOR DIAGRAM OF PLAIN SERIES MOTOR

and leads it by a small angle γ due to the iron losses as explained on page 217. This exciting flux Φ_d sets up an e.m.f. in the stator winding which lags the flux by 90° and is given by

$$E_{tsd} = (\sqrt{2})\pi f_1 T_s \Phi_d \text{ volts}$$

where T_s is the effective stator turns and is equal to the actual turns if, as is usual with these small machines, salient poles are used.

The stator current also sets up a leakage flux which gives rise to an e.m.f. which can be conveniently treated as a leakage reactance drop Ix_{L1} leading the current by 90° in the usual way.

The current I also sets up the quadrature-axis flux Φ_q in phase with the exciting flux. This flux sets up a transformer e.m.f. in the armature (from eq. (2.8a), page 21)—

$$E_{taq} = (\sqrt{2})f_1(T_a/a)\Phi_q \text{ volts}$$

T_a being the total turns on the armature.

The rotation of the armature in the flux Φ_d sets up a rotational e.m.f. between the brushes—

$$E_{rad} = (\sqrt{2})f_r(T_a/a)\Phi_d \text{ volts}$$

This e.m.f. has its peak value at the same moment as the peak value of the flux; drawing the dots and crosses on Fig. 13.1 in accordance with the conventions mentioned on page 13 and using Fleming's left-hand rule shows the direction of rotation to be clockwise. The right-hand rule can now be used to find the direction of the e.m.f. in the conductors, and it will be found to be opposed to the current. The e.m.f. E_{rad} is thus at 180° to the flux producing it.

The applied voltage has to overcome all these e.m.f.'s and also the resistance drop Ir which is in phase with the current.

The coils short-circuited by the brushes are linked with the full exciting flux, and are rotating in the quadrature flux, and therefore have e.m.f.'s induced in them as described later; these e.m.f.'s cause a circulating current I_b which lags by a considerable angle behind the fluxes as shown. A further component of primary current I_b' is necessary to neutralize the m.m.f. of this, as in the ordinary transformer, so that the total current is I as shown.

It can be seen that the power factor of the motor is inevitably less than unity, but the effect of the iron loss and the brush circulating current is to improve it slightly, although at the expense of efficiency.

Circle Diagram

The transformer e.m.f.'s are all proportional to the fluxes and therefore, if saturation is neglected, to the current. The sum of these e.m.f.'s can therefore be treated as a reactance drop Ix where x is the total reactance of the motor, due to the quadrature-flux linkages with the armature turns and the exciting and leakage flux linkages with the stator turns.

If OA, in Fig. 13.3, is drawn to represent the constant applied voltage V, the intersection of OB, representing the transformer e.m.f.'s, and BA, representing the rotational e.m.f. and the Ir drop, lies on a circle since the angle OBA equals 90°.

The current complexor is parallel to BA so that angle BAO is the phase angle, ϕ. If a vertical OD is drawn through O, the angle DOB is also ϕ, so that if OD is made to represent the position of the applied voltage complexor, then OB can represent the current complexor both in magnitude and phase. If the original voltage scale were 1 cm = Y volts, then the current scale for OB is 1 cm = Y/x amperes.

If BB' represents the Ir drop, then $B'A$ represents the rotational e.m.f. As the speed falls, i.e. as the load rises, $B'A$ gets smaller and B moves round the circle towards A, the power factor decreasing continuously.

Motor Input. If OB represents the current, a perpendicular

dropped from B on to OA represents the power component of current, and therefore the input to a scale of 1 cm $= VY/x$ watts.

TORQUE. As torque $\propto I\Phi \propto I^2 \propto OB^2$, and since triangles OBA and OEB are similar, $OB/OE = OA/OB$ and

$$OB^2 = OE \, . \, OA = V \, . \, OE$$

i.e. torque is proportional to OE.

The torque can be expressed as a percentage of the full-load value, or a torque scale can be found by calculating the value for a particular current and flux from eq. (12.1).

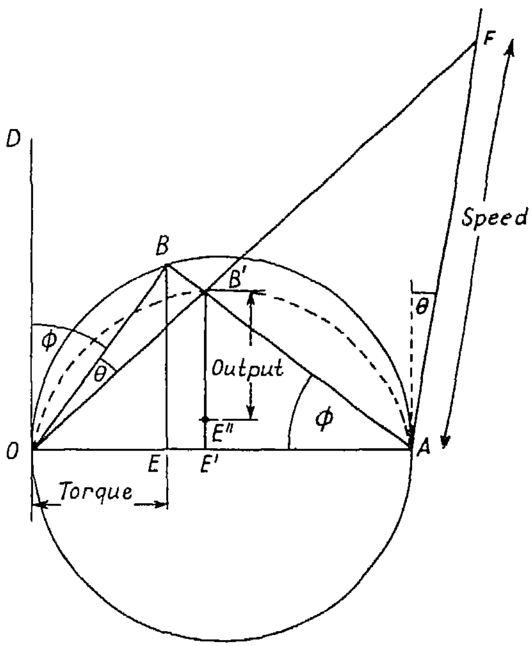


FIG. 13.3. SIMPLE CIRCLE DIAGRAM FOR SERIES MOTOR

OUTPUT. If iron, friction and windage losses are neglected, or allowed for as described later, the output can be written—

$$\text{Output} = \text{input} - \text{copper loss}$$

The copper loss is

$$I^2r = I \, . \, Ir$$

$$\propto OB \, . \, BB'$$

$$\propto \text{area of triangle } OBB'$$

Hence

$$\text{output} \propto \text{area of triangle } OAB - \text{area of triangle } OBB'$$
$$\propto \text{area of triangle } OB'A$$
$$\propto B'E' \text{ since base } OA \text{ is constant.}$$

The ratio

$$BB'/OB = Ir/I = r = \tan\theta$$

so that θ, and therefore the angle $OB'A$ ($= 90° + \theta$), are constant and the point B' moves on a circular locus as shown dotted with its centre slightly below OA.

The output, neglecting iron, friction and windage losses, is thus proportional to the vertical ordinate $B'E'$, the scale being the same as for the input.

The iron, friction and windage losses cannot be represented geometrically on the diagram, but if they are obtainable from design or test data they can be represented by the vertical ordinate $E'E''$ and the output is then $B'E''$.

SPEED. The rotational e.m.f. is proportional to speed $\times$ flux, i.e. $B'A \propto$ speed $\times$ OB so that speed $\propto B'A/OB$.

If OB' is projected to cut, at F, a line drawn through A at an angle θ to the vertical, the triangles OAF and $OB'A$ are similar so that $B'A/OB' = FA/OA$.

Since OA is constant, speed is proportional to FA. It can be expressed as a percentage of the full-load speed, or a scale can be found by calculating the speed from the rotational e.m.f. equation on page 222 for a particular value of current.

That part of the circle lying below the horizontal corresponds to operation as a generator.

From the above circle diagram a set of typical characteristics can be drawn as in the following example.

Example 13.1. A 1/5 h.p., 250-V, 50-c/s universal motor has a total resistance of 30 Ω and a total reactance of 155 Ω. Using the circle diagram and neglecting iron, friction and windage loss, draw curves of horse-power, torque, speed and power factor to a base of current.

Choosing a scale of, say, 1 cm = 20 V the line OA forming the diameter of the circle can be drawn and a semi-circle constructed on it. The current scale is then 1 cm = 20/155 = 0·129 A.

Points B at various positions round the semi-circle can then be located, and for each position of B the corresponding current measured. By drawing the vertical through O the phase angles can also be measured for each current, and the power factors determined.

For one current the Ir drop can be calculated and the point B' located, enabling the circle $OB'A$ to be drawn. The power scale is 1 cm = 250 × 0·129 = 32·2 watts = 0·043 h.p.

Vertical ordinates from B' to E' on the base line give the output to this scale and enable the h.p. curve to be plotted as shown in Fig. 13.4, and the full-load position (0·2 h.p.) determined. The line

OE at the full-load position gives full-load torque, and other torques can be expressed as a fraction of this.

The line AF can now be drawn at an angle θ ($= BOB'$) to the vertical, this angle being the same for any position of B. Projecting OB' for the full-load position to cut AF gives the full-load speed.

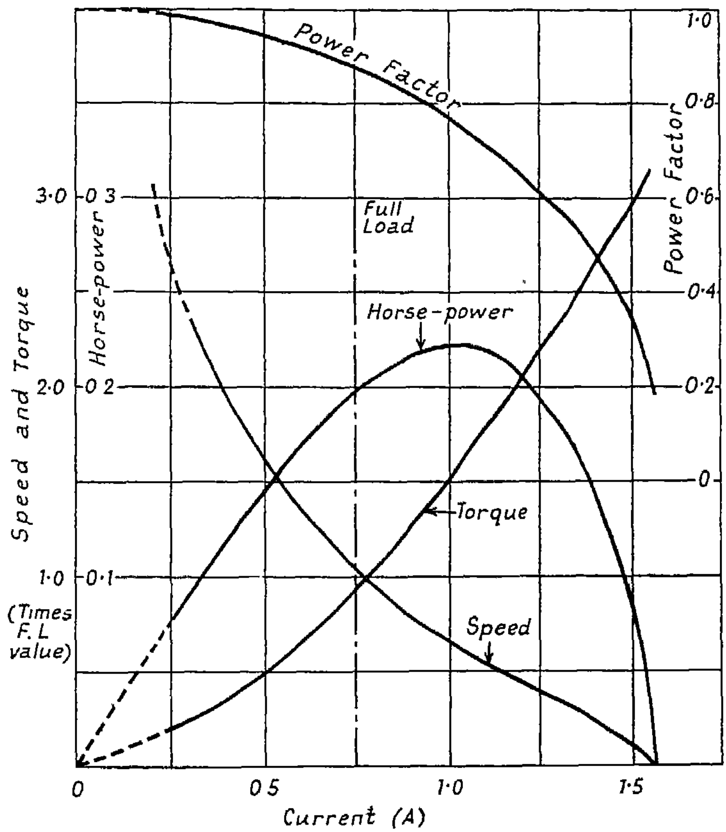


FIG. 13.4. CHARACTERISTICS OF 1/5 H.P. SERIES MOTOR

and other speeds may be expressed as a percentage of this. The complete set of curves is shown in Fig. 13.4, and they are seen to be generally similar in shape to those of a d.c. series motor.

Commutation

Reference to Fig. 13.1 shows that the coils short-circuited by the brushes are linked with the exciting flux Φ_d and are rotating in the

quadrature flux Φ_q. As described in Chapter 3 (page 31) there will thus be three e.m.f.'s induced in them—

the transformer e.m.f. E_{tbd} lagging the flux Φ_d by 90°
the rotational e.m.f. E_{rbq} in phase with Φ_q
the reactance e.m.f. E_{xb} in phase with the current

These e.m.f.'s are shown in Fig. 13.5 and their resultant E_b causes a circulating current I_{3d} lagging slightly behind E_b as shown. This

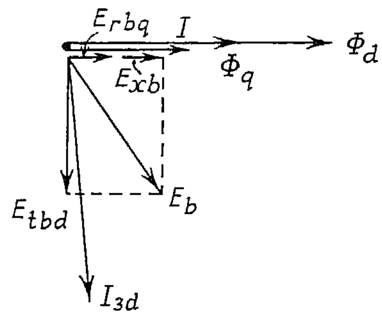


FIG. 13.5. E.M.F.'S IN COILS SHORT-CIRCUITED BY BRUSHES

current sets up additional loss and also influences the behaviour of the motor to some extent.

With fractional-horse-power motors, provided the voltage E_b does not exceed about 7 volts per coil, the impedance of the coil and the brush contact resistance are sufficiently high to keep the circulating current down to values which will not cause trouble, and no special steps are taken to assist the commutation process. Some compensation can be obtained by a brush shift of 10°–20° against the rotation.

With larger motors the problem must be examined in greater detail as in the next chapter.

Operation on A.C. and D.C. Supplies

If the motor has to run on either a.c. or d.c. supplies as when driving portable apparatus such as vacuum cleaners or electric drills, it is desirable that the speed-torque curve should be nearly the same in both cases.

It is shown in eq. (12.1) that for equal current and peak flux values the torque with alternating current is $1/\sqrt{2}$ times that with direct current; the peak flux value for the same current is, with alternating current, $\sqrt{2}$ times that with direct current, so that the torque with direct current is the same as that with alternating current of the same r.m.s. value.

Since the peak flux with alternating current is $\sqrt{2}$ times that with direct current of the same value, the peak a.c. rotational e.m.f. is $\sqrt{2}$ times the d.c. rotational e.m.f.; the r.m.s. a.c. rotational e.m.f.

is therefore the same as the d.c. value for the same speed. Since both e.m.f.'s are proportional to speed the relations between the a.c. and d.c. speeds may be written—

$$E_{rad}/E_{dc} = n_{ac}/n_{dc}$$

Referring to the complexor diagram of Fig. 13.2 it can be seen that approximately

$$E_{rad} = V \cos \phi - Ir$$

If run on direct current,

$$E_{dc} = V - Ir$$

so that

$$n_{ac}/n_{dc} = \frac{\cos \phi - Ir/V}{1 - Ir/V} \quad . \quad . \quad . \quad (13.1a)$$

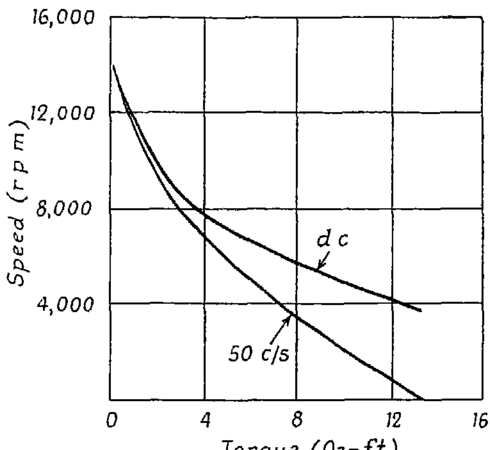


FIG. 13.6. SPEED-TORQUE CURVES OF UNIVERSAL MOTOR (1/4 H.P., 8,000 R.P.M.)

The full-load value of Ir/V is generally between 0·1 and 0·15 so that, if this is neglected, the speed ratio becomes

$$n_{ac}/n_{dc} = \cos \phi \quad . \quad . \quad . \quad (13.1b)$$

For a given current value the peak flux on alternating current may be somewhat less than $\sqrt{2}$ times the d.c. value because of saturation, so that the a.c. speed will be closer to the d.c. speed than given by the above relations. Curves for a typical motor are shown in Fig. 13.6.

Analytical Treatment

The six general eqs. (12.3*a*) to (12.3*f*) can be simplified for the plain series motor since circuits $1q$, $2d$ and $3q$ do not exist and $1d$ and $2q$ are connected in series carrying the main current I. The circuits thus become as in Fig. 13.7 and only the following three equations occur—

Stator winding in direct axis (exciting winding)

$$V_{1d} = Ir_1 + jx_{L1}I + jx_{md}(I + I_{3d}') \quad . \quad (13.2a)$$

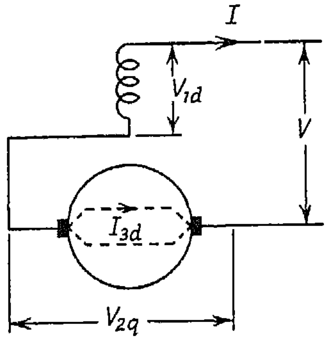


Fig. 13.7. Circuits of Simple Series Motor

Rotor winding in quadrature axis (armature winding)

$$V_{2q} = Ir_2 + jx_{L2}I + jx_{mq}I + Sx_{md}(I + I_{3d}') \quad . \quad (13.2b)$$

Brush circuit in direct axis

$$0 = I_{3d}'r_3' + jx_{L3}'I_{3d}' + jx_{md}(I + I_{3d}') + Sx_{mq}I \quad (13.2c)$$

These contain four unknown quantities, V_{1d}, V_{2q}, I and I_{3d}' so that another equation must be obtained from the boundary conditions, i.e.

$$V = V_{1d} + V_{2q} \quad . \quad . \quad . \quad (13.2d)$$

Solving these gives, from (13.2*a*, *b* and *d*),

$$V = I\{r_1 + r_2 + Sx_{md} + j(x_{L1} + x_{L2} + x_{md} + x_{mq})\} + I_{3d}'(Sx_{md} + jx_{md}) \quad . \quad (13.3)$$

and from (13.2*c*)

$$I_{3d}' = -I(Sx_{mq} + jx_{md})/\{r_3' + j(x_{L3}' + x_{md})\} \quad . \quad (13.4)$$

Putting $r_1 + r_2 = r$, the total motor resistance

and $x_{L1} + x_{L2} = x_L$, the total leakage reactance

and substituting in (13.3) for I_{3d}' from (13.4) gives

$$V = I\left\{r + Sx_{md} + \mathrm{j}(x_L + x_{md} + x_{mq}) - \frac{(Sx_{mq} + \mathrm{j}x_{md})(Sx_{md} + \mathrm{j}x_{md})}{r_3' + \mathrm{j}(x_{L3}' + x_{md})}\right\} \quad . \quad (13.5)$$

The last term of the expression represents the effect of the currents in the coils short-circuited by the brushes; if r_3' and x_{L3}' are so great as to make these currents negligible this term becomes zero. It can often be neglected, in which case the following relations can be obtained—

$$I = V/(r + Sx_{md} + \mathrm{j}x) = V/\sqrt{\{(r + Sx_{md})^2 + x^2\}} \quad (13.6)$$

where $x = x_L + x_{md} + x_{mq}$.

This lags the voltage by $\tan^{-1}\{x/(r + Sx_{md})\}$ so that the power factor is

$$\cos\phi = (r + Sx_{md})/\sqrt{\{(r + Sx_{md})^2 + x^2\}} \quad . \quad (13.7)$$

Power Output and Torque. The mechanical output is the product of the rotational e.m.f., the current and the cosine of the phase angle between them (eq. (12.2)). The angle in this case is zero so that

$$P = SIx_{md} \,.\, I = Sx_{md}V^2/\{(r + Sx_{md})^2 + x^2\} \text{ watts} \quad (13.8)$$

and the torque is

$$TM = P/S = x_{md}V^2/\{(r + Sx_{md})^2 + x^2\} \text{ synch. watts} \quad . \quad (13.9)$$

Effect of Iron Loss. The equations so far given take no account of the iron loss. This loss is approximately proportional to the square of the flux, and therefore to the square of the current so that it can be represented by an additional resistance r_c.

The flux Φ_d, instead of being given by $Ix_{md}/2\pi fT_1$ will be given by $I(r_c + \mathrm{j}x_{md})/2\pi fT_1$ so that the e.m.f. of rotation will be

$$E_{rad} = -\mathrm{j}S(r_c + \mathrm{j}x_{md}) = S(x_{md} - \mathrm{j}r_c)$$

Substituting this for the terms involving S in the previous equations gives—

Current,

$$I = V/\sqrt{\{(r + Sx_{md})^2 + (x - Sr_c)^2\}} \text{ amperes} \quad . \quad . \quad (13.10)$$

Power factor,

$$\cos\phi = (r + Sx_{md})/\sqrt{\{(r + Sx_{md})^2 + (x - Sr_c)^2\}} \quad . \quad . \quad (13.11)$$

Output,

$$P = Sx_{md}V^2/\{(r + Sx_{md})^2 + (x - Sr_c)^2\} \text{ watts} \quad . \quad (13.12)$$

Torque,

$$TM = x_{md}V^2/\{(r + Sx_{md})^2 + (x - Sr_c)^2\} \text{ synch. watts} \,.\, (13.13)$$

Example 13.2. Calculate the performance of a 1/6-h.p., 230-V, 2-pole universal motor when running at 50 c/s. The motor has the following parameters: $r = 64\ \Omega$, $x = 140\ \Omega$, $x_{md} = 112\ \Omega$ and $r_c = 32\ \Omega$.

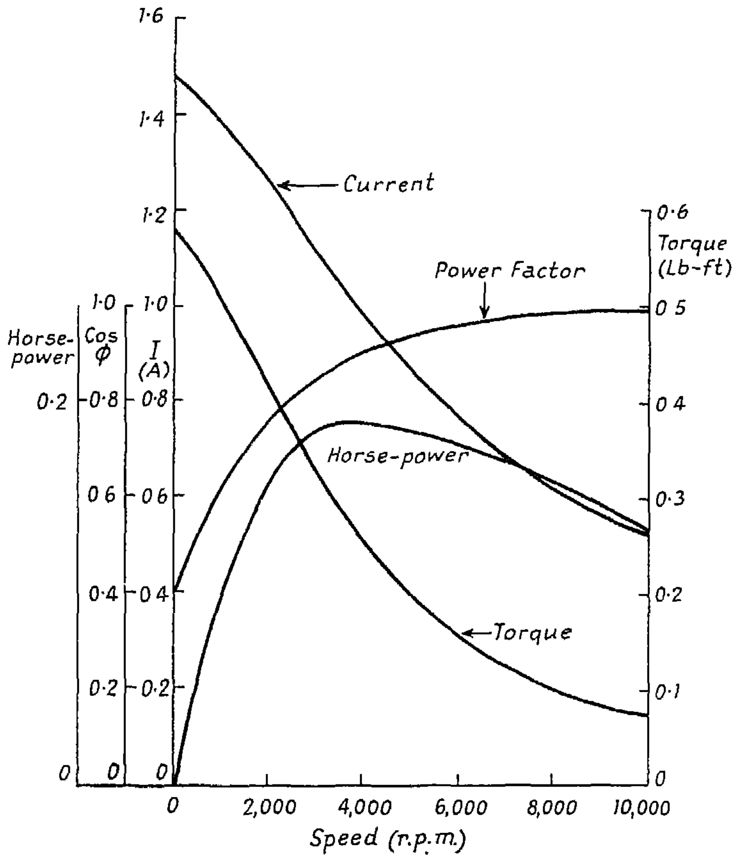


FIG. 13.8. CHARACTERISTICS OF 1/6 H.P. SERIES MOTOR

Taking a speed of 4,000 r.p.m. ($S = 4{,}000/3{,}000 = 1·33$)

$$I = 230/\sqrt{\{(64 + 1·33 \times 112)^2 + (144 - 1·33 \times 32)^2\}}$$
$$= 230/236 \qquad = 0·975 \text{ A}$$
$$\cos\phi = (64 + 1·33 \times 112)/236 \qquad = 0·90$$
$$P = 1·33 \times 112 \times 230^2/236^2 = 142 \text{ W} \qquad = 0·190 \text{ h.p.}$$
$$TM = 112 \times 230^2/236^2 = 106 \text{ synch. watts}$$
$$= 106 \times 0·117/50 \qquad = 0·248 \text{ Lb-ft}$$

To obtain the net output the friction and windage loss must be subtracted from the above; this may be about 40 W for a motor of this size at 4,000 r.p.m. so that the net output = 102 W = 0·137 h.p. The overall efficiency is thus

$$102/(230 \times 0{\cdot}975 \times 0{\cdot}90) = 50{\cdot}5 \text{ per cent}$$

Similar calculations can be made for other speeds giving the characteristics shown in Fig. 13.8.

Determination of Machine Parameters

FROM DESIGN DATA. Resistances can be calculated from wire sizes and lengths in the usual way while brush drop and brush resistance can be computed from the curves of Fig. 3.10 (page 41). Leakage reactances can be calculated from the slot and overhang permeances, and the mutual reactances can be found from

$$x_{md} = (\sqrt{2})\pi f T \Phi_d / I \text{ and } x_{mq} = (\sqrt{2})\pi f T \Phi_q / I$$

The quadrature flux, Φ_q, is difficult to determine for a salient-pole machine but it is relatively small and may be taken as about $0{\cdot}2\Phi_d$.

FROM TEST. Measurements of rotor impedance and power input with the rotor stationary and with special narrow brushes so that no coils are short-circuited by them gives $z = r + jx$ from which r and x may be determined. If the above test is repeated with normal brushes the additional power consumed represents approximately that due to the loss in the short-circuited coils.

Design Features

The general construction of a small a.c. series motor is similar to that of a d.c. motor of similar size except that the magnet system is laminated. There are, however, certain special features concerned with its commutation and the maintenance of a good power factor, which means similarity of speed when running on alternating and direct current.

DESIGN FOR GOOD POWER FACTOR. Reference to the complexor diagram of Fig. 13.2 shows that, neglecting resistance and leakage reactance drops, the phase angle is given by $\tan^{-1} (E_{tsd} + E_{taq})/E_{rad}$. To obtain a small phase angle and, consequently, a good power factor E_{tsd} and E_{taq} should therefore be small and E_{rad} large; this leads to the following requirements—

Small E_{tsd}:	*few stator turns*	low supply frequency	low Φ_d
Small E_{taq}:	few rotor turns	low supply frequency	*low* Φ_q
Large E_{rad}:	many rotor turns	*high rotational frequency*	high Φ_d

It can be seen that most of these items are conflicting and, assuming a given supply frequency, only those in italics can be used to design a motor with a good power factor. To keep the number of stator turns to a minimum necessitates the minimum possible air

gap and a value between 1/100 and 1/200 of the rotor diameter, i.e. 0·15 to 0·03 in., is usual; to avoid excessive flux distortion on load, however, which would lead to high iron loss, the ratio of stator to rotor turns should not be less than a value between 0·7 and 1·0.

The quadrature flux can be kept low by using salient poles which gives a high-reluctance flux path in the quadrature axis.

The rotational speed is generally high, values up to 10,000 or 15,000 r.p.m. being common.

Attention to these points enables motors with satisfactory "universal" characteristics to be built, although for low-speed motors (below 4,000 r.p.m.) where E_{rad} is small it may be necessary to fit a field tapping for use when running on alternating current.

COMMUTATION. To secure good commutation the e.m.f. induced in the short-circuited coils by the direct-axis flux should be kept below about 7 V. It is found that coils of 20 or 30 turns enable this to be done and this will necessitate between 12 and 24 commutator segments.

OUTPUT COEFFICIENT. As with other types of machine, the output coefficient is a suitable starting point for a design. The output of the motor is

$$P = E_{rad} I \cos \psi \text{ watts}$$

ψ being the angle between current and flux, allowing for the iron and brush circulating-current loss. Cos ψ is not usually less than 0·96 and may be regarded as 1·0.

Thus
$$P = (\sqrt{2})(T_a/a)pn\Phi I$$

Since $\Phi = \bar{B}\pi DL/2p$ and $(I/2a)2T_a = ac\pi D$ this becomes

$$P = (\pi^2/\sqrt{2})\bar{B}acD^2Ln$$

so that
$$G = P/D^2Ln = (\pi^2/\sqrt{2})\bar{B}ac \quad . \quad . \quad (13.14)$$

Typical values of the specific magnetic and electric loadings are $\bar{B} = 0{\cdot}25 - 0{\cdot}35$ Wb/m² and $ac = 6{,}000 - 10{,}000$ A-conductors/m. These figures give values of output coefficient (watts/D^2Ln) between $10{\cdot}5 \times 10^3$ and 25×10^3, D and L being expressed in metres.

Performance and Applications

Typical performance curves are shown in Figs. 13.4 and 13.8; power factor is good and starting torque is 3 to 4 times full-load torque.

The usual applications of the universal motor are for vacuum cleaners, sewing machines, portable drills, mechanical computing machines and other small-power drives. The series speed-torque characteristic may be a definite advantage in certain cases, e.g. on a drill where a small drill will run at a high speed, and on a vacuum cleaner since the motor speed will rise under conditions which decrease air volume, thereby tending to counteract the effect. For

circumstances where a universal motor is desired but a constant speed is necessary a governor-controlled motor can be employed; the governor operates a switch above a certain speed and inserts a resistor in series with the motor; in this way speeds can be held constant up to about 1·5 times full-load.

Accuracy is essential in estimating the power required by the driven load; with an induction motor little ill effect results if a 1/4 h.p. motor is used for a drive requiring only 1/6 h.p. but a series motor under such conditions would run at quite the wrong speed.

Since these motors are widely used in domestic premises, radio interference is a serious problem, and suppression methods as described in Chapter 3 (page 41) are desirable.

EXERCISES 13

1. A 2-pole, 50-c/s, 230-V, f.h.p. series motor has 400 field turns and 1,200 armature turns. At full load the current is 1 A, the exciting flux is 0·0008 Wb and the speed is 7,000 r.p.m. The total resistance is 20 Ω and the leakage reactance is 30 Ω. Draw the complexor diagram and find—

(a) The transformer e.m.f. induced in the armature by the quadrature flux.
(b) The magnitude of the quadrature flux.
(c) The h.p. output.
(d) The speed at which it would run if supplied with a direct voltage of 230 and taking 1 A.

2. A universal f.h.p. series motor has a resistance of 30 Ω and a total inductance of 0·5 H. When connected to a 250-V, d.c. supply and loaded to take 0·8 A it runs at 2,000 r.p.m. Estimate the speed and power factor when connected to a 250-V, a.c. supply and loaded to the same current.

3. A $\frac{1}{2}$-h.p., 10,000-r.p.m., 50-c/s, 115-V series motor with a cylindrical stator has a total resistance of 1·6 Ω and a total reactance of 10 Ω. Plot, to a base of current, curves of speed, torque, h.p. and power factor using (a) a graphical method based on the complexor diagram and (b) an analytical method.

4. The data refer to a $\frac{1}{16}$-h.p., 2-pole, 50-c/s series motor having salient poles—

Total resistance = 23·6 Ω.
Total leakage reactance = 68 Ω.
Mutual reactance = 121 Ω in direct axis.
Mutual reactance = 72 Ω in quadrature axis.
Iron, friction and windage loss = 40 W.

Calculate the current, power factor and output when running at 6,000 r.p.m. Neglect the current in the coils short-circuited by the brushes.

5. A 1-ph., 2-pole, series motor has 16 slots each carrying 4 coil sides with 8 turns per coil. The pole covers 0·67 of the pole-pitch. If the full-load current is 5·6 A draw the armature m.m.f. wave and estimate the minimum number of field turns necessary to avoid reversal of the resultant m.m.f. under the trailing pole-tip. If the number of field turns used is 40 per cent more than the above and the length of the air-gap at the pole-tip is 0·03 in. estimate the flux density under the leading pole-tip. Neglect saturation and fringing.

6. Work out the main features of a design for a $\frac{1}{10}$-h.p., 230-V, 50-c/s, 2-pole, 6,000-r.p.m., universal motor. Use specific magnetic and electric loadings of 0·3 Wb/m² and, 7,000 A-conductors/m respectively.

CHAPTER 14

THE COMPENSATED SERIES MOTOR

For motors above the fractional-horse-power size it is necessary to have better power factor and better commutation than are obtainable with the plain series motor described in the last chapter. By attention to the appropriate details it is possible to build motors giving 1,000 h.p. or more, the chief application of such motors being for single-phase traction such as is employed on the railways of Central Europe and Scandinavia.

The Compensating Winding

Reference to the complexor diagram of Fig. 13.2 (page 221) shows that the low power factor is in part due to the voltage induced in the rotor winding by the quadrature flux. The special feature of

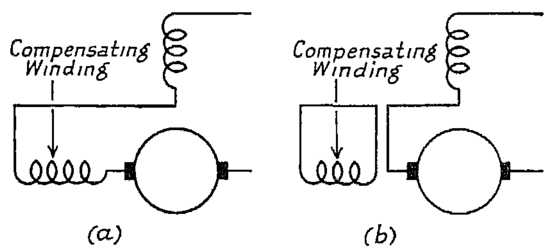


Fig. 14.1. Compensated Series Motors
(a) Conductively compensated.
(b) Inductively compensated.

the compensated series motor is the compensating winding fitted on the stator to neutralize the rotor m.m.f. and thus eliminate the quadrature flux and the e.m.f. E_{taq}.

For compensation to be correct at all loads the compensating winding must carry currents proportional to the main current, and two methods of connexion are available as shown in Fig. 14.1. In the *conductively compensated* machine the winding carries the main current so that the effective turns per pole T_q' are given by $IT_q' = (I/2a)/(T_a'/2p)$. With the *inductively compensated* machine the compensating current is induced in the winding by transformer action as in a transformer with a short-circuited secondary winding. Any convenient number of turns can therefore be used; this may lead to a simpler design, although compensation cannot be quite complete due to the leakage fluxes. In both cases the turns must be uniformly distributed over the whole of the stator periphery if complete compensation is to be achieved and a typical arrangement,

including the compoles to be described later, is shown in Fig. 14.2. The exciting winding is usually placed in slots in order to minimize leakage flux, but it may be concentrated in a single large slot.

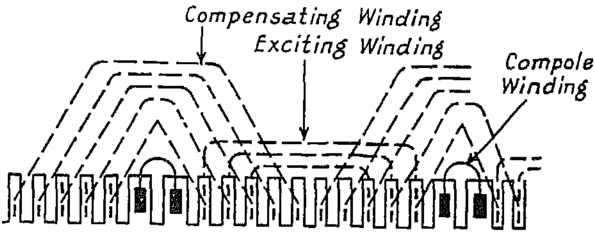


FIG. 14.2. ARRANGEMENT OF WINDINGS IN SLOTS

Complexor Diagram

Assuming the quadrature flux to be completely neutralized, the complexor diagram can be drawn as shown in Fig. 14.3, this being in most respects similar to that for the plain series motor shown in Fig. 13.2 (page 221). The main current I leads the exciting flux Φ_d by a small angle, usually between 3° and 9° due to iron loss and

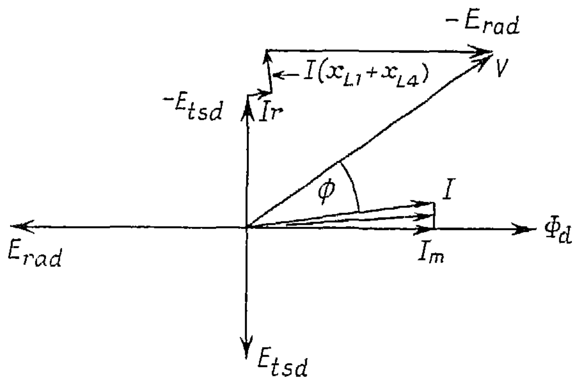


FIG. 14.3. COMPLEXOR DIAGRAM OF COMPENSATED SERIES MOTOR

brush circulating currents. E.m.f.'s E_{tsd} in the stator winding and E_{rad} in the rotor winding are set up as before, together with the stator leakage reactance drops and the resistance drop for the whole machine. There are, in addition, leakage reactance drops due to the rotor and compensating windings. In the conductively compensated machine these drops appear across their respective windings, while in the inductively compensated machine the drop in the compensating winding is reflected into the rotor winding.

Circle Diagram

The circle diagram described on page 222 for the plain series motor applies also to the compensated motor; with these larger motors it is, however, desirable to be able to construct the diagram from simple test data and this can be done as follows. Selecting a point O and drawing a horizontal line through it as in Fig. 14.4 gives

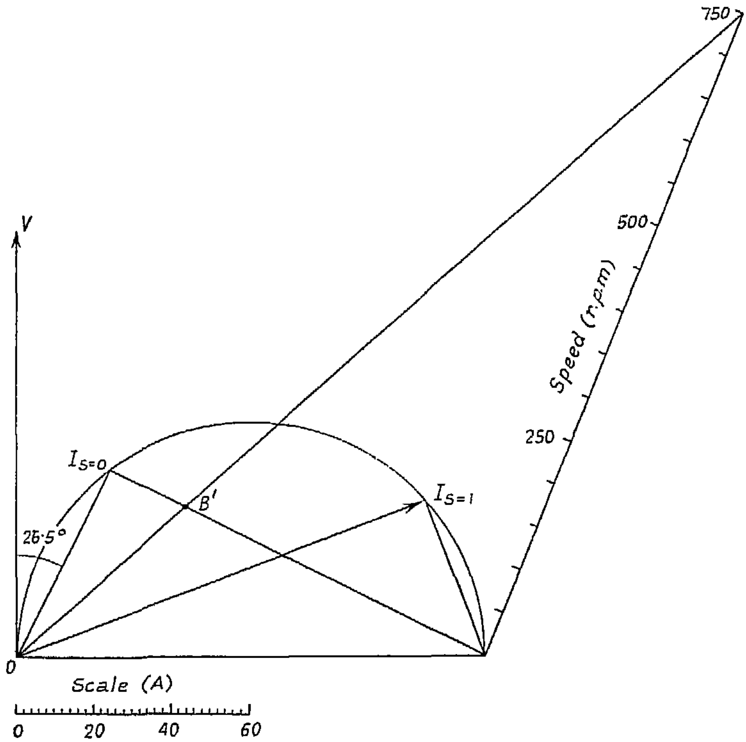


FIG. 14.4. CIRCLE DIAGRAM FOR 10 H.P. COMPENSATED SERIES MOTOR

the line along which lies the diameter of the circle. A locked rotor test at normal voltage would give the line $OI_{s=1}$—in practice it is not possible to carry out this test at normal voltage on account of the heavy currents which would occur, particularly in the coils short-circuited by the brushes, and a current of about half full-load value should not be exceeded, the full-voltage value being obtained by proportion. The circle can then be drawn through O and the point $I_{s=1}$ with its centre on the horizontal. The power input and power factor for various input currents can thus be determined. To find the speed a second point on the speed line must be determined, and synchronous speed may conveniently be selected.

At synchronous speed ($f_r = f_1$)

$$E_{tsd} = (\sqrt{2})\pi T_s f_1 \Phi_d \text{ volts}$$

and

$$E_{rad} = (\sqrt{2})(T_a/a) f_1 \Phi_d \text{ volts}$$

Neglecting resistance and leakage reactance drops we have

$$\tan\phi = E_{tsd}/E_{rad} = \pi \,.\, T_s a/T_a$$

If, therefore, the turns ratio T_s/T_a and the number of pairs of parallel paths can be found, the power factor at synchronous speed can be determined and the point $I_{s=0}$ located on the circle. The number of pairs of parallel paths, a, is equal to the number of pairs of poles for a simple lap winding or to 1 for a simple wave winding. The type of winding can usually be seen by inspection, the lap winding being almost universal. The turns ratio can be obtained by a simple test involving the application of a suitable low voltage V_s to the stator winding, and the measurement of the voltage between adjacent commutator segments for a pair of segments in the neutral zone, i.e. linked with the whole exciting flux. If this segment voltage is V_c then $V_s/V_c = T_s/T_c$.

If the number of commutator segments (which may be counted) is C, then $T_a = CT_c$ so that

$$(T_s/T_a)C = V_s/V_c$$

and

$$\tan\phi = (V_s/V_c)(\pi a/C)$$

Setting off $I_{s=0}B'$ equal to the resistance drop corresponding to the current $OI_{s=0}$ enables the synchronous speed point to be located on the speed line and a speed scale constructed.

To find the output the iron, friction and windage losses must be determined or estimated—they can be found on test by running the motor with separate excitation as in the case of a d.c. series motor.

Example 14.1. The following test figures were taken on a 10-h.p., 25-c/s, 4-pole, 225-V compensated series motor having a lap-connected rotor winding.

LOCKED-ROTOR TEST. 60 V, 30·5 A, 0·645 kW.

Hence $\cos\phi = 0{\cdot}358 \quad (\phi = 69^\circ)$

and current at normal voltage $= 30{\cdot}5 \times 225/60 = 115$ A.

The point $I_{s=1}$ can thus be located and the circle drawn as shown in Fig. 14.4. The effective resistance of the motor is

$$645/30{\cdot}5^2 = 0{\cdot}695\ \Omega$$

VOLTS PER COIL. With 26 V applied to the stator the volts per coil was 1·54 V. The number of commutator segments was counted to be 212.

Thus at synchronous speed

$$\tan\phi = (\pi \times 26 \times 2)/(212 \times 1{\cdot}54) = 0{\cdot}5$$

$$\therefore \quad \phi = 26{\cdot}5^\circ$$

Setting this angle off on the diagram gives the point $I_{s=0}$ from which the current at synchronous speed is measured to be 56 A. The resistance drop is thus $56 \times 0{\cdot}695 = 39$ V and can be set off giving $I_{s=1}B'$. Projecting OB' to cut the speed line gives the synchronous speed point (750 r.p.m.) and enables the speed scale to be constructed as shown.

Commutation

Since, in the compensated motor, the quadrature flux has been eliminated, the only e.m.f.'s induced in the coils short-circuited by the brushes are the reactance e.m.f. E_{xb} in phase with the current and the transformer e.m.f. E_{tbd} lagging the flux Φ_d by 90° as described

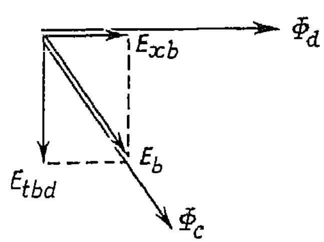


FIG. 14.5. BRUSH E.M.F.'S AND COMPOLE FLUX

on page 33 and as shown in Fig. 14.5. The transformer e.m.f. is proportional to flux, and therefore approximately to current, while the reactance e.m.f. is proportional to both current and speed.

Since the impedances of the short-circuited coils are relatively smaller with the larger motors the currents induced by the transformer e.m.f. may cause bad commutation if the e.m.f. E_b is allowed to exceed 3 or 4 volts per coil, and this imposes a limit to the flux per pole which may be used in designing the motor. Early motors, particularly those for 25 c/s, made use of resistance connectors (page 37) for limiting the brush circulating currents, but with traction motors there is danger of these being burnt out due to delay in the starting of a heavy train. Modern motors therefore usually employ compoles, the flux from which induces an e.m.f. in opposition to those mentioned above in much the same way as in a d.c. machine.

USE OF COMPOLES. The e.m.f. induced in the short-circuited coil by the compole flux is in phase with, or in phase opposition to, that flux and for complete neutralization must be equal and opposite to E_b. The phase of the compole flux Φ_c must therefore be as shown in Fig. 14.5.

Various ways of connecting the compole are possible—if connected across the supply the compole flux will lag the applied voltage by 90° so that it will induce an e.m.f. having components opposed to both E_{tbd} and E_{xb}. The flux will be constant so that the component

opposing E_{tbd} will be proportional to speed instead of to exciting flux, while the component opposing E_{xb} will be proportional to speed instead of to the speed and current.

Alternatively, the compole may be connected across the rotor; the flux is then approximately constant, so that the e.m.f. induced by it is proportional to speed and is opposed to E_{tbd} with no component opposed to E_{xb}.

Neither of these connexions fully meets the requirements, and a better although not perfect alternative is the shunted compole now in common use.

THE SHUNTED COMPOLE. The compole is shunted by a resistor as shown in Fig. 14.6(*a*) and connected in the main circuit. The current in the compole lags the main current as shown in Fig. 14.6(*b*) and the resulting compole flux, which is in phase with the compole current, will induce e.m.f.'s having components opposed to both E_{tbd} and E_{xb}. Since flux is proportional to current the component opposing E_{xb} will be proportional to both current and speed, so that neutralization of this can be complete but the component opposed to E_{tbd} can neutralize it at only one value of speed, and not at all at starting. The compole is normally designed to give a flux lagging by between 30° and 50° behind the main flux and to give complete neutralization at normal load. Commutation will thus be satisfactory at normal loads but poor at starting. The loss in the resistors is usually between 1 and 2 per cent of the output.

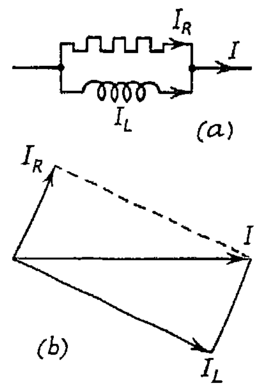


FIG. 14.6.
SHUNTED COMPOLE
(*a*) Circuit diagram.
(*b*) Current complexors.

To give good commutation over a wider range of speed, two settings of the compole resistor may be used. Also, to improve commutation at starting, the main field may be weakened by a tapping; this results in a higher starting current for a given torque but reduces the circulating current and gives a net improvement.

Harmonics in the current wave tend to flow in the ohmic resistor so that their effect is not compensated by the compole flux; this is avoided by shunting a part of the resistor with an inductor so that the shunting effect decreases with an increase of frequency.

Analytical Treatment

Expressions for both types of compensated motor can be obtained by inserting the appropriate boundary conditions in the general eqs. (12.3) on pages 218 and 219. If complete compensation is assumed, however, the conditions are similar to those of the plain series motor outlined on page 228 but with x_{mq} made zero and the

resistance and leakage reactance of the compensating winding included in the rotor impedance.

Control

The most common application of the compensated series motor is for traction, and speed control is therefore necessary. The usual method is by a variable voltage obtained from tappings on a transformer, this transformer being necessary in any case to step the

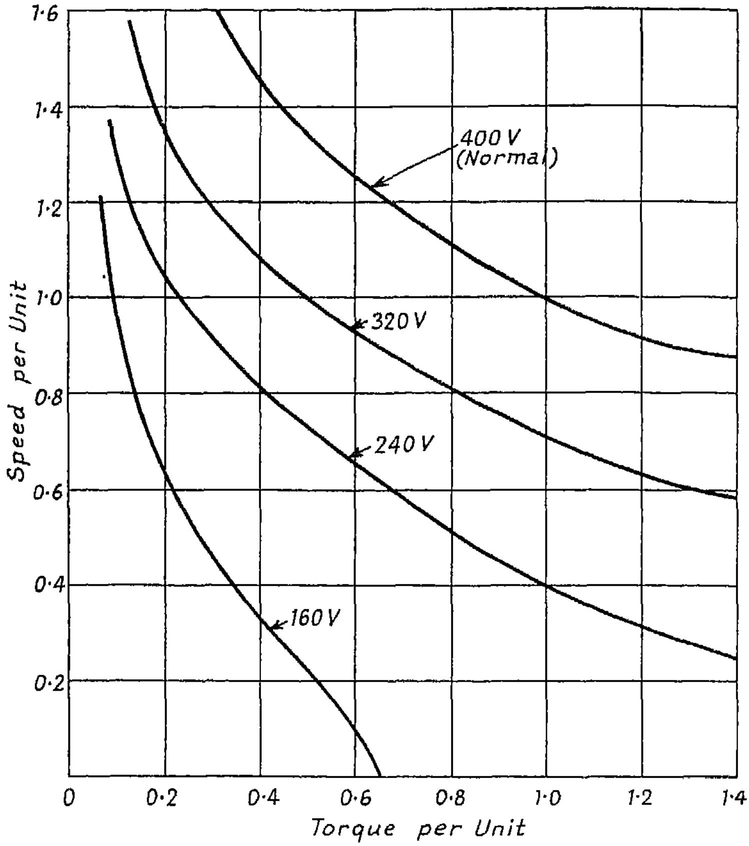


FIG. 14.7. SPEED-TORQUE CHARACTERISTICS WITH VOLTAGE CONTROL

voltage down to the low value required for the motors. Typical locomotives have between 10 and 20 tappings giving motor characteristics as shown in Fig. 14.7.

The tappings may be arranged on the low- or high-voltage windings of the transformer—the former involves heavy-current tapping switches and, since a single switch cannot handle more than

about 1,000 A, special arrangements to ensure the division of the current through several tapping switches have sometimes to be employed; although high-voltage tappings may lead to insulation difficulties, the modern tendency is to use them for high-power locomotives.

In the design of the tapping switches an essential feature is to ensure a smooth acceleration, and interruption of current during a tap change is quite inadmissible. Some form of on-load tap-changing equipment is therefore necessary.*

Braking

For traction purposes either rheostatic or regenerative braking is desirable and, as with d.c. motors,† both may be effected.

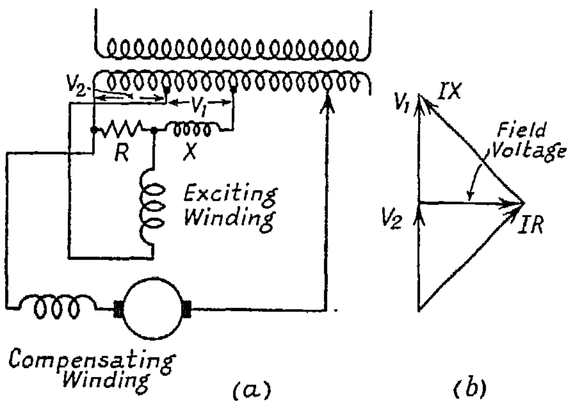


FIG. 14.8. REGENERATIVE BRAKING WITH PHASE-SHIFT NETWORK FOR FIELD VOLTAGE
(a) Connexion diagram.
(b) Complexor diagram.

RHEOSTATIC BRAKING. The motors are connected as separately-excited generators and loaded on resistors, the fields being excited from a low-voltage tapping on the main transformer or from one of the motors operating as a series generator. In the former case the braking energy will be at supply frequency while, in the latter, direct currents will be generated.

REGENERATIVE BRAKING. To enable the motor to act as a generator and return power to the line, it is necessary to ensure that the frequency of the regenerated power is of supply frequency, and the field winding must therefore be excited from the supply. Also the power factor of the regenerated current must be near to unity so that the current must be approximately in phase opposition to the e.m.f.

* Say, *The Performance and Design of A.C. Machines*, Ch. V (Pitman).
† Taylor, *Utilization of Electric Energy*, Ch. II (English Universities Press).

(E_{rad}) and the flux. To ensure the correct phase for the flux, therefore, the voltage applied to the exciting winding must be 90° displaced from the applied voltage. One arrangement is shown in Fig. 14.8, the correct phase for the voltage applied to the field being obtained by suitable choice of values for R and X as shown by the complexor diagram. Alternative arrangements giving a similar result employ a capacitor in series or parallel with the field windings or a separate exciter.

An alternative arrangement using an auxiliary inductor in series with the motor is shown in Fig. 14.9(*a*). The transformer voltage V_a is equal to the sum of the rotational e.m.f. E_{rad} and the e.m.f. E_x induced in the choke ($= -IX$). The voltage V_e applied to the field

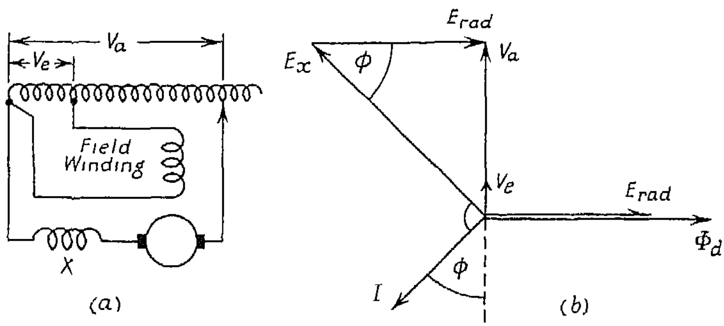


FIG. 14.9. REGENERATIVE BRAKING USING AUXILIARY CHOKE

(*a*) Connexion diagram.
(*b*) Complexor diagram.

winding is in phase with V_a, and the exciting flux Φ_d lags this by 90° as shown in the complexor diagram, Fig. 14.9(*b*). Since E_{rad} is in phase with the flux, the e.m.f. induced in the inductor must be as shown and lags the current by 90°. It is thus seen that there is a component of the current opposed to V_a, and power is therefore being supplied to the transformer at a power factor of

$$\cos \phi = E_{rad}/E_x$$

The power regenerated is $V_a I \cos \phi$.

Since $I = E_x/X$ the power may be written

$$P = V_a(E_x/X)(E_{rad}/E_x) = V_a E_{rad}/X$$

Braking torque is proportional to

$$P/n = V_a E_{rad}/nX \propto V_a n \Phi_d/nX \propto V_a V_e/X$$

i.e. it is constant for any given transformer tapping and can be controlled by varying V_a using the tappings normally employed for speed control.

A further improvement on this involves a phase control circuit for the field winding as in the previous scheme, so that Φ_d can be made to lag V_e by less than 90° with a corresponding improvement in power factor.

Design Features

Governing factors in the design of compensated series motors are the necessity of securing a good power factor and a low value of e.m.f. induced in the coils short-circuited by the brushes.

OUTPUT COEFFICIENT. The output coefficient is again a suitable starting point for a design and, as shown on page 232 for the plain series motor, is given by—

$$G = \text{kW}/D^2Ln = (\pi^2/\sqrt{2})\bar{B}_{ac} \times 10^{-3} \times \eta'$$

where η' is the efficiency allowing only for losses included in the rotor power.

Typical values of specific loadings are discussed later.

POWER FACTOR. A good power factor is more essential with the compensated than with the plain series motor on account of its larger size. Reference to the complexor diagram of Fig. 14.3 shows that the phase angle is approximately (neglecting resistance and leakage reactance drops) given by

$$\tan \phi = E_{tsd}/E_{rad}$$

To attain a high power factor, therefore, E_{rad} should be large and E_{tsd} should be small, and this leads to the following conditions—

Large E_{rad}:	many rotor turns	high f_r	high Φ_d
Small E_{tsd}:	few stator turns	low f_1	low Φ_d

The flux requirements are conflicting but from the others it can be seen that a high speed and large number of poles, a low supply frequency, few stator turns and many rotor turns are desirable. If the full-load speed is made three to five times synchronous speed and attention is given to the other points, full-load power factors between 0·9 and 0·95 are obtainable. It may be noted that the improvement effected by a large number of poles may be partially off-set by increased leakage due to the small number of slots per pole. The power factor at starting is, of course, very low, between 0·1 and 0·2.

COMMUTATION. A further limitation is imposed as with all a.c. commutator motors, by the maximum permissible induced e.m.f. in the coils short-circuited by the brushes. This e.m.f. should not exceed 3 to 4 volts and is given by

$$E_{tvd} = (\sqrt{2})\pi f_1 T_c \Phi_d \text{ volts}$$

To keep this to a low value a small flux per pole, a low frequency and a minimum number of turns per coil are thus required. With motors of more than a few h.p. single-turn coils are universal so

that the maximum permissible flux per pole is given by the values of Table 3.2 (page 34).

FREQUENCY. For industrial purposes the frequency is fixed at 50 or 60 c/s but for traction it is possible to use a separate supply network operating at a lower frequency. Railways in Central Europe and Scandinavia use 15 or $16\frac{2}{3}$ c/s and a.c. railways in America use 25 c/s, so that motors with a good power factor and a reasonably large flux per pole are available. Motors operating at 50 c/s have, however, recently been built for use on 50 c/s traction schemes in France and elsewhere.

ROTOR WINDING. *In addition to using single-turn coils, a lap* winding is desirable so that there shall be only one turn between adjacent commutator segments. To minimize the reactance voltage the same rules as for a d.c. machine should be observed, i.e. an odd integral number of slots per pole, an odd number of turns per slot, a fractional pitch, the use of split coils to minimize mutual induction between coils undergoing commutation, and the use of equalizing connexions.

FIELD AT/ROTOR AT RATIO. A small number of field turns and a large number of rotor turns have been seen to be desirable, and ratios of 0·2 to 0·3 can be adopted since the distorting effect of the armature m.m.f. is eliminated by the compensating winding.

AIR GAP. The need for keeping down the field m.m.f. results in the minimum possible air gap, and values between 2 and 4 mm (0·08 and 0·16 in.) are usual compared to 4 to 6 mm for a d.c. machine of similar size.

SPECIFIC LOADINGS. The requirement of a low field m.m.f. necessitates low flux densities and therefore a low specific magnetic loading; values (at the peak of the wave) between 0·4 and 0·7 Wb/m^2 are usual. The elimination of flux distortion by the armature m.m.f. enables the electric loading to be pushed well up to the heating limit, and values between 35,000 and 60,000 A-conductors/m are usual, the higher values being used if forced ventilation is available.

COMMUTATOR. The use of single-turn coils leads to a large number of commutator segments (up to 500). The smallest practicable segment width is about 0·45 cm but even with this the diameter of the commutator will be almost as great as that of the rotor; i.e. about 0·9D compared to about 0·6D for a d.c. machine.

BRUSHES. To avoid short-circuiting more than two coils the brush *width should not exceed* 2 *segments, i.e.* $\frac{3}{8}$ *to* $\frac{1}{2}$ in. Even if high-resistance brushes are used the circulating currents induced in the short-circuited coils may be comparable with the full-load current so that the permissible load current under the brush should not be more than about half the values given in Table 3.3 (page 41), i.e. 15 to 20 $A/in.^2$ To carry the heavy currents a large number of brushes may therefore be required; the large number of poles enables, however, a correspondingly large number of brush arms to be used so that a long commutator is not necessary.

Output Limitations

The output is given by

$$P = E_{rad}I = (\sqrt{2})(T_a/a)pn_r\Phi_d I$$

Hence

$$\text{output per pole} = P/2p = (1/\sqrt{2})(T_a/a)n_r\Phi_d I$$

Inserting $$(I/2a)2T_a = \pi Dac = (v/n_r)ac$$

and $$\Phi_d = E_{tbd}/\sqrt{2}\pi f_1 T_c = E_{tbd}/\sqrt{2}\pi f_1 \text{ (since } T_c = 1)$$

gives $$\text{output/pole} = v \times ac \times E_{tbd}/2\pi f_1 \text{ watts}$$

Inserting limiting values

$$v = 30 \text{ m/sec}$$
$$ac = 50{,}000 \text{ A-conductors/m}$$
$$E_{tbd} = 6 \text{ V}$$
$$\text{h.p. per pole} = 1{,}925/f_1$$

The limiting outputs are thus 115 h.p. at $16\frac{2}{3}$ c/s, 77 h.p. at 25 c/s and 38·5 h.p. at 50 c/s.

It can be seen that a low frequency is desirable if large motors are required and also that a large number of poles, e.g. up to 24, may be needed, especially for 50 c/s motors.

LIMITING VOLTAGE. The necessity of using single-turn coils imposes *a limit to the rotor voltage* which can be used. The rotor voltage may be written

$$E_{rad} = (\sqrt{2})(T_a/a)pn_r\Phi_d = (T_a/a)pn_r E_{tbd}/\pi f_1$$

Since single-turn coils are used the number of commutator segments C is equal to T_a so that

$$Cw_c = T_a w_c = \pi D_c = v_c/n_r$$

where w_c = width of segment,

v_c = peripheral speed of commutator.

Thus $$E_{rad} = (v_c/w_c)(p/a)(E_{tbd}/\pi f_1)$$

Inserting the limiting values gives—

$$E_{rad} = (3{,}000/0{\cdot}75)\,.\,1\,.\,6/\pi f_1 = 7{,}600/f_1$$

The limiting rotor voltages, which are slightly less than the terminal voltages as seen from the complexor diagram, are thus—

Frequency (c/s)	$16\frac{2}{3}$	25	50
Voltage . . .	455	302	152
Field turns per pole .	22	15	8

It is thus evident that for traction purposes, where a high-voltage is desirable for the trolley wire, a transformer will have to be employed on the locomotives.

Reference to the complexor diagram shows that for a good power factor the field voltage should not exceed about $\frac{1}{4}$ of the rotor voltage at full load; since the voltage per turn must not exceed 4 to 5 volts (the same as for the short-circuited coils), the maximum permissible number of field turns per pole is fixed and given above.

Modern motors can be built with a weight of about 8 Lb/h.p. which is little more than that of a d.c. traction motor.

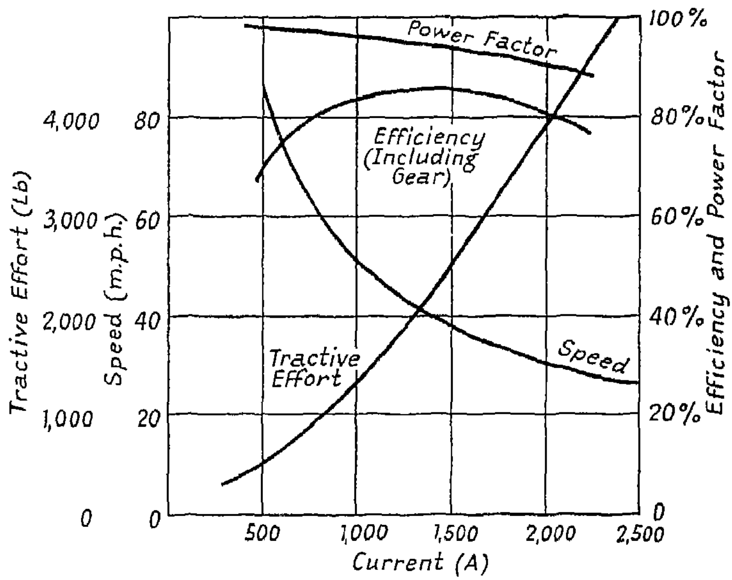


FIG. 14.10. TYPICAL CHARACTERISTICS OF 300-H.P. SINGLE-PHASE RAILWAY MOTOR

Performance and Applications

Typical performance curves for a 300-h.p., 170-V railway motor are shown in Fig. 14.10. It is seen that the power factor approaches unity over the working range, but the efficiency is a few per cent lower than a d.c. motor of corresponding output, this being due largely to commutation losses.

Almost the only application of the large single-phase series motor is for railway traction on the lines of Central Europe, Scandinavia and, to a lesser extent, U.S.A. The low voltage for which the motor must be designed necessitates a transformer on the locomotive; this, however, enables a high trolley-wire voltage to be employed (11–16 kV) with a consequent reduction in distribution losses as compared to the standard d.c. systems. The design difficulties

arising from commutation have, in the past, required the use of a low frequency ($16\frac{2}{3}$ c/s in Europe and 25 c/s in U.S.A.) so that the railway distribution network has had to be supplied from special low-frequency generating stations or through frequency converters from the 50 c/s industrial network. Modern developments, however, have enabled satisfactory 50 c/s motors to be designed so that the railway network can be fed directly from the 50 c/s industrial network; locomotives using these motors are, however, in competition with rectifier locomotives in which the 50 c/s supply is transformed and rectified on the locomotive and fed to ordinary d.c. motors.

EXERCISES 14

1. A 16-pole, 1,250-h.p., 465-r.p.m., $16\frac{2}{3}$-c/s compensated series motor has a flux per pole at full load of 0·063 Wb and its stator winding has 48 turns arranged in two parallel paths. The rotor winding is lap connected with 576 turns. Calculate, using appropriate values of distribution factor, the stator and rotor e.m.f.'s when running at full load and normal speed.

2. Draw the complexor diagram and find the applied voltage and power factor of a 25-c/s, 6-pole compensated series motor running at 1,000 r.p.m. and taking 60 A, the motor having the following particulars—

    Useful flux per pole at 60 A = 0·0075 Wb.
    Total stator flux per pole at 60 A = 0·0078 Wb.
    Wave-connected armature with 848 conductors and 2 turns per coil.
    Field winding = 15 turns per pole, all poles being in series.
    Total resistance = 0·31 Ω.
    Armature leakage reactance = 0·53 Ω.
    Friction, windage and iron-loss component of current = 5 A.
    Component of current supplying loss in short-circuited coils = 8 A.

Find also the e.m.f. in the short-circuited coils.

3. A 145-h.p., 275-V, 25-c/s, 6-pole compensated series motor has a lap-connected armature with 252 single-turn coils. The field coils have 3 turns per pole and all poles are in series.

    Total resistance = 0·03 Ω.
    Total leakage reactance = 0·063 Ω.
    Full-load current = 500 A.
    Full-load flux = 0·05 Wb.
    Iron-loss component of full-load current = 18·5 A.
    Component of full-load current supplying loss in short-circuited coils = 37·5 A.

Draw the complexor diagram and plot, to a base of current, curves of speed, power factor and torque.

4. A 16-pole 1,000-A compensated series motor has 576 single-turn coils forming a lap-connected armature winding. The compensating winding is arranged in two parallel paths and is located in 5 slots per pole, the slot-pitch being 1/120 of the stator periphery. Determine a suitable number of turns for the compensating winding and draw the m.m.f. waves for the armature and compensating windings. What is the maximum uncompensated m.m.f.?

5. A 50-c/s compensated a.c. series motor has a limiting peripheral speed for its commutator of 33 m/sec, a segment width of 4 mm and a maximum permissible e.m.f. in the short-circuited coil of 3 V. Estimate the flux per pole, the rotor voltage and the number of field turns per pole. If the electric loading is 50,000 A-conductors/m how many poles would be needed for a motor to give 200 h.p.? Assume a simple lap winding.

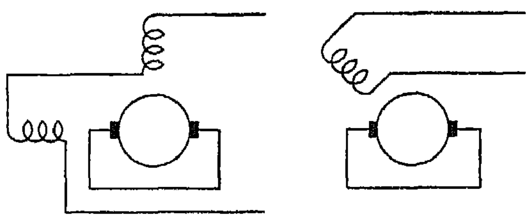
CHAPTER 15

THE REPULSION MOTOR

General Principle

A further simple way of arranging the circuits of a single-phase motor is shown in Fig. 15.1(*a*) and gives the plain repulsion motor. Two stator windings with their axes at 90° are provided, while the rotor has one pair of brushes located along the axis of one of the stator windings. The brush axis is generally known as the *quadrature* or *transformer* axis while the axis at 90° to this is the *direct, exciting* or *speed* axis.

The current in the direct-axis stator winding sets up an exciting flux Φ_d along this axis while the same current flowing in the quadrature-axis stator winding sets up an m.m.f. which, together with the m.m.f. of the rotor winding, sets up a quadrature-axis flux Φ_q. These fluxes set up e.m.f.'s in the rotor and, since the rotor winding is short-circuited on itself, currents flow which interact with the exciting flux to produce torque.



(*a*) (*b*)

FIG. 15.1. TYPES OF REPULSION MOTOR

(*a*) Double-winding motor.
(*b*) Single-winding motor.

Since the flux is produced by the main current, as in the series motor, the repulsion motor has a series characteristic; the motor is commonly used in sizes up to about 5 h.p. where such a characteristic is required and also it is used in connexion with the starting of single-phase induction motors as described on page 297.

MOTOR WITH SINGLE STATOR WINDING. The above arrangement can be simplified by combining the two stator windings into a single winding as shown in Fig. 15.1(*b*). The exciting- and quadrature-axis components of the m.m.f. of this winding correspond to the m.m.f.'s of the two separate windings of Fig. 15.1(*a*). Except where reversal of rotation is required this type, being simpler, is generally used.

Brush Position

The brushes may be made adjustable or continuously movable, and it is therefore necessary to consider the effect of brush position. As mentioned above, the single-winding motor is the most common and is considered in what follows.

HIGH-IMPEDANCE BRUSH POSITION. If the brushes are placed with their axis at 90° to that of the stator winding as shown in Fig. 15.2(*a*), there will be no mutual induction between the two windings, no e.m.f. induced in the rotor winding and no rotor current. The only m.m.f. acting will be that due to the stator current, and this will be simply a magnetizing m.m.f. so that the impedance of the motor will be high. Since there is no rotor current there can be no torque, and the motor will not run with the brushes

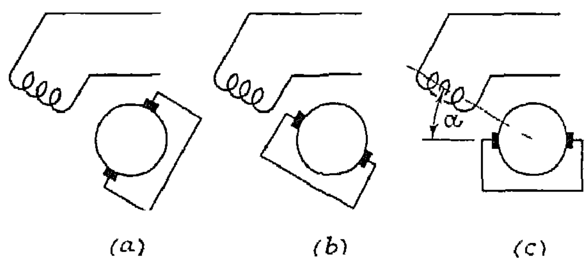


FIG. 15.2. BRUSH POSITIONS

(*a*) High-impedance position.
(*b*) Low-impedance position.
(*c*) Running position.

in this position. The conditions are similar to those of a transformer on open circuit, and this brush position is commonly called the open-circuit, high-impedance or neutral position. It should be noted that the coils short-circuited by the brushes are fully linked with the flux, and it is therefore not desirable to leave the motor switched on with the brushes in this position.

LOW-IMPEDANCE BRUSH POSITION. If the brushes are moved to the position shown in Fig. 15.2(*b*) where their axis is coincident with that of the stator winding the mutual induction between the windings is a maximum, and any flux produced by the currents in the windings completely links both of them. An e.m.f. will thus be induced in the rotor winding and a rotor current set up. The conditions are similar to those of a transformer on short circuit, with a low impedance and heavy current, and this position is known as the short-circuit or low-impedance position. As the axes of the two m.m.f.'s are coincident there is again no torque and the motor will not run.

RUNNING POSITION. If the brushes are displaced by an angle α from the low-impedance position the stator m.m.f. can be resolved into two components, $IT_s \cos \alpha$ along the quadrature axis and

$IT_s \sin \alpha$ along the direct axis as shown in Fig. 15.3. The latter produces a direct-axis or exciting flux Φ_d and the interaction between this and the rotor current gives torque so that the motor will start and run.

The directions of the two m.m.f.'s are in opposition when in the low-impedance position and the conditions may then be represented

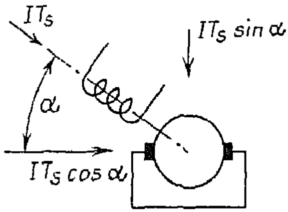


FIG. 15.3. RESOLUTION OF STATOR M.M.F.

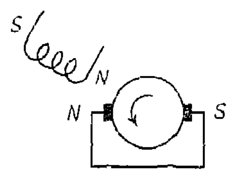


FIG. 15.4. DIRECTION OF ROTATION

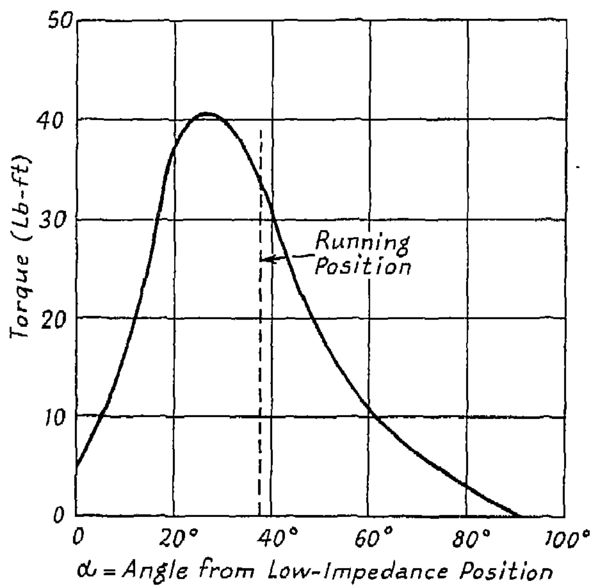


FIG. 15.5. EFFECT OF BRUSH POSITION ON STARTING TORQUE FOR $1\frac{1}{2}$ H.P. 1,500 R.P.M. MOTOR

as in Fig. 15.4. It can be seen that there are two N poles adjacent and these will repel each other—the direction of rotation will thus be the same as that in which the brushes are moved from the low-impedance position.

STARTING AND SPEED CONTROL. The magnitude of the torque available, and therefore the speed, depend on the position of the

brushes. The magnitude of the starting torque with different brush positions for a typical 1½ h.p. motor is shown in Fig. 15.5 while Fig. 15.6 shows typical speed-torque curves for various brush positions. The position giving the maximum torque is usually with α between 15° and 40°.

It can thus be seen that starting, speed control and reversal can be effected by movement of the brushes without any auxiliary

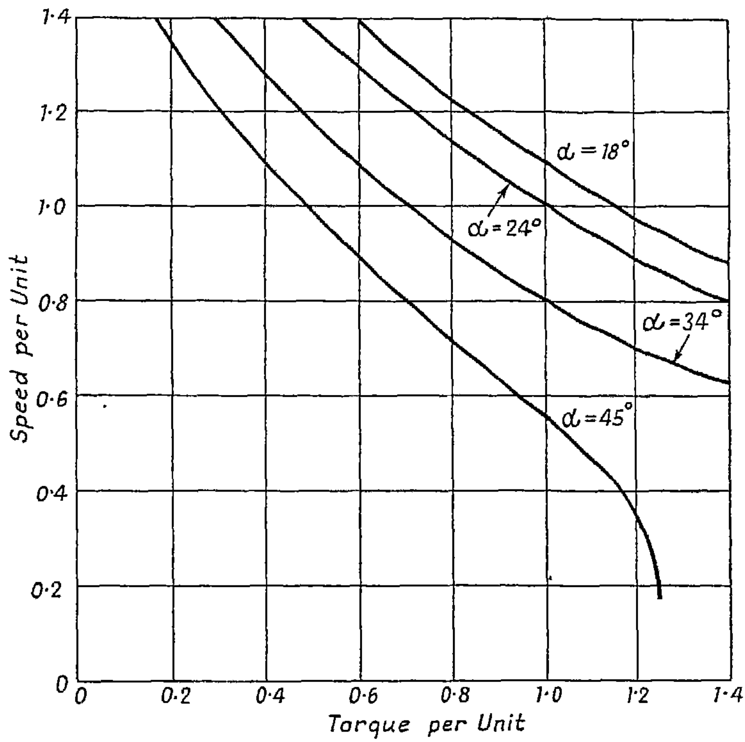


FIG. 15.6. SPEED-TORQUE CURVES OF PLAIN REPULSION MOTOR

starting or control equipment. It should be noted, however, that power factor and commutation are poor if the brushes are moved far from the optimum position, so that the majority of simple repulsion motors operate with a fixed brush position.

If starting is, however, carried out by brush shifting, the starting position will, on account of the lower current taken, be the high-impedance position, and the direction of rotation will thus be opposite to that of the movement of the brushes, a fact which gives rise to the name *repulsion* motor.

Complexor Diagram

By neglecting resistance and leakage reactance drops and the magnetizing current necessary to set up the quadrature flux Φ_q a simplified complexor diagram can be drawn upon which a more complete diagram can subsequently be based.

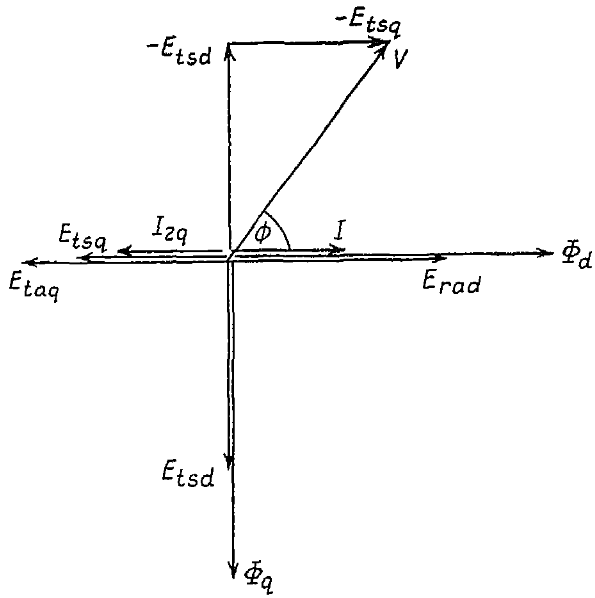


FIG. 15.7. SIMPLIFIED COMPLEXOR DIAGRAM FOR REPULSION MOTOR

SIMPLIFIED DIAGRAM. The complexor Φ_d of Fig. 15.7 represents the exciting flux. The rotor conductors are rotating in this and have set up in them a rotational e.m.f.

$$E_{rad} = (\sqrt{2})(T_a/a)f_r\Phi_d \text{ volts}$$

which is in phase with Φ_d.

The flux Φ_q sets up a transformer e.m.f. in the rotor

$$E_{taq} = (\sqrt{2})\pi f_1(T_a/a)\Phi_q \text{ volts}$$

Neglecting impedance drops, there are no other e.m.f.'s in the rotor so that $E_{taq} = E_{rad}$ and they are opposite in phase as shown. The transformer e.m.f. must, however, lag the flux Φ_q, which is producing it, by 90° so that this flux must be drawn vertically downwards and can be seen to lag Φ_d by 90°.

Assuming, for simplicity, a double-winding motor (Fig. 15.1(a)), the quadrature flux Φ_q induces a transformer e.m.f. in the quadrature-axis stator winding,

$$E_{tsq} = (\sqrt{2})\pi T_{sq}f_1\Phi_q \text{ volts}$$

lagging Φ_q by 90°. Also the exciting flux Φ_d induces a transformer e.m.f.

$$E_{tsd} = (\sqrt{2})\pi T_{sd} f_1 \Phi_d \text{ volts}$$

in the direct-axis stator winding, lagging Φ_d by 90°. Again neglecting impedance drops, the applied voltage must be equal and opposite to the sum of these two stator e.m.f.'s as shown.

Since, in the direct axis, there is no m.m.f. other than that produced by the main current I, the m.m.f. due to this sets up the exciting flux, and the current I must therefore be in phase with Φ_d.

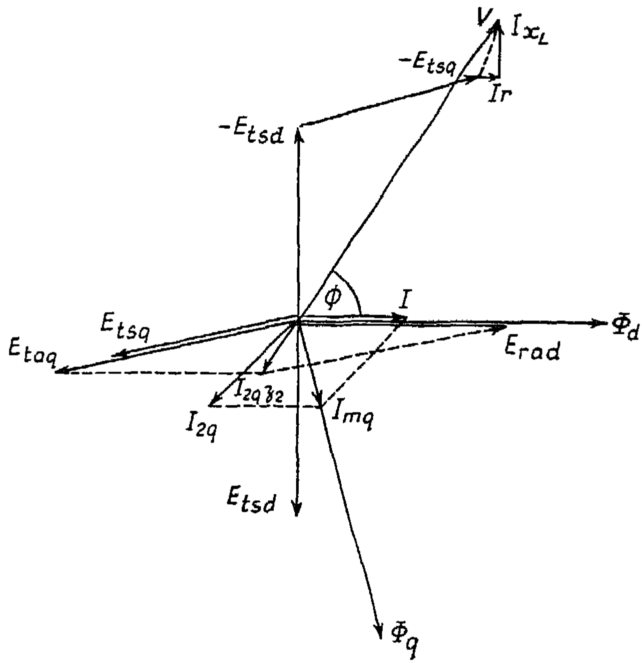


FIG. 15.8. MORE COMPLETE COMPLEXOR DIAGRAM

In the quadrature axis the flux is produced by the resultant of the m.m.f.'s due to I and the rotor current I_{2q}. Since magnetizing current to set up this flux is neglected, the currents I and I_{2q} (referred to the stator) must be equal and opposite as in a short-circuited transformer. I_{2q} can thus be drawn and it can be seen that it is opposed to E_{rad} which is correct for motor operation. $E_{rad}I_{2q}$ gives the rotor power which, neglecting rotor iron, friction and windage losses, is equal to the mechanical output.

MORE COMPLETE COMPLEXOR DIAGRAM. A more complete complexor diagram based on the above can now be drawn as in Fig. 15.8. The flux Φ_d, the rotational e.m.f. E_{rad} and the current I can be

drawn as before. If the impedance drop $I_{2q}z_2$ be allowed for, the e.m.f.'s E_{rad} and E_{taq} will no longer be equal and opposite but will have a resultant equal to $I_{2q}z_2$ as shown. The e.m.f. E_{taq} is thus advanced slightly in phase and the flux Φ_q, which must lead E_{taq} by 90°, is advanced by an equal angle and is no longer at 90° to Φ_d. If I_{mq} is the magnetizing current (referred to the stator) required to set up Φ_q it is in phase with Φ_q and will be the resultant of I and I_{2q} as shown; I_{2q} lags $I_{2q}z_2$ by the impedance angle of the rotor. The applied voltage can be found as before but with the inclusion of the stator resistance and leakage reactance drops.

For a single-winding motor the diagram is similar except that the e.m.f.'s induced in the stator winding by the two fluxes are—

$$E_{tsq} = (\sqrt{2})\pi T_s f_1 \Phi_q \cos \alpha \text{ and } E_{tsd} = (\sqrt{2})\pi T_s f_1 \Phi_d \sin \alpha$$

A refinement can be added by allowing for iron losses, and the loss in the coils short-circuited by the brushes which cause the fluxes to lag, by small angles, their respective magnetizing currents.

Relation between Direct (Exciting) and Quadrature Fluxes

Considering the rotor circuit and neglecting resistance and leakage reactance drops, it has been seen that

$$E_{rad} = E_{taq}$$

i.e. $$(\sqrt{2})\pi f_r T_a' \Phi_d = (\sqrt{2})\pi f_1 T_a' \Phi_q$$

so that $$f_r \Phi_d = f_1 \Phi_q \text{ and } \Phi_q/\Phi_d = f_r/f_1$$

At synchronous speed, therefore, the two components of the flux are equal in magnitude and displaced by 90° from each other in time and space. The resultant field in the air gap is therefore a true rotating field of constant magnitude.

Below synchronous speed the exciting flux is the greater while above it the quadrature flux is the greater—there will still be a rotating field under these conditions but it does not remain constant in magnitude as it rotates, i.e. it is an elliptic field (page 12). At standstill the quadrature field is, neglecting resistance and leakage reactance drops, zero so that the resultant is a pulsating field.

Commutation

As in the series motor the commutation process is complicated by the fact that the coils short-circuited by the brushes have induced in them e.m.f.'s additional to the reactance e.m.f. produced by the changing current. Reference to Fig. 15.1 shows that the short-circuited coils are linked with the exciting flux Φ_d and will therefore have induced in them a transformer e.m.f. $E_{tbd} = (\sqrt{2})\pi T_c f_1 \Phi_d$ lagging Φ_d by 90°.

Also during the commutating period the short-circuited coils are rotating in the quadrature flux Φ_q and have induced in them a

rotational e.m.f. $E_{rbq} = (\sqrt{2})\pi T_c f_r \Phi_q$ which is in phase opposition to Φ_q. Since Φ_d and Φ_q are at 90° in time the e.m.f.'s E_{tbd} and E_{rbq} are in phase opposition, and the resultant e.m.f. in the short-circuited coils is

$$\begin{aligned} E_b &= E_{tbd} - E_{rbq} \\ &= (\sqrt{2})\pi T_c(f_1\Phi_d - f_r\Phi_q) \\ &= (\sqrt{2})\pi T_c f_1 \Phi_d(1 - f_r\Phi_q/f_1\Phi_d) \\ &= E_{tbd}\{1 - (f_r/f_1)^2\} \end{aligned}$$

E_b will lag Φ_d by 90° if $E_{tbd} > E_{rbq}$, below synchronous speed, and lead it above synchronous speed, i.e. its direction will reverse as the speed passes through synchronism.

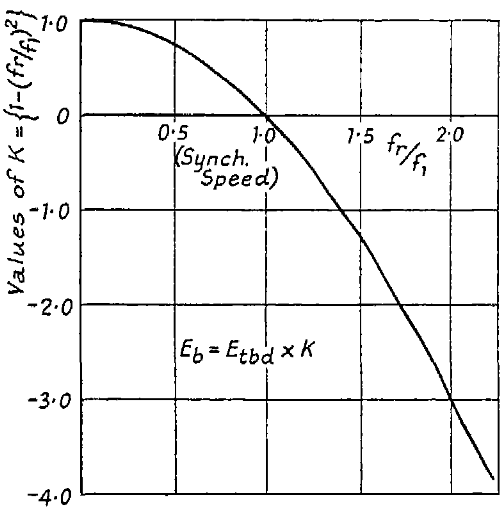


FIG. 15.9. E.M.F. INDUCED IN SHORT-CIRCUITED COILS

At synchronous speed, when $f_r = f_1$ the term in the curly brackets above becomes zero and there is no e.m.f. induced in the coil; conditions are therefore similar to those obtaining in a d.c. machine with only the reactance e.m.f. to be considered. At other speeds the resultant e.m.f. is given by the curve of Fig. 15.9 which shows the resultant in terms of E_{tbd}, the e.m.f. induced at standstill. It can be seen that above 1·4 times synchronous speed the resultant e.m.f. exceeds the value at standstill, and the machine is therefore not suitable for high-speed running—it is, in fact, desirable to limit the speed to the range between about 0·5 and 1·3 times synchronous speed.

The commutation problem may also be viewed from the point of view of the rotating field—it has been explained that if the motor is running at synchronous speed there is in the gap a rotating field

which is also moving at that speed and in the same direction. This field is constant in magnitude and is moving at the same speed as the conductors so that it can induce no e.m.f. in them.

EFFECT OF CURRENT IN SHORT-CIRCUITED COILS ON TORQUE. Whereas in the series motor the current induced in the short-circuited coils has little effect on the behaviour, the effect in the repulsion motor may be considerable.

The e.m.f. E_b lags the exciting flux Φ_d by 90° at speeds below synchronism as shown by the full complexor in Fig. 15.10; the circulating current I_c lags E_b by an angle depending on the resistance

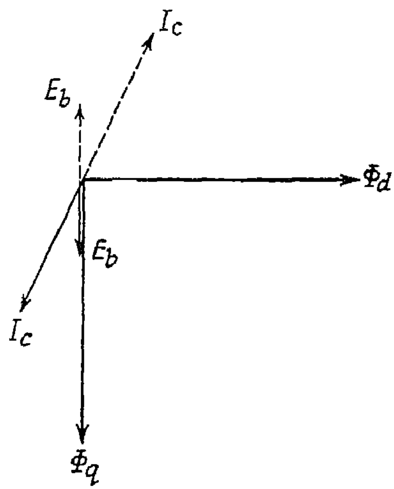


FIG. 15.10. CIRCULATING CURRENT IN COILS SHORT-CIRCUITED BY BRUSHES

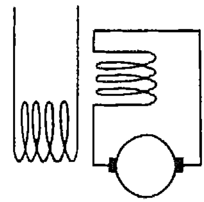
and reactance of the short-circuited coil. It can be seen that I_c has a considerable component in phase with Φ_q, and this will produce a torque which adds to the main torque produced by the interaction of I_{2q} and Φ_d. Above synchronous speed the directions of E_b and I_c are reversed as shown dotted, and the torque produced therefore subtracts from the main torque. The circulating currents thus improve the performance of the motor below synchronous speed and impair it above synchronous speed.

IMPROVEMENT OF COMMUTATION. Attempts have been made to improve the commutation at high speeds. Compoles may be fitted which reduce the quadrature flux Φ_q in the commutating zone—this reduces E_{tbq} which preponderates at high speeds so that the curve of Fig. 15.9 falls off less steeply above synchronism. Compoles of course, can be used only on motors with fixed brush position.

Another method, due to Atkinson, is to reduce the phase angle between Φ_q and Φ_d so that there is a component of Φ_q in phase with Φ_d; this induces a rotational e.m.f. in the short-circuited coil which

opposes the reactance e.m.f. and, since this is important at high speeds, some improvement in commutation is effected. The reduction of phase angle is brought about by connecting the exciting winding in series with the rotor as shown in Fig. 15.11. Referring to the simplified complexor diagram of Fig. 15.7 the stator e.m.f.

FIG. 15.11. ATKINSON'S MODIFICATION TO IMPROVE COMMUTATION



E_{tsd} now appears in the rotor circuit, and the rotor e.m.f. has to be advanced considerably in phase in order to overcome it, with a corresponding advance in the phase of Φ_q.

Power Factor

From the simplified complexor diagram of Fig. 15.7 it can be seen that the phase angle is given by

$$\tan\phi = E_{tsd}/E_{tsq}$$

$$= \frac{(\sqrt{2})\pi T_d f_1 \Phi_d}{(\sqrt{2})\pi T_q f_1 \Phi_q}$$

But $$\Phi_d/\Phi_q = f_1/f_r$$

so that $$\tan\phi = (T_d/T_q)(f_1/f_r)$$

In order to obtain satisfactory commutation f_r/f_1 must, as already explained, be approximately equal to unity so that $\tan\phi = T_d/T_q$, i.e. to obtain a small phase angle T_q must be made large relative to T_d, and in practice it is usually between two and four times T_d for a double-winding motor.

With a single-winding motor $T\cos\alpha$ must be two to four times $T\sin\alpha$ so that α may be between 15° and 40°.

Analytical Treatment

By inserting the appropriate boundary conditions in the general equations of page 217, the equations representing the performance of the motor can be found. The solution of these becomes rather cumbersome if the brush circulating currents are taken into account, so that although these are included in the initial statement of the equations they are omitted from the solutions.

DOUBLE-WINDING MOTOR. The boundary conditions are—

$$I_{1d} = I_{1q} = I \qquad V_{2q} = 0$$

$$I_{2d} = I_{3q} = 0 \qquad V_{2d} = V_{3d} = 0$$

$$V = V_{1d} + V_{1q}$$

Substituting these in the general equations gives—

Stator winding in direct axis—

$$V_{1d} = r_{1d}I + jx_{L1d}I + jx_{md}(I + I_{3d}) \quad . \quad (15.1a)$$

Stator winding in quadrature axis—

$$V_{1q} = r_{1q}I + jx_{L1q}I + jx_{mq}(I + I_{2q}) \quad . \quad (15.1b)$$

Rotor winding in quadrature axis—

$$0 = r_{2q}I_{2q} + jx_{L2q}I_{2q} + jx_{mq}(I + I_{2q}) - Sx_{md}(I + I_{3d}) \quad . \quad (15.1c)$$

Brush circuit in direct axis—

$$0 = r_{3d}I_{3d} + jx_{L3d}I_{3d} + jx_{md}(I + I_{3d}) + Sx_{mq}(I + I_{2q}) \quad . \quad (15.1d)$$

Neglecting the brush circuit and assuming $x_{mq} = x_{md} = x_m$ as is usual in a symmetrically built machine, the relation between voltage and current can be found as follows from (15.1*a*, *b* and *c*).

From (15.1*c*) the rotor current is

$$I_{2q} = I\,\frac{Sx_m - jx_m}{r_2 + j(x_{L2} + x_m)}$$

Hence

$I + I_{2q}$ (= I_{mq}, the magnetizing current for the quadrature flux)

$$= I\left\{1 + \frac{Sx_m - jx_m}{r_2 + j(x_{L2} + x_m)}\right\}$$

$$= I\,\frac{r_2 + jx_{L2} + Sx_m}{r_2 + j(x_{L2} + x_m)}$$

Thus the applied voltage is

$$V = V_{1d} + V_{1q}$$

$$= I\left\{(r_{1d} + r_{1q}) + j(x_{L1d} + x_{L1q}) + jx_m + jx_m\,\frac{r_2 + jx_{L2} + Sx_m}{r_2 + j(x_{L2} + x_m)}\right\}$$

$$= I\left\{r + j(x_L + x_m) + jx_m\,\frac{r_2 + jx_{L2}}{r_2 + j(x_{L2} + x_m)} + \frac{jSx_m^{\,2}}{r_2 + j(x_{L2} + x_m)}\right\}$$

where r = total stator resistance,

x_L = total stator leakage reactance.

If resistances and leakage reactances are neglected this becomes

$$V = I(Sx_m + jx_m)$$

SINGLE-WINDING MOTOR. If x_m is the mutual reactance between the rotor and stator windings when their axes are coincident, the mutual reactance between the rotor quadrature-axis circuit and the stator when the brushes are displaced by an angle α is $x_m \cos \alpha$ and between the rotor direct-axis circuit and the stator is $x_m \sin \alpha$.

Furthermore the proportion of the stator current producing an effect on the quadrature-axis circuit is $I \cos \alpha$ and on the direct-axis circuit is $I \sin \alpha$.

The boundary conditions can thus be written—

$I_{1d} = I \sin \alpha$ $\quad$ V_1 and V_3 are together replaced by the applied voltage V

$I_{1q} = I \cos \alpha$

$I_2 = I_6 = 0$ $\quad$ $V_2 = V_4 = 0$

Inserting these in the general equations gives—

Stator winding—

$$V = r_s I + jx_{Ls} I + jx_m \sin \alpha (I \sin \alpha + I_{3d}) + jx_m \cos \alpha (I \cos \alpha + I_{2q}) \quad . \quad (15.2a)$$

where r_s and x_{Ls} are the resistance and leakage reactance of the stator winding.

Rotor winding in quadrature axis—

$$0 = r_2 I_{2q} + jx_{L2} I_{2q} + jx_m (I \cos \alpha + I_{2q}) - Sx_m (I \sin \alpha + I_{3d}) \quad . \quad (15.2b)$$

Brush circuit in direct axis—

$$0 = r_{3d} I_{3d} + jx_{L3} I_{3d} + jx_m (I \sin \alpha + I_{3d}) + Sx_m (I \cos \alpha + I_{2q}) \quad . \quad (15.2c)$$

Again neglecting the brush circuit, the relation between voltage and current can be found from (15.2*a*, *b* and *c*).

From (15.2*b*), the rotor current is

$$I_{2q} = I \frac{Sx_m \sin \alpha - jx_m \cos \alpha}{r_2 + j(x_{L2} + x_m)}$$

Therefore

$$V = I \left\{ r_s + jx_{Ls} + jx_m \sin^2 \alpha + jx_m \cos^2 \alpha + jx_m \cos \alpha \frac{Sx_m \sin \alpha - jx_m \cos \alpha}{r_2 + j(x_{L2} + x_m)} \right\}$$

$$= I \left\{ r_s + j(x_{Ls} + x_m) + \frac{x_m^2 \cos^2 \alpha}{r_2 + j(x_{L2} + x_m)} + \frac{jSx_m^2 \sin \alpha \cos \alpha}{r_2 + j(x_{L2} + x_m)} \right\} \quad (15.3)$$

If resistances and leakage reactances are neglected this becomes—

$$V = I\{Sx_m \sin \alpha \cos \alpha + j(x_m - x_m \cos^2 \alpha)\} \quad . \quad (15.4)$$

Current Locus

Equation (15.3) shows that the impedance of the motor can be written in the form $z = z' + Sz''$, i.e. a constant impedance plus an impedance which is proportional to speed. If the real and quadrature components of this are plotted on a coordinate diagram as shown in Fig. 15.12 the resulting impedance locus is a straight line. Inverting this gives the admittance locus and this is a circle. The

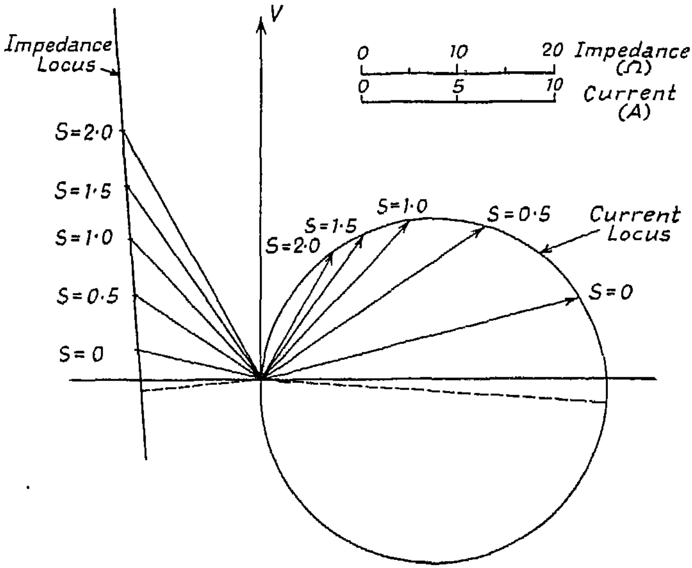


FIG. 15.12. CURRENT LOCUS OF REPULSION MOTOR

current locus, which is, at a constant applied voltage, proportional to admittance, is therefore also a circle as shown. The above circle diagram is illustrated in the following example.

Example 15.1. $1\frac{1}{2}$-h.p., 50-c/s, 4-pole, 230-V repulsion motor having the following particulars—

$$x_m = 50\ \Omega \qquad r_s = 1{\cdot}1\ \Omega$$
$$x_{Ls} = 5\ \Omega \qquad r_2 = 2{\cdot}7\ \Omega$$
$$x_{L2} = 5\ \Omega \qquad \alpha = 15°$$

$$z = 1{\cdot}1 + \mathrm{j}55 + \frac{2{,}500 \times 0{\cdot}966^2}{2{\cdot}7 + \mathrm{j}55} + \frac{\mathrm{j}S2{,}500 \times 0{\cdot}259 \times 0{\cdot}966}{2{\cdot}7 + \mathrm{j}55}$$

$$= 3{\cdot}18 + \mathrm{j}12{\cdot}8 + S(11{\cdot}3 + \mathrm{j}0{\cdot}556)$$

Plotting this for various values of S gives the impedance locus of

Fig. 15.12. Inverting points on this gives the admittance and current locus—for instance for $S = 1$

$$z = 14{\cdot}48 + j13{\cdot}36 = 19{\cdot}7\ \Omega$$

Therefore $$y = 1/19{\cdot}7 = 0{\cdot}0508 \text{ mho}$$

and the current is $230 \times 0{\cdot}0508 = 11{\cdot}7$ A.

Design Features

If the motor is run near synchronous speed the commutating conditions are good, and a large flux per pole can be used without the necessity of a low frequency such as is required for the series motor. Large motors can thus be built, if desired, for operation from a 50- or 60-c/s supply.

The rotor winding is usually short-pitched by one or two slots in order to improve commutation, and the stator generally has a fully wound single-layer winding in order to give an m.m.f. wave of comparable shape.

Since the rotor is not connected to the supply both stator and rotor can be wound for any desired voltage.

Decreasing the leakage reactance has the effect of reducing the slope of the speed-torque curve, usually a desirable feature, so that a fairly large number of stator slots is generally employed.

A decrease of total rotor resistance or an increase of the resistance of the coils short-circuited by the brushes also reduces the slope of the speed-torque curve—to secure both these effects simultaneously a large number of rotor coils and commutator segments can be used so that the brush resistance is a small percentage of the total resistance but a large percentage of the coil resistance.

Semi-shunt Repulsion Motor

The high no-load speed of the plain repulsion motor may be a disadvantage, and a motor with a much flatter characteristic (e.g. 40 per cent drop from no load to full load) can be obtained by putting an additional set of brushes on the direct axis and connecting them to a small stator winding on the quadrature axis.

Applications

Due to the limited speed range the repulsion motor is less suited for single-phase traction than the series motor and its application is therefore limited to industrial drives where a three-phase supply is not conveniently available, i.e. to motors of not more than about 5 h.p. For lifts, hoists and certain other drives it has the advantage over the single-phase induction motor of a large starting torque, and, with the brush shifting motor, starting and some measure of speed control can be effected by moving a single lever attached to the brush rocker. Reversing motors are generally fitted with a double stator winding with a reversing switch in one of them.

Another common application is in the repulsion-start single-phase induction motor in which the repulsion principle is incorporated in the single-phase induction motor in order to give it a high starting torque as described in Chapter 18 (page 311).

EXERCISES 15

1. A 240-V, 50-c/s, 4-pole, double-wound repulsion motor with identical stator windings has the following parameters—

$x_m = 60\ \Omega$ $\qquad r_2 = 2{\cdot}7\ \Omega$ (referred to primary)
$r_1 = 0{\cdot}55\ \Omega$ $\qquad x_{L2} = 3{\cdot}0\ \Omega$ (referred to primary)
$x_{L1} = 2{\cdot}28\ \Omega$

Calculate the input current, power factor and torque when running at 2,000 r.p.m.

2. A $1\frac{1}{2}$-h.p., 50-c/s, 4-pole, 230-V, single-winding repulsion motor has the following particulars—

Mutual reactance = 50 Ω
Stator leakage reactance = 5 Ω
Stator resistance = 1·1 Ω
Rotor leakage reactance = 5 Ω
Rotor resistance = 2·7 Ω
Brush displacement from low-impedance position = 15°

Determine the current and power factor when running at 1,400 r.p.m.

3. Develop an expression for the current of a rotor-excited repulsion motor (Fig. 15.11) in terms of the supply voltage, S, and the motor parameters.

4. In a 4-pole, 50-c/s repulsion motor the maximum e.m.f. in the short-circuited coils at starting is to be limited to (*a*) 4 V and (*b*) 2 V, with full-load flux. Draw curves showing the e.m.f.'s in the coil at other speeds and determine the range of speed over which the e.m.f. is not more than 1·5 V, assuming the flux to remain constant at the full-load value.

CHAPTER 16

FURTHER TYPES OF SINGLE-PHASE COMMUTATOR MOTOR

THE series and the repulsion motors are the only two types of single-phase commutator motor that are now extensively manufactured; other types, however, notably the *compensated-repulsion* motor, the *Deri brush-shifting repulsion* motor, the *series-repulsion* motor and the *single-phase shunt* motor are, or have been in the past, used to a limited extent and therefore merit attention.

Compensated-repulsion Motor

This is a modification of the double-winding repulsion motor (Fig. 15.1(a), page 248) in which the exciting flux Φ_d is produced by passing the main current through additional brushes on the commutator instead of through a stator winding, the simplest arrangement being shown in Fig. 16.1(a). Across these additional exciting brushes is produced a rotational e.m.f. which does not exist in the plain repulsion motor and which, to a greater or less extent depending on the speed, neutralizes the transformer e.m.f. of the exciting winding, thereby giving a better power factor.

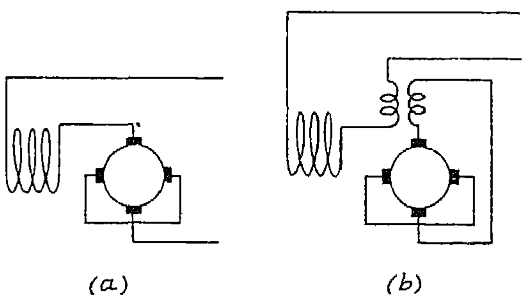


FIG. 16.1. COMPENSATED-REPULSION MOTOR

(a) Simple motor.
(b) Motor with auxiliary transformer.

The supply circuit is now connected directly to the commutator, and a feature of the plain repulsion motor, namely that the rotor can be designed independently of the supply voltage, has been lost. It can be regained, however, by using a series transformer as shown in Fig. 16.1(b) without altering the characteristics of the motor.

This type of motor was developed independently by Latour in France and by Winter and Eichberg in Germany and is, from the initials of the inventors, commonly referred to as the W.E.L. motor.

COMPLEXOR DIAGRAM. The supply current I flowing through the rotor exciting brushes sets up the exciting flux Φ_d, and this induces between the exciting brushes a transformer e.m.f. E_{tad} given by

$$E_{tad} = (\sqrt{2})\pi T_a' f_1 \Phi_d \text{ volts}$$

lagging Φ_d by 90°; this corresponds exactly to the e.m.f. E_{tsd} which appears across the direct-axis stator winding of the plain repulsion motor. Again as in the plain repulsion motor there will be, between the quadrature-axis brushes, a rotational e.m.f. E_{rad} due to the rotation of the rotor in the flux Φ_d and given by

$$E_{rad} = (\sqrt{2})(T_a/a)f_r\Phi_d = (\sqrt{2})\pi T_a' f_r \Phi_d \text{ volts}$$

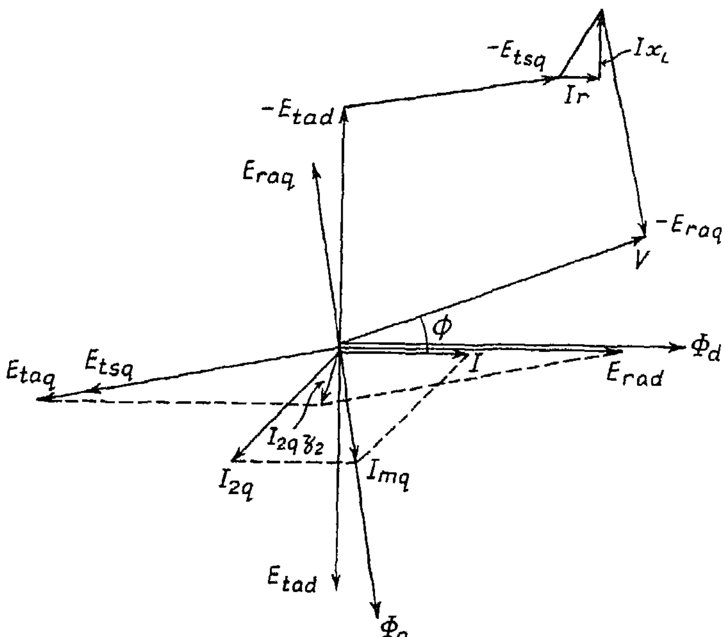


FIG. 16.2. COMPLEXOR DIAGRAM OF COMPENSATED-REPULSION MOTOR

and also a transformer e.m.f. E_{taq} due to the quadrature flux and given by

$$E_{taq} = (\sqrt{2})\pi T_a' f_1 \Phi_q \text{ volts}$$

Since these brushes are short-circuited the resultant of these two is equal to the impedance drop $I_{2q}z_2$, and also the flux Φ_q leads E_{taq} by 90° as shown in Fig. 16.2. The quadrature flux also induces a transformer e.m.f.

$$E_{tsq} = (\sqrt{2})\pi T_s f_1 \Phi_q \text{ volts}$$

in the stator winding.

So far the diagram is similar to that for the plain repulsion motor. There is, however, the e.m.f. induced across the exciting brushes due to rotation in the flux Φ_q which is given by

$$E_{raq} = (\sqrt{2})\pi T_a' f_r \Phi_q$$

and is in phase opposition to Φ_q as shown. The applied voltage V thus has to overcome $-E_{tsq}$, $-E_{tad}$ and $-E_{raq}$ and also the resistance and leakage reactance drops of the stator winding and the rotor winding in the direct axis. It can be seen that the effect of the additional rotational e.m.f. E_{raq} is to improve the power factor by opposing E_{tad}.

As in the plain repulsion motor, neglecting rotor resistance and leakage reactance drops, $E_{taq} = E_{rad}$ so that

$$\Phi_d/\Phi_q = f_1/f_r$$

Consider now the two e.m.f.'s appearing across the direct-axis brushes,

$$E_{tad}/E_{raq} = f_1\Phi_d/f_r\Phi_q = (f_1/f_r)^2$$

i.e. E_{raq} will equal E_{tad} at synchronous speed, and neglecting resistance and leakage reactance drops, the power factor will then be unity. Below synchronous speed $E_{raq} < E_{tad}$ and the power factor will be lagging, while above synchronous speed $E_{raq} > E_{tad}$ and the power factor will be leading. It can be seen that the effect of the impedance drops is to make the power factor unity at a speed somewhat above synchronism.

It may be noted that there is no need to keep E_{tad} to a minimum in order to obtain a good power factor, and the ratio field AT/rotor AT can be made larger than in a repulsion motor or compensated series motor; there will thus be more iron and less copper in the machine.

COMMUTATION. So far as the quadrature-axis brushes are concerned the commutating conditions are exactly as in the plain repulsion motor, i.e. good near synchronous speed. In the case of the direct-axis brushes there is in the short-circuited coil a transformer e.m.f.

$$E_{tbq} = (\sqrt{2})\pi T_c f_1 \Phi_q \text{ volts}$$

and a rotational e.m.f.

$$E_{rbd} = (\sqrt{2})\pi T_c f_r \Phi_d \text{ volts}$$

these two being opposed to each other. Since $f_1\Phi_q = f_r\Phi_d$ these two e.m.f.'s neutralize each other at all speeds and commutation at the direct-axis brushes is therefore always good.

ANALYTICAL TREATMENT. As with the other types of machine the behaviour can be calculated by using the general equations of page 217. Omitting the brush circuits the boundary conditions become—

$$I_{2d} = I_{1q} = I$$
$$V_{2q} = 0$$
$$V = V_{2d} + V_{1q}$$

so that, assuming $x_{md} = x_{mq} = x_m$ the equations can be written—
Stator quadrature-axis circuit—

$$V_{1q} = r_1 I + jx_{L1}I + jx_m(I + I_{2q}) \quad . \quad (16.1)$$

Rotor direct-axis circuit—

$$V_{2d} = r_2 I + jx_{L2}I + jx_m I + Sx_m(I + I_{2q}) \quad . \quad (16.2)$$

Rotor quadrature-axis circuit—

$$0 = r_2 I_{2q} + jx_{L2}I_{2q} + jx_m(I + I_{2q}) - Sx_m I \quad . \quad (16.3)$$

From (16.3)

$$I_{2q} = I\,\frac{Sx_m - jx_m}{r_2 + j(x_{L2} + x_m)}$$

$$I + I_{2q} = I\left\{1 + \frac{Sx_m - jx_m}{r_2 + j(x_{L2} + x_m)}\right\}$$

$$= I\,\frac{r_2 + jx_{L2} + Sx_m}{r_2 + j(x_{L2} + x_m)}$$

Hence

$$V = rI + jx_L I + jx_m I + (Sx_m + jx_m)(I + I_{2q})$$

where $r = r_1 + r_2$ and $x_L = x_{L1} + x_{L2}$

$$= I\left\{r + j(x_L + x_m) + \frac{(Sx_m + jx_m)(r_2 + jx_{L2} + Sx_m)}{r_2 + j(x_{L2} + x_m)}\right\}$$

Neglecting resistance and leakage reactance,

$$V = I\left\{jx_m + \frac{(Sx_m + jx_m)Sx_m}{jx_m}\right\}$$

$$= I(jx_m + Sx_m - jS^2x_m)$$

Current Locus Diagram. The above expression can be used to determine the current locus diagram as with the plain repulsion motor. Consider a 200-V, 110-h.p., 25-c/s motor having a synchronous speed of 750 r.p.m. and a reactance x_m of 0·4 Ω. The impedance for various values of speed can be found as in Table 16.1.

Table 16.1

Speed (r.p.m.)	0	225	375	600	750	900	1,125	1,500	2,250
S . . .	0	0·3	0·5	0·8	1·0	1·2	1·5	2·0	3·0
Sx_m .	0	0·12	0·2	0·32	0·4	0·48	0·6	0·8	1·2
S^2x_m .	0	0·036	0·1	0·256	0·4	0·576	0·9	1·6	3·6
$j(x_m - S^2x_m)$.	0·4	0·364	0·3	0·144	0	− 0·176	− 0·5	− 1·2	− 3·2

Plotting the real (Sx_m) and quadrature ($j(x_m - S^2x_m)$) components of the impedance on a coordinate diagram gives the impedance locus as shown in Fig. 16.3. Inverting this gives the admittance locus and this, to another scale, represents the current locus. For instance at 600 r.p.m. the impedance is 0·353 Ω and the admittance is 1/0·353 = 2·84 mhos; the current is thus 200 × 2·84 = 568 A.

It can be seen that, since the impedance locus is not a straight line, the current locus is not a circle. It can also be seen that above synchronous speed (since resistance and leakage reactances have

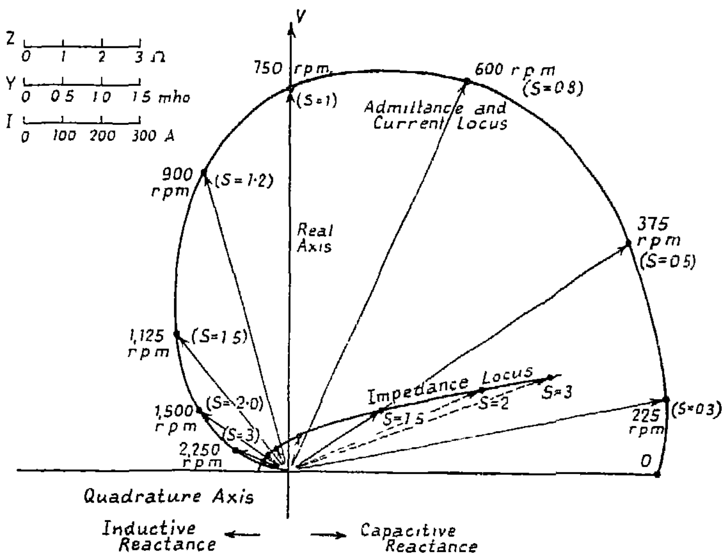


FIG. 16.3. CURRENT LOCUS FOR COMPENSATED-REPULSION MOTOR

been neglected) the power factor leads due to jS^2x_m (the rotational e.m.f. in the direct axis) exceeding jx_m (the transformer e.m.f. in the direct axis).

MISCELLANEOUS FEATURES AND APPLICATIONS. The good power factor of the compensated-repulsion motor might seem to make it preferable to the plain repulsion or compensated-series types; also since the exciting flux can be controlled by means of the excitation-circuit transformer of Fig. 16.1(*b*) the flux per pole at starting can be reduced so that larger values of flux per pole can be used in the design. These advantages are, however, obtained only at the expense of greater commutator friction and resistance losses due to the extra set of brushes, a greater rotor copper loss due to the uneven current distribution in the rotor and a non-sinusoidal exciting flux due to the triangular rotor m.m.f. wave which is producing it. The motor has therefore disappeared from ordinary

commercial use although in the past it was used to some extent as a traction motor; the early electrification of the London, Brighton and South Coast Railway originally employed this type of motor, although even before the system was changed to direct current it had been superseded by the compensated-series type.

Series-repulsion (Doubly-fed) Motor

As its name implies this motor is a combination of the series and repulsion motors in which power is fed to the rotor both by induction

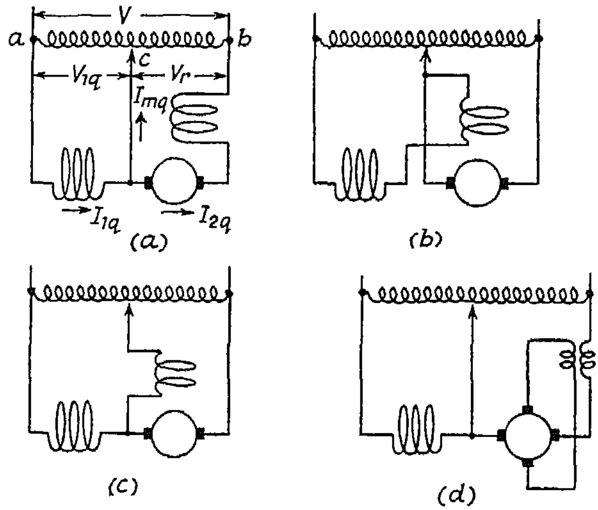


FIG. 16.4. CONNEXIONS FOR SERIES-REPULSION MOTORS

and conduction; the purpose of the arrangement is to secure better commutation than with other types, particularly at super-synchronous speeds. Four practicable methods of making the connexions are shown in Fig. 16.4.

In all these it can be seen that the rotor is not short-circuited along the quadrature axis so that the resultant e.m.f. at the brushes need not be zero; there is thus no need for the quadrature- and direct-axis fluxes to bear any fixed relation to each other and to the speed as is the case with the plain repulsion motor. The possibility of varying the quadrature flux, which is proportional to the voltage V_{1q} across the quadrature stator winding, thus enables it to be made of such a value as to secure good commutation at all speeds, the appropriate variation being carried out by varying the tapping position c.

COMPLEXOR DIAGRAM. Referring to the diagram of Fig. 16.4(a) it can be seen that if the connexion to the tapping point c is removed or adjusted to a position at which it carries no current, the machine

becomes a compensated-series motor, and the quadrature-axis flux will, neglecting leakage reactance, be zero, the windings being such that their m.m.f.'s are equal and opposite. If the connexion to b is removed the current in the quadrature-axis stator winding will be simply the magnetizing current I_{mq} producing the quadrature flux Φ_q; when running, the current I_{1q} will thus be the sum of I_{mq} and I_{2q}.

A complexor diagram, neglecting resistance and leakage reactance drops, can now be drawn as in Fig. 16.5. The rotor current I_{2q}

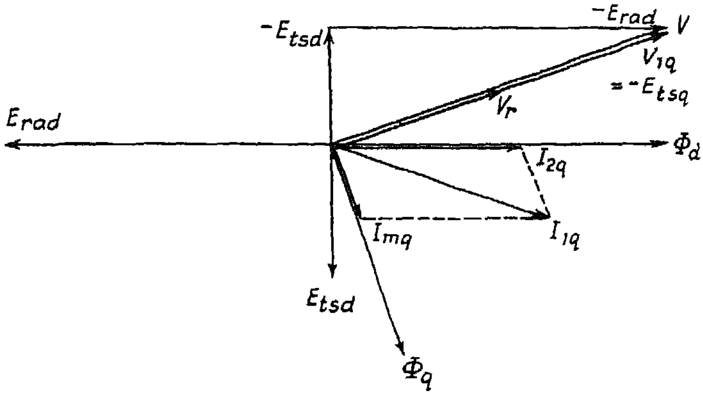


FIG. 16.5. COMPLEXOR DIAGRAM OF SERIES-REPULSION MOTOR (Connexion of Fig. 16.4.(a))

passes through the exciting winding and sets up a flux Φ_d in phase with it. This flux sets up a transformer e.m.f. E_{tsd} in the exciting winding lagging Φ_d by 90° and a rotational e.m.f. E_{rad} in phase opposition to Φ_d and, at super-synchronous speeds, considerably greater than E_{tsd}. The quadrature flux induces transformer e.m.f.'s E_{tsq} and E_{taq} in the stator and rotor windings, but these are in opposition and therefore, assuming a 1:1 turns ratio, neutralize each other so that the terminal voltage V is equal to the sum of $-E_{tsd}$ and $-E_{rad}$ as shown. The terminal voltage is also equal to the sum of its two components V_{1q} and V_r; V_{1q} however is equal to $-E_{tsq}$, the transformer e.m.f. induced in the quadrature-axis stator winding by the quadrature flux Φ_q, so that Φ_q can be drawn lagging 90° behind V. I_{mq} is in phase with Φ_q so that I_{1q} may be drawn as $I_{mq} + I_{2q}$.

COMMUTATION. To secure complete neutralization of the transformer e.m.f. E_{tbd} in the coils short-circuited by the brushes, the rotational e.m.f. resulting from rotation in the quadrature flux must be equal to it at all speeds, i.e.

$$E_{rbq} = E_{tbd}$$

i.e. $$(\sqrt{2})\pi T_c f_r \Phi_q = (\sqrt{2})\pi T_c f_1 \Phi_d$$

$\therefore$ $$\Phi_q/\Phi_d = f_1/f_r$$

In the plain repulsion motor $\Phi_q/\Phi_d = f_r/f_1$ so that, as already explained (page 254), neutralization is complete only at synchronous speed, being too great at super-synchronous speeds and too low at sub-synchronous speeds.

From the complexor diagram of the series-repulsion motor it can be seen that V is approximately equal to E_{rad} so that

$$V_{1q} = (\sqrt{2})\pi T_q f_1 \Phi_q$$

$$V \simeq (\sqrt{2})\pi T_a' f_r \Phi_d$$

$$\therefore \qquad V_{1q}/V = (\Phi_q/\Phi_d)(f_1/f_r)$$

assuming a 1:1 turns ratio.

For the required relation to hold V_{1q}/V must thus be made equal to $(f_1/f_r)^2$, i.e. V_{1q} must be varied inversely as the square of the speed. At high speeds therefore V_{1q} must be small and point c will be towards the left, at synchronous speed $V_{1q} = V$ and c and b coincide, while at sub-synchronous speeds V_{1q} should be greater than V and the transformer winding should be extended beyond b. Sub-synchronous running is, however, usually required only at starting, and also an increase of Φ_q by making $V_{1q} > V$ would unduly saturate the magnetic circuit, so that for starting and for speeds below synchronism the machine is usually connected as a plain repulsion motor.

Other Connexions. The connexion of Fig. 16.4(b) gives a lower torque due to the rotor current I_{2q} and the exciting flux not now being in phase, and it also has less satisfactory commutation. In Fig. 16.4(c) the exciting winding carries the difference between the stator and rotor currents and also gives a performance inferior to that of Fig. 16.4(a). The final arrangement, Fig. 16.4(d), has the motor excited from the rotor instead of the stator as in the compensated-repulsion motor (page 263). It was explained in connexion with that motor that, due to the particular relations between the quadrature and exciting fluxes, commutation at the exciting brushes was excellent. This flux relation does not exist in the series-repulsion motor so that commutation is liable to be poor at the exciting brushes—this and the extra commutation losses due to the additional brushes make this arrangement much inferior to the other types.

Applications. This motor was used in the early days of a.c. traction on account of its better commutation at high speeds, but the extra complication and the need for switching to repulsion-motor operation for starting have caused it to be superseded by the compensated-series motor.

Deri Repulsion Motor with Fixed and Movable Brushes

It has been seen that the speed of the plain repulsion motor can be varied by brush shifting, but reference to the curves of Figs. 15.5 and 15.6 shows that near the normal running position the motor

is rather sensitive to brush movement, i.e. a small movement produces a considerable change of speed. The Deri motor employs a double set of brushes in order to overcome this and also gives some improvement in power factor and commutation.

The arrangement is shown in Fig. 16.6—brushes F_1 and F_2 are fixed while M_1 and M_2 are movable, being attached to a common brush rocker and moved together.

If the brushes are moved so that M_2 is adjacent to F_1 and M_1 adjacent to F_2 as shown in Fig. 16.6(a), the whole winding is short-circuited with the brush axis coincident with the axis of the stator winding; conditions are thus similar to the plain repulsion motor in the low-impedance or short-circuit position.

If the brushes are moved so that M_1 is adjacent to F_1 and M_2 adjacent to F_2 as in Fig. 16.6(b) no part of the rotor winding is

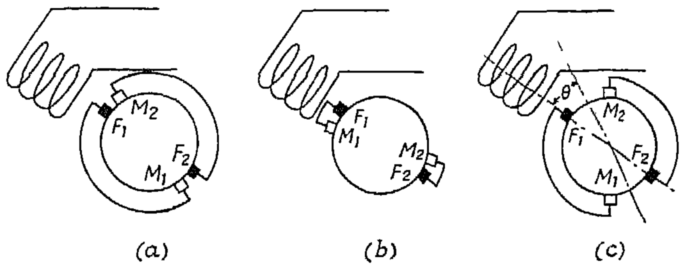


FIG. 16.6. DERI REPULSION MOTOR

(a) Low-impedance position.
(b) High-impedance position (starting).
(c) Running position.

short-circuited (other than the coils short-circuited by the brushes themselves), and conditions are similar to those with the plain repulsion motor in the high-impedance position; there is no rotor current and no torque.

Intermediate between these two is the running position shown in Fig. 16.6(c). The direction of the rotor m.m.f. produced by that part of the winding between M_1 and F_1 and between M_2 and F_2 is as shown by the dotted line inclined at θ to the stator axis. In this position there is a torque just as in the plain repulsion motor, but in order to obtain a shift of θ of the rotor axis a brush shift of 2θ is required, i.e. twice that of the ordinary motor. A typical set of speed-torque curves is shown in Fig. 16.7.

POWER FACTOR. That part of the rotor winding between brushes M_2 and F_1 and between M_1 and F_2 gives rise to an m.m.f. acting at 90° to that of the main part of the rotor winding, i.e. it is acting along the exciting axis. The conditions are thus to some extent similar to those obtaining in the compensated-repulsion motor (page 263), and a slight improvement of power factor over that of a plain repulsion motor is obtained.

COMMUTATION. Both sets of brushes, when in the running position, are linked with some flux, and therefore circulating currents are set up in the short-circuited coils; at starting, however, in the high-impedance position the short-circuited coils lie along the stator axis and do not therefore link with any part of the flux so that commutation at starting is much better than in the plain repulsion motor.

APPLICATIONS. This type of motor has been used for traction since good commutation can be obtained over a slightly greater

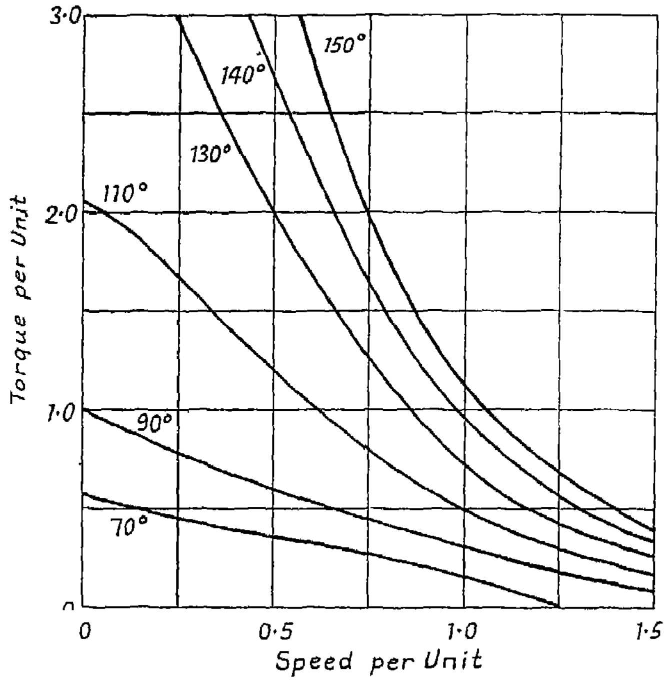


FIG. 16.7. CHARACTERISTICS OF DERI MOTOR

speed range than with the plain repulsion motor but, like other types, it has been superseded for this purpose by the compensated series motor.

It has, however, been used to a considerable extent in smaller sizes in Central Europe as a crane motor and for other variable-speed industrial drives. Speed control is obtained by brush shifting, and a braking torque is obtained by moving the brushes backwards beyond the starting (high-impedance) position; under these conditions a resistor should be inserted in the stator circuit to prevent self-excitation. When used for cranes it is also desirable to limit the maximum speed and this can be done by short-circuiting, either

directly or through a resistor, the two pairs of brushes—this causes the motor to behave similarly to an induction motor and limits the speed to about synchronism, the necessary short-circuiting being done by a centrifugal switch.

Single-phase Shunt Motor

Although the single-phase series motor is similar in principle and construction to the d.c. series motor, the single-phase shunt motor is totally different in principle to its d.c. counterpart and is so called only because it has a "shunt" characteristic. A d c. shunt motor,

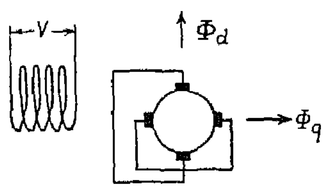


FIG. 16.8. SHUNT COMMUTATOR MOTOR (COMMUTATOR INDUCTION MOTOR)

if supplied from an a.c. source, would take a shunt-field current lagging by nearly 90° behind the rotor current and so would produce only a negligible torque; attempts have been made to remedy this by a capacitor in series with the field to produce a resonant circuit but these have not been commercially successful.

Ll. B. Atkinson, in 1898, showed that it was possible to build a reasonably satisfactory commutator motor having a shunt characteristic, the connexions being as shown in Fig. 16.8; it has a simple stator winding and two sets of short-circuited brushes at 90° to each other, one set being along the stator-winding axis.

COMPLEXOR DIAGRAM. When the motor is running an exciting flux Φ_d is set up by currents in the brush axis which is at 90° to the stator winding axis, i.e. in the direct axis. A quadrature flux Φ_q is set up by the resultant of the stator m.m.f. and the rotor m.m.f. in the quadrature axis; rotation in this flux sets up an e.m.f. E_{raq} at the direct-axis brushes and this produces the magnetizing current giving the exciting flux Φ_d.

Referring to Fig. 16.9, the exciting flux Φ_d sets up a transformer e.m.f. E_{tad} at the direct-axis brushes and lagging Φ_d by 90°. There will be a magnetizing current I_{2d} setting up Φ_d and in phase with it and this current gives an impedance drop $I_{2d}z_2$ which leads I_{2d} by a small angle. The only other e.m.f. between the direct-axis brushes is the rotational e.m.f. mentioned above, due to the flux Φ_q so that, since $I_{2d}z_2$ is the resultant of E_{tad} and E_{raq}, the latter must be drawn as shown together with the flux Φ_q which is in phase with it. It is thus seen that the two fluxes are nearly at 90° to each other in time as well as being at 90° in space.

Consider now the e.m.f.'s in the quadrature axis. There is a transformer e.m.f. E_{taq} due to the flux Φ_q and lagging Φ_q by 90° and also a rotational e.m.f. E_{rad} due to rotation in the flux Φ_d and in phase opposition to Φ_d. The resultant of these two is the impedance drop $I_{2q}z_2$ due to the current I_{2q}. The quadrature-axis current I_{2q} thus lags this drop by a small angle as shown. A magnetizing com-

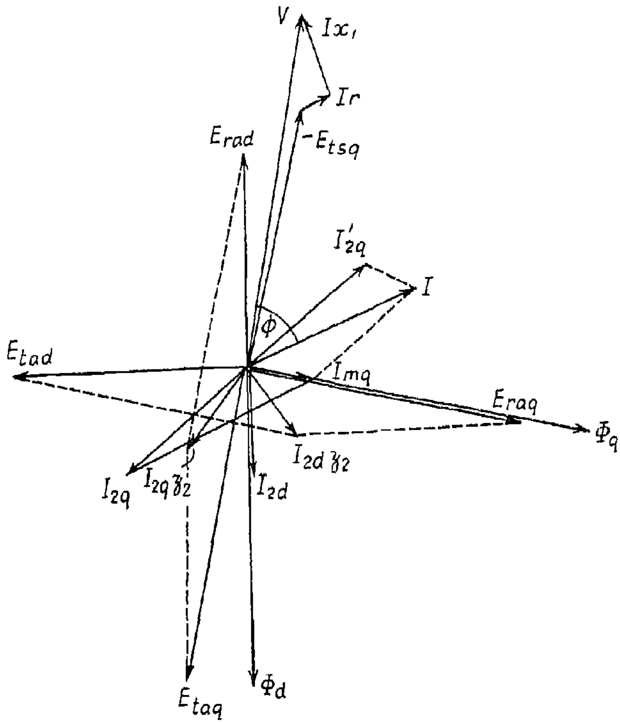


FIG. 16.9. COMPLEXOR DIAGRAM OF SHUNT MOTOR

ponent of current I_{mq} sets up the flux Φ_q, and is in phase with it so that the primary current is the sum of I_{mq} and the value of I_{2q} reflected into the stator, i.e. $I = I_{mq} + I_{2q}'$.

A back e.m.f. E_{tsq} is induced in the stator winding by the flux Φ_q, and the primary voltage has to overcome this as well as the resistance and leakage reactance drops. It can be seen that the power factor is inevitably rather poor.

SPEED. Neglecting rotor impedance drops it can be seen that in the direct axis $E_{raq} = E_{tad}$, i.e.

$$(\sqrt{2})\pi T_a' f_r \Phi_q = (\sqrt{2})\pi T_a' f_1 \Phi_d$$

Therefore $$f_r\Phi_q = f_1\Phi_d$$

Also, in the quadrature axis $E_{rad} = E_{tnq}$, i.e.

$$(\sqrt{2})\pi T_a' f_r \Phi_d = (\sqrt{2})\pi T_a' f_1 \Phi_q$$

Therefore $$f_r\Phi_d = f_1\Phi_q$$

For these two conditions to hold simultaneously f_r must equal f_1, and Φ_d must equal Φ_q. The motor must therefore run at synchronous speed. The effect of the impedance drops, neglected above, is, however, to cause a slight speed drop as the load is increased.

Since Φ_d is equal to Φ_q and the two fluxes are almost in space and time quadrature there will be a rotating field of almost constant magnitude moving in the same direction as the rotor.

TORQUE. When running, the torque is produced by the interaction of the quadrature-axis current and the exciting flux, this being the main torque and proportional to $I_{2q}\Phi_d \cos \widehat{I_{2q}\Phi_d}$.

There is also a torque due to the interaction of the direct-axis current and the quadrature flux which opposes the above and is proportional to $I_{2d}\Phi_q \cos \widehat{I_{2d}\Phi_q}$. The angle $\widehat{I_{2d}\Phi_q}$ is, however, nearly 90° so that this torque is relatively small.

Since the direct-axis flux is produced by the current due to the rotational e.m.f. E_{raq} there can be no exciting flux, and therefore no torque, when the motor is stationary, i.e. there is no starting torque.

COMMUTATION. On account of the equality of the two fluxes Φ_d and Φ_q the rotational and transformer e.m.f.'s at both sets of brushes neutralize each other, and commutation is therefore comparable to that in a d.c. machine.

SUMMARY OF PROPERTIES OF SIMPLE SHUNT MOTOR. It can be seen that the motor in the simple form of Fig. 16.8 runs at approximately synchronous speed, has no starting torque, has a poor power factor but commutates satisfactorily. Its operating characteristics are thus similar to those of the single-phase induction motor and it is, in fact, sometimes referred to as a *commutator induction motor*.

On account of the cost and complication of the commutator this simple form of the motor cannot compete commercially with the single-phase induction motor but it is possible, however, to improve its characteristics by various devices which improve the power factor, give it some starting torque and enable its speed to be controlled. It is the last feature that, in the past, has caused a considerable amount of attention to be devoted to the machine since there is no other single-phase motor which gives a variable speed with a shunt characteristic.

IMPROVEMENT OF POWER FACTOR. An improvement of power factor can be effected by injecting into the exciting circuit a small e.m.f. E_j in phase with the terminal voltage; this can be done from a separate transformer as shown in Fig. 16.10(a), by a separate

winding on the stator as in Fig. 16.10(b) or by a tapping on the main stator winding as in Fig. 16.10(c).

The effect of this injected e.m.f. can be studied by extending the complexor diagram of Fig. 16.9 to that of Fig. 16.11. The e.m.f.

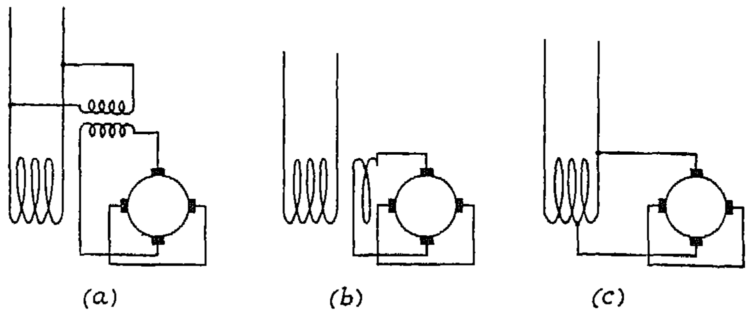


FIG. 16.10. METHODS OF OBTAINING PHASE COMPENSATION

(a) Transformer.
(b) Auxiliary stator winding.
(c) Tapping on stator winding

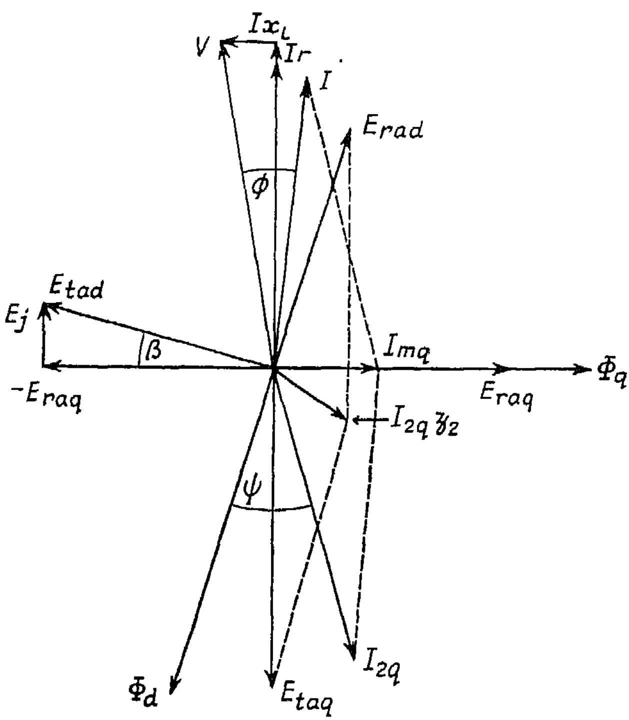


FIG. 16.11. COMPLEXOR DIAGRAM WITH PHASE COMPENSATION

E_{tad} now has to overcome $-E_{rad} + E_j$ and is therefore displaced by an angle β from its previous position. The flux Φ_d sets up this e.m.f. and is therefore similarly displaced. E_{rad} is in phase opposition to Φ_d so that $I_{2q}z_2$, which is the resultant of E_{rad} and E_{taq}, is advanced in phase as shown. The quadrature-axis rotor current lags slightly behind this impedance drop, but it can be seen that it has also been advanced, leading to a considerable advance in the phase of main stator current I and a consequent improvement of power factor. It may be noted that the angle ψ, upon which depends the torque, has not been greatly changed in magnitude although its sign has been altered; too great an injected e.m.f. would, however, cause ψ to be reduced with a resulting reduction of torque.

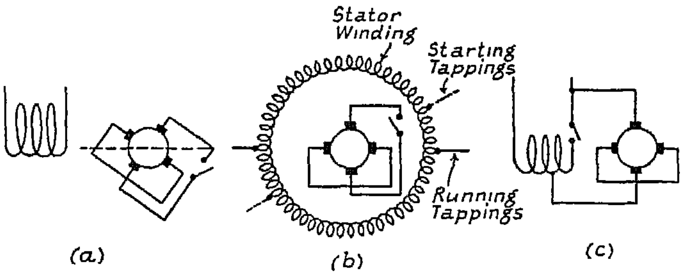


FIG. 16.12. METHODS OF STARTING.

(a) As repulsion motor.
(b) Tapped stator winding
(c) As compensated-repulsion motor.

STARTING. The simplest method of starting is to open the rotor exciting circuit and shift the brushes 15° to 20° from the quadrature axis as indicated in Fig. 16.12(a).* The motor then starts as a repulsion motor with a good starting torque, and when up to synchronous speed the exciting circuit may be closed after which the motor continues to run with its shunt characteristic; the brushes may be left in their shifted position although commutation is not so good as if they are shifted back to the quadrature axis.

Instead of shifting the brushes, which may introduce constructional complications, starting tappings may be provided on the stator winding as shown in Fig. 16.12(b) which shifts the axis of the stator winding.

A third method, applicable to a motor having phase compensation by a stator tapping, is shown in Fig. 16.12(c). With the switch open the motor starts as a compensated-repulsion motor.

Any of these methods gives about $2\frac{1}{2}$ times full-load torque with full-load current.

SPEED CONTROL. Much ingenuity has been expended in devising methods of obtaining a satisfactory method of speed control since

* In this and succeeding diagrams the phase compensation device is omitted for simplicity.

there is no other single-phase motor with a shunt characteristic with which speed control can be obtained. The two basic methods employed are analogous to field control and armature voltage control of a d.c. shunt motor; in the former the current in the exciting axis is varied and in the latter a voltage is injected into the rotor circuit.

SPEED CONTROL BY FIELD VARIATION. One arrangement is shown in Fig. 16.13(*a*) in which there is an additional tapped stator winding

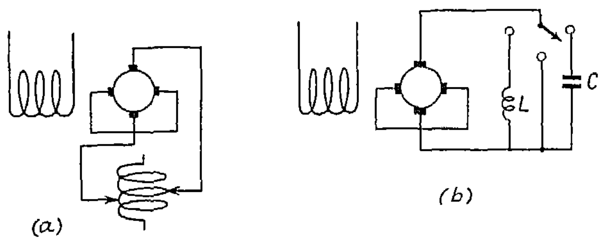


FIG. 16.13. SPEED CONTROL BY FIELD VARIATION

(*a*) Tapped field.
(*b*) Inductor or capacitor.

in the direct axis. Referring to the complexor diagram of Fig. 16.9 the rotational e.m.f. E_{raq} appearing at the direct-axis brushes now has to overcome both the transformer e.m.f. E_{tad} plus the e.m.f. E_{tsd} induced in the tapped stator winding. Thus, neglecting resistance and leakage reactance drops,

$$\begin{aligned} E_{raq} &= E_{tad} \pm E_{tsd} \\ &= E_{tad}(1 \pm E_{tsd}/E_{tad}) = E_{tad}(1 \pm T_{sf}'/T_a') \end{aligned}$$

where T_{sf}' is the effective number of turns on the tapped stator winding.

Hence $$f_r\Phi_q = f_1\Phi_d(1 \pm \rho)$$
where $$\rho = T_{sf}'/T_a'$$

Also, at the quadrature-axis brushes,

$$E_{taq} \simeq E_{rad}$$

$$f_1\Phi_q = f_r\Phi_d$$

Hence $$(f_r/f_1)^2 = 1 \pm \rho$$

and $$f_r = f_1\sqrt{(1 \pm \rho)}$$

The no-load speed is thus proportional to $\sqrt{(1 \pm \rho)}$, and a family of speed-torque curves as shown in Fig. 16.14 can be obtained. The curves relate to an 8 h.p. motor, and the figures on the curves give the number of turns on the tapped winding.

An alternative arrangement, due to Creedy, is shown in Fig. 16.13(*b*)—an inductor L connected in series with the direct-axis brushes reduces the exciting current, and hence the flux, without altering its phase position since the rotor is highly reactive: this reduction of flux raises the speed to values above synchronism. If a capacitor C is used instead of the inductor it will neutralize some

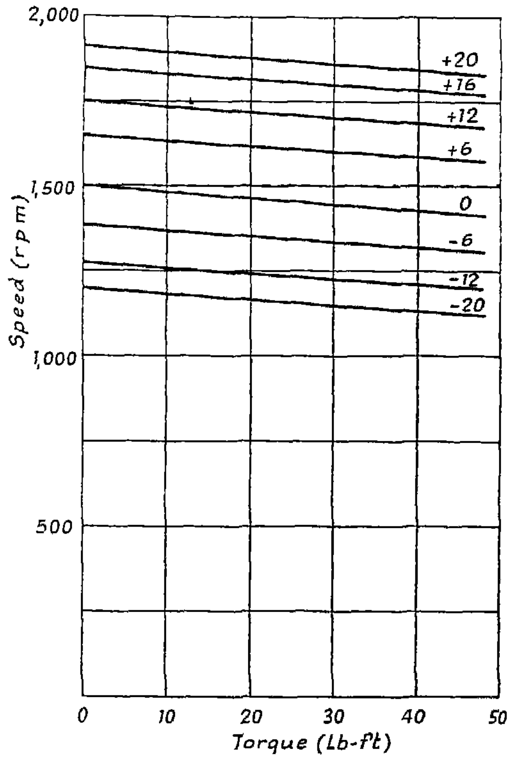


FIG. 16.14. SPEED-TORQUE CHARACTERISTICS OF SHUNT MOTOR

of the rotor reactance and result in an increase of exciting current so that the speed is reduced below synchronism. It may be noted that a resistor for limiting the current would be impracticable since, in addition to introducing losses, it would cause a phase shift of the current and so reduce the available torque.

SPEED CONTROL BY ROTOR VOLTAGE VARIATION. An e.m.f. V_r can be injected into the quadrature-axis circuit as shown in Fig. 16.15, the arrangement being somewhat similar to that of the series-repulsion motor. Neglecting resistance and leakage reactance drops, the rotational e.m.f. E_{rad} at the quadrature-axis brushes must now

equal $E_{taq} \pm V_r$, i.e. if V_r is additive to E_{taq} the speed must rise so that E_{rad} may balance the increase.

If the turns ratio is 1:1 between stator and rotor

$$E_{rad} = E_{taq} = V_s$$

so that

$$E_{rad} = E_{taq} \pm V_r = E_{taq}(1 \pm V_r/E_{taq}) = E_{taq}(1 \pm V_r/V_s)$$

Hence

$$f_r \Phi_d = f_1 \Phi_q (1 \pm V_r/V_s)$$

In the direct axis

$$E_{tad} = E_{raq}$$

$$\therefore \qquad f_1 \Phi_d = f_r \Phi_q$$

so that

$$f_r = f_1 \sqrt{(1 \pm V_r/V_s)}$$

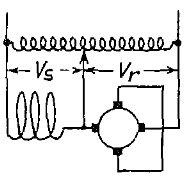


FIG. 16.15.
SPEED CONTROL BY VARIATION OF ROTOR VOLTAGE

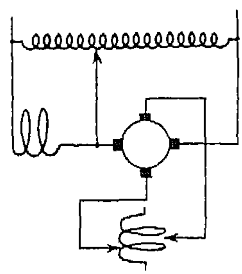


FIG. 16.16.
SPEED CONTROL BY COMBINED FIELD AND VOLTAGE VARIATION

The no-load speed is thus proportional to $\sqrt{(1 \pm V_r/V_s)}$ and a set of characteristics similar to those of Fig. 16.14 can be obtained.

COMBINATION OF FIELD- AND ROTOR-VOLTAGE CONTROL. By combining the above two schemes the arrangement of Fig. 16.16 is obtained which has some advantage in commutation.

Referring to the direct axis,

$$f_r \Phi_q = f_1 \Phi_d (1 \pm \rho)$$

Referring to the quadrature axis,

$$f_r \Phi_d = f_1 \Phi_q (1 \pm V_r/V_s)$$

Combining these gives

$$f_r = f_1 \sqrt{(1 \pm V_r/V_s)(1 \pm \rho)}$$

If the various tappings are adjusted so that the ratio V_r/V_s is equal to ρ then $f_r = f_1(1 \pm \rho)$.

COMMUTATION WITH SPEED CONTROL. In the plain motor it has been seen that $\Phi_d = \Phi_q$ and commutation is good at both sets of brushes since the rotational and transformer e.m.f.'s neutralize each other. A departure from synchronous speed by field control or voltage control destroys the above relation so that the brush e.m.f.'s no longer neutralize each other and commutation deteriorates.

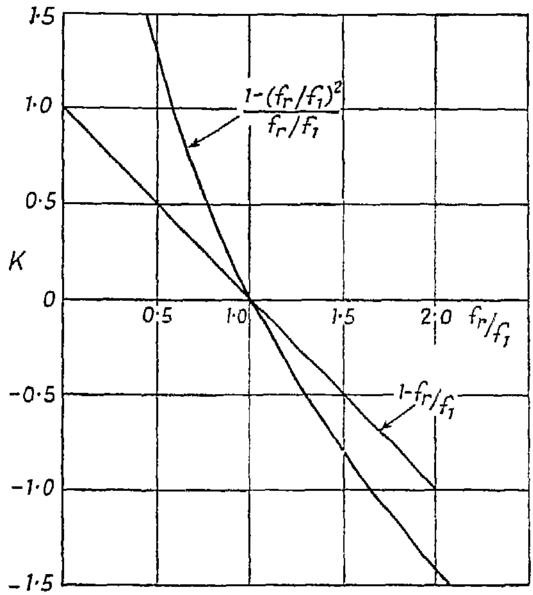


FIG. 16.17. VALUES OF BRUSH-E.M.F. FUNCTIONS

The resultant brush e.m.f. at the quadrature brushes is

$$\begin{aligned} E_b &= E_{tbd} - E_{rbq} \\ &= (\sqrt{2})\pi T_c(f_1\Phi_d - f_r\Phi_q) \\ &= (\sqrt{2})\pi T_c f_1\Phi_q(f_1/f_r - f_r/f_1) \\ &= \text{constant}\,\frac{1-(f_r/f_1)^2}{f_r/f_1} \end{aligned}$$

assuming Φ_q constant.

The variable function is plotted in Fig. 16.17 which may be compared with that of Fig. 15.9 relating to the plain repulsion motor; in this case it can be seen that super-synchronous running is preferable to sub-synchronous running.

With speed variation by combined field and voltage control the commutation is somewhat better since if

$$f_r = f_1(1 \pm \rho)$$

then $$\Phi_d = \Phi_q$$

and $$E_b = (\sqrt{2})\pi T_c f_1 \Phi_q (1 - f_r/f_1)$$
$$= \text{constant } (1 - f_r/f_1)$$

The variable function is also plotted on Fig. 16.17 from which it can be seen that a greater range of speed for a given value of brush e.m.f. can be obtained.

APPLICATIONS. Motors of this type having two or three speeds have been built in the past in sizes up to about 30 h.p. and used for lifts, hoists, printing presses and other drives. The poor commutation resulting from the use of either field or voltage control of speed and the complication of combined field and voltage control have, however, militated against its survival as a commercially practicable type of motor.

The simpler single-speed motor with phase compensation also had considerable use due to its power factor being better than that of the single-phase induction motor—the development of the capacitor type of induction motor (page 297), which is simpler and more robust as well as having a satisfactory power factor, has, however, rendered it obsolete.

CHAPTER 17

THE SINGLE-PHASE INDUCTION MOTOR

THE extended use of electric drive for low-power constant-speed apparatus such as small machine tools, domestic apparatus, and agricultural machinery in circumstances where a three-phase supply is not readily available has led to a large demand for single-phase induction motors in sizes ranging from a fraction of a horse-power up to about 5 h.p.

A feature of the single-phase induction motor is that it has no inherent starting torque, and some special device must be incorporated to make it self-starting. Although such devices are entirely satisfactory they complicate the motor somewhat and make it more expensive than a three-phase motor of the same output; for sizes greater than those indicated above it is therefore usually desirable to arrange for a three-phase supply to be available although, in special circumstances, single-phase induction motors as large as 100 h.p. have been installed.

Theories of Operation

The action of the single-phase induction motor can be explained in either of the two following ways—

1. Rotating-field theory.
2. Cross-field theory.

ROTATING-FIELD THEORY. In this the air-gap m.m.f. is resolved into two rotating components moving at synchronous speed in opposite directions; the treatment can thus be carried out on each component separately by the methods used for the polyphase machine.

CROSS-FIELD THEORY. The air-gap flux is resolved into two pulsating components along, and at 90° to, the stator-winding axis; the treatment thus follows the lines of that used for single-phase commutator motors.

Rotating-field Theory

The space distribution of m.m.f. produced by a single-phase stator winding is shown in Fig. 1.3, page 4. If harmonics are neglected this may be represented by the expression, from eq. 1.2(a)—

$$F = \frac{4}{\pi}\frac{\sin\gamma}{\gamma}\hat{F}_m \sin\alpha \sin\omega t \qquad . \qquad . \qquad (17.1)$$

The term $\sin\alpha$ indicates that the m.m.f. varies sinusoidally in space, α being measured from the centre of the winding, while the term $\sin\omega t$ indicates that it also varies sinusoidally with time.

This sinusoidally-distributed m.m.f. can be represented by a space vector pointing along the axis as shown in Fig. 17.1 and varying in magnitude between OA and OA' or $\hat{F}$ and $-\hat{F}$ in accordance with $F = \hat{F}\sin\omega t$ where $\hat{F} = (4/\pi)(\sin\gamma/\gamma)\hat{F}_m$.

RESOLUTION INTO TWO ROTATING FIELDS. The expression (17.1) can be expanded into

$$F = (\hat{F}/2)\{\cos(\alpha - \omega t) + \cos(\alpha + \omega t)\}$$

which represents two m.m.f. waves of constant magnitude moving at equal speeds in opposite directions and each having a maximum

FIG. 17.1. VECTOR REPRESENTATION OF ROTATING FIELD

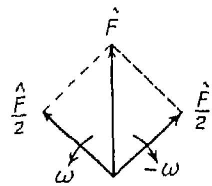


FIG. 17.2. RESOLUTION OF PULSATING M.M.F. INTO TWO ROTATING M.M.F.'S

value of $\hat{F}/2$. The speed of rotation is a movement past a pole-pair in one cycle, i.e. synchronous speed.

The single pulsating m.m.f. can thus be resolved into two rotating m.m.f.'s moving at synchronous speed in opposite directions, each m.m.f. being half the maximum amplitude of the original pulsating m.m.f.

These m.m.f.'s can be represented vectorially as in Fig. 17.2. The sum of the two rotating m.m.f.'s at any moment gives the pulsating m.m.f. which always acts along the vertical axis.

In developing the theory of the motor on these lines the m.m.f. which is moving in the same direction as the rotation of the rotor is referred to as the *forward* m.m.f. and the related quantities are given the suffix f while the m.m.f. moving in the opposite direction to the rotor is referred to as the *backward* m.m.f. and the related quantities are given the suffix b. The two m.m.f.'s are considered to set up two rotating fluxes Φ_f and Φ_b.

FREQUENCY OF ROTOR CURRENTS. Suppose the rotor is moving at a speed n_r ($< n_1$) in the same direction as the forward field. The speed relative to this field will be $n_1 - n_r$ rev/sec as in the polyphase induction motor, and e.m.f.'s and currents will be set up in the rotor having a frequency of $f_f = p(n_1 - n_r) = sf_1$, i.e. slip frequency.

The speed relative to the backward field will be $n_1 + n_r$, and the frequency of the e.m.f.'s and currents set up by this field will be

$$f_b = p(n_1 + n_r)$$
$$= (2 - s)f_1$$

Currents of two different frequencies may thus be considered to be set up in the rotor—for instance a 4-pole, 50-c/s motor running at 1.300 r.p.m. will carry rotor currents of frequencies—

$$f_f = 50 \times (1{,}500 - 1{,}300)/1{,}500 = 6{\cdot}67 \text{ c/s}$$

and

$$f_b = 50 \times (1.500 + 1{,}300)/1{,}500 = 93{\cdot}2 \text{ c/s}$$

In Fig. 17.3 is shown an oscillogram of the rotor current of a single-phase motor which clearly shows the current components of two different frequencies.

The rotor currents induced by the forward field, which have a frequency of sf_1, set up a rotating m.m.f. moving at $n_f = sf_1/p = sn_1$ relative to the conductors and in the same direction; the conductors

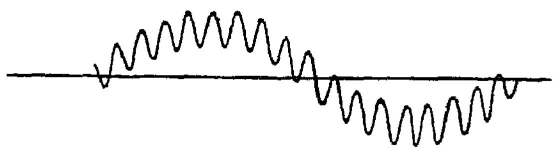


FIG. 17.3. OSCILLOGRAM OF ROTOR CURRENT OF SINGLE-PHASE INDUCTION MOTOR

are, however, moving at a speed of n_r so that the speed of the m.m.f. relative to the fixed stator is $n_f + n_r = n_1$, i.e. synchronous speed as in the polyphase motor.

Similarly the currents induced by the backward field set up a rotating m.m.f. moving at $n_b = (2 - s)n_1$ relative to the conductors and in the opposite direction, so that, since the conductors are already moving at n_r, the actual speed of the m.m.f. relative to the stator will be $n_r - (2 - s)n_1 = - n_1$, i.e. synchronous speed backwards.

Both rotor m.m.f.'s thus move at the same speed as the corresponding stator m.m.f.'s and therefore produce supply-frequency reactions in the stator.

EQUIVALENT CIRCUIT. With the motor stationary the equivalent circuit is exactly similar to that of a transformer on short-circuit as shown in Fig. 17.4(a). If the pulsating field is assumed to be resolved into two equal rotating fluxes moving in opposite directions and each of half the peak magnitude of the pulsating flux, the effect of each can be represented by separate impedance groups as shown in Fig. 17.4(b), E_f and E_b being equal at standstill.

When the machine is running at slip s the slip to be introduced into the forward-field group is s, thereby altering the resistive term to $(r_2'/2)/s$ as in the ordinary 3-ph. induction motor. The backward-field slip is $(2 - s)$ so that the resistance term becomes $(r_2'/2)/(2 - s)$; it may be noted that r_2' for the backward field may be 1·2 to 1·4 times r_2' for the forward field on account of skin effect due to the higher frequency of the backward-field currents. The complete equivalent circuit is thus as shown in Fig. 17·4(c).

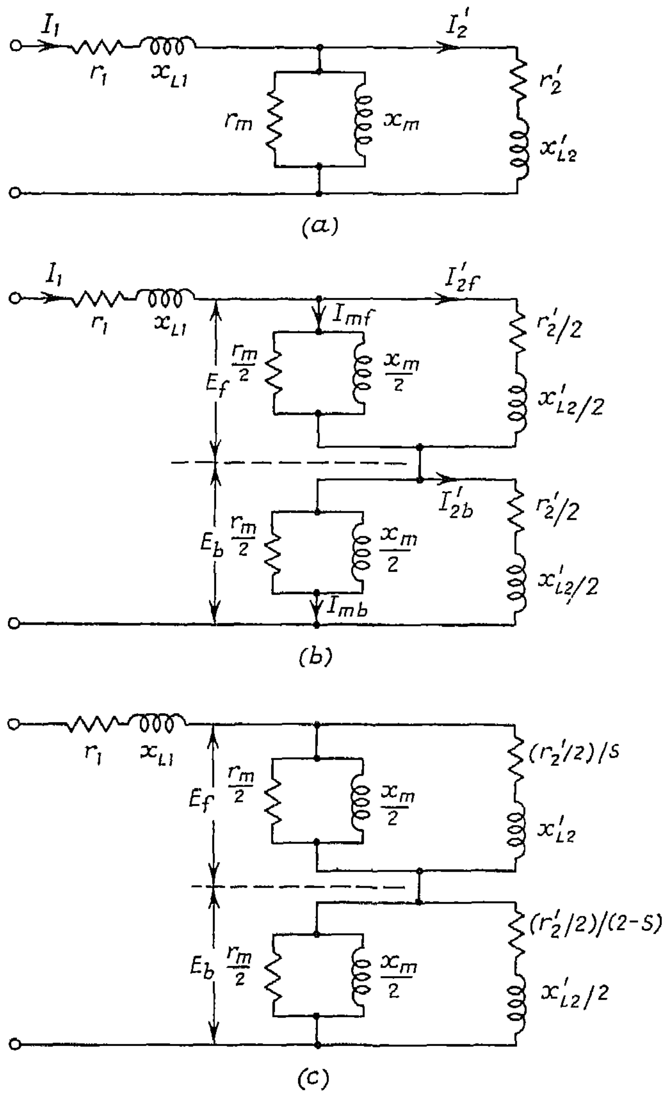


FIG. 17.4. EQUIVALENT CIRCUIT (ROTATING-FIELD THEORY)

(a) Equivalent circuit at standstill.
(b) Equivalent circuit at standstill with effect of forward and backward flux components.
(c) Equivalent circuit at slip s.

An inspection of the circuit shows that the total impedance of the forward branch will, when running, be considerably higher than that of the backward branch [since $(r_2'/2)/s$ is greater than $(r_2'/2)/(2 - s)$]. E_f is thus greater than E_b and the forward flux is greater than the backward flux. The variations in these quantities with speed are shown, for a typical motor, in Fig. 17.6.

Iron loss is represented by the resistor $r_m/2$, to facilitate calculations this can be omitted from the equivalent circuit without serious error and the iron loss included with the friction and windage loss and subtracted from the gross output.

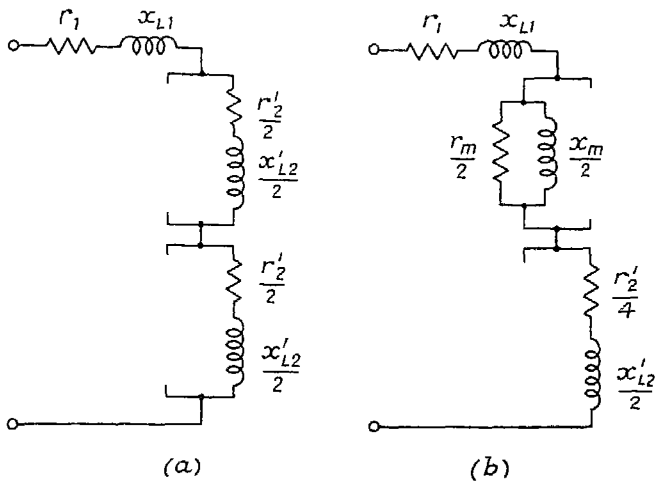


FIG. 17.5. EQUIVALENT CIRCUITS FOR EXPERIMENTAL DETERMINATION OF CONSTANTS

(a) Locked-rotor test ($s = 1$).
(b) No-load test ($s = 0$)

As in the polyphase induction motor the torque can be found from the rotor input—

Torque due to forward field $= I_{2f}^2(r_2'/2)/s$ synch. watts

in the direction of rotation.

Torque due to backward field $= I_{2b}^2(r_2'/2)/(2 - s)$ synch. watts

against the direction of rotation.

The net torque is the difference between these. It can be seen from the equivalent circuit and from the curves of Fig. 17.6 that at standstill the two torques are equal so that there is no net torque, i.e. no starting torque; also if $s = 0$ there is some negative torque, so that the motor will run at slightly less than synchronous speed on no load.

CALCULATION OF MOTOR PARAMETERS. The various parameters arising in the equivalent circuit can be calculated from the dimensions in the same way as for a three-phase motor.

EXPERIMENTAL DETERMINATION OF MOTOR PARAMETERS. As with the polyphase motor the parameters can be determined from no-load and locked-rotor tests. Consider the motor stationary—the flux will be small so that the magnetizing current can be neglected and the equivalent circuit reduces to that of Fig. 17.5(a). Readings of voltage, current and power thus give—

$$R = r_1 + r_2' \text{ and } X = x_{L1} + x_{L2}'$$

It is not possible to separate x_{L1} and x_{L2}', but common practice is to assume $x_{L1} = x_{L2}'$. The stator resistance r_1 can be determined from a d.c. measurement so that r_2' can be found.

If the motor is run on no load it can be assumed that $s = 0$ so that the equivalent circuit becomes that of Fig. 17.5(b) since $(r_2'/2)/s$ becomes infinitely great and $(r_2'/2)/(2 - s)$ becomes small relative to the magnetizing impedance. As r_1, r_2' and x_{L2}' have already been found, the values of r_m and x_m can be determined from the voltage, current and power measurements. The procedure is illustrated in the following example.

Example 17.1. The following test results were obtained on a 1/6-h.p., 220-V, 50-c/s, 6-pole, single-phase induction motor.

Stator winding resistance $= 11{\cdot}4\ \Omega$

Locked-rotor test: $V = 220$ V, $I = 5{\cdot}8$ A, $W = 850$ W

No-load test: $V = 220$ V, $I = 1{\cdot}36$ A, $W = 64$ W

Using the rotating-field theory draw, to a base of speed, curves of current, E_f and E_b (proportional to the forward and backward flux components), and the forward, backward and total torques, and also, to a base of output, the speed, current, power factor and efficiency.

From the locked-rotor test—

$$Z = 220/5{\cdot}8 = 38\ \Omega$$

$$R = 850/5{\cdot}8^2 = 25{\cdot}2\ \Omega$$

$$X = \sqrt{(38^2 - 25{\cdot}2^2)} = 28{\cdot}6\ \Omega$$

Hence $x_{L1} = x_{L2}' = 28{\cdot}6/2 = 14{\cdot}3\ \Omega$

and $x_{L2}'/2 = 7{\cdot}15\ \Omega$

$$r_2' = 25{\cdot}2 - 11{\cdot}4 = 13{\cdot}8\ \Omega$$

so that $r_2'/2 = 6{\cdot}9\ \Omega$

From the no-load test—

$$Z = 220/1{\cdot}36 = 162\ \Omega$$

$$R = 64/1{\cdot}36^2 = 34{\cdot}6\ \Omega$$

$$X = \sqrt{(162^2 - 34{\cdot}6^2)} = 159\ \Omega$$

Correct handling of the iron, friction and windage loss is difficult. In the equivalent circuit of Fig. 17.5(*b*) $r_m/2$ represents the iron loss, the friction and windage loss being regarded as mechanical output and subtracted from the gross output to give the actual net output. The test figures, however, do not differentiate between the iron loss and the friction and windage loss, so that either the whole loss must be treated as iron loss and the values of $r_m/2$ and $x_m/2$ calculated accordingly or it must all be regarded as friction and windage loss and subtracted from the gross output. The latter procedure is simpler and is adopted in what follows, $r_m/2$ being omitted from the calculations. From the test figures the total loss on no load is 64 W so that the iron, friction and windage loss is

$$64 - 1{\cdot}36^2(11{\cdot}4 + 6{\cdot}9) = 30{\cdot}2 \text{ W}$$

With $r_m/2$ omitted from Fig. 17.5(*b*)

$$\begin{aligned} x_m/2 &= 159 - 14{\cdot}3 - 7{\cdot}15 \\ &= 137{\cdot}5\ \Omega \end{aligned}$$

For a slip s of 0·06

$$\begin{aligned} (r_2'/2)/s + \mathrm{j}x_{L2}'/2 = 6{\cdot}9/0{\cdot}06 + \mathrm{j}7{\cdot}15 &= 115 + \mathrm{j}7{\cdot}15 \\ &= 115\underline{/3{\cdot}6^\circ}\ \Omega \end{aligned}$$

Putting this in parallel with $x_m/2$ gives for the total forward impedance

$$\begin{aligned} Z_f = (115\underline{/3{\cdot}6^\circ} \times 137{\cdot}5\underline{/90^\circ})/(115 + \mathrm{j}144{\cdot}6) &= 85{\cdot}5\underline{/41{\cdot}8^\circ} \\ &= 63{\cdot}5 + \mathrm{j}56{\cdot}5\ \Omega \end{aligned}$$

For the backward-flux part of the circuit, $(2 - s) = 1{\cdot}94$ so that

$$\begin{aligned} (r_2'/2)/(2 - s) + \mathrm{j}x_{L2}'/2 = 6{\cdot}9/1{\cdot}94 + \mathrm{j}7{\cdot}15 &= 3{\cdot}56 + \mathrm{j}7{\cdot}15 \\ &= 8{\cdot}0\underline{/63{\cdot}5^\circ}\ \Omega \end{aligned}$$

Putting this in parallel with $x_m/2$ gives for the total backward impedance

$$\begin{aligned} Z_b = (8{\cdot}0\underline{/63{\cdot}5^\circ} \times 137{\cdot}5\underline{/90^\circ})/(3{\cdot}56 + \mathrm{j}144{\cdot}6) &= 7{\cdot}6\underline{/64{\cdot}9^\circ} \\ &= 3{\cdot}22 + \mathrm{j}6{\cdot}9\ \Omega \end{aligned}$$

The total impedance of the motor is thus—

$$\begin{aligned} z_1 + Z_f + Z_b &= 11{\cdot}4 + \mathrm{j}14{\cdot}3 + 63{\cdot}5 + \mathrm{j}56{\cdot}5 + 3{\cdot}22 + \mathrm{j}6{\cdot}9 \\ &= 78{\cdot}1 + \mathrm{j}77{\cdot}7 \\ &= 110\underline{/45^\circ} \end{aligned}$$

Hence the total motor current is

$$220/(110\underline{/45^\circ}) = 2{\cdot}0 \text{ A at } 0{\cdot}707 \text{ power factor}$$

E.m.f. across forward impedance (proportional to Φ_f) $= 2{\cdot}0 \times 85{\cdot}5$
$= 171$ V

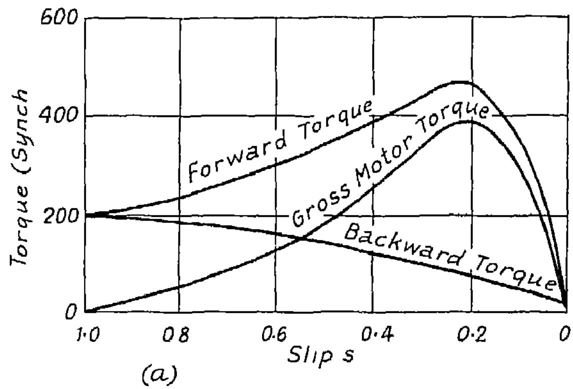
E.m.f. across backward impedance (proportional to Φ_b) $= 2{\cdot}0 \times 7{\cdot}6$
$= 15{\cdot}2$ V

Forward component of rotor current,

$$I_{2f} = 2{\cdot}0 \times 137{\cdot}5/(115 + j144{\cdot}6)$$
$$= 1{\cdot}49 \text{ A}$$

Backward component of rotor current,

$$I_{2b} = 2{\cdot}0 \times 137{\cdot}5/(3{\cdot}56 + j144{\cdot}6)$$
$$= 1{\cdot}9 \text{ A}$$



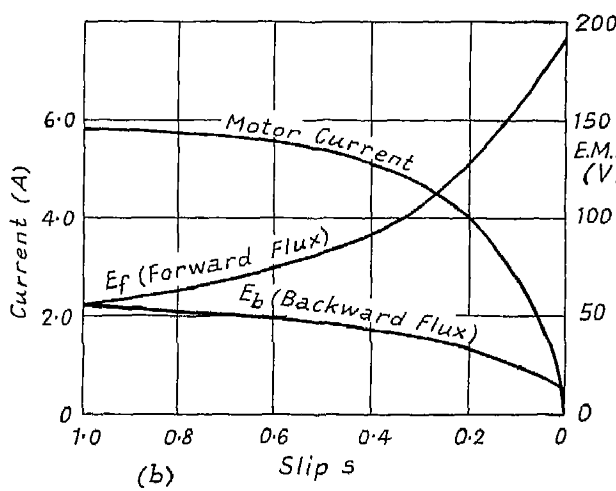


FIG. 17.6. TORQUES AND FLUXES (EXAMPLE 17.1.)
(a) Torque.
(b) Current and fluxes.

Forward torque $= 1{\cdot}49^2 \times 115 = 255$ synch. W

Backward torque $= 1{\cdot}9^2 \times 3{\cdot}56 = 12{\cdot}8$ synch. W

Gross motor torque $= 255 - 12{\cdot}8 = 242$ synch. W

Net motor torque (subtracting iron, friction and windage loss)

$$= 242 - 30{\cdot}2 = 211{\cdot}8 \text{ synch. W}$$

Output $= 211{\cdot}8(1 - 0{\cdot}06) = 198$ W $= 0{\cdot}266$ h.p.

1/6 h.p. is 125 W so that this is about $1\frac{1}{2}$ times full load.

Input $= 220 \times 2{\cdot}0 \times 0{\cdot}707 = 311$ W

Efficiency $= 198/311 = 63{\cdot}5$ per cent

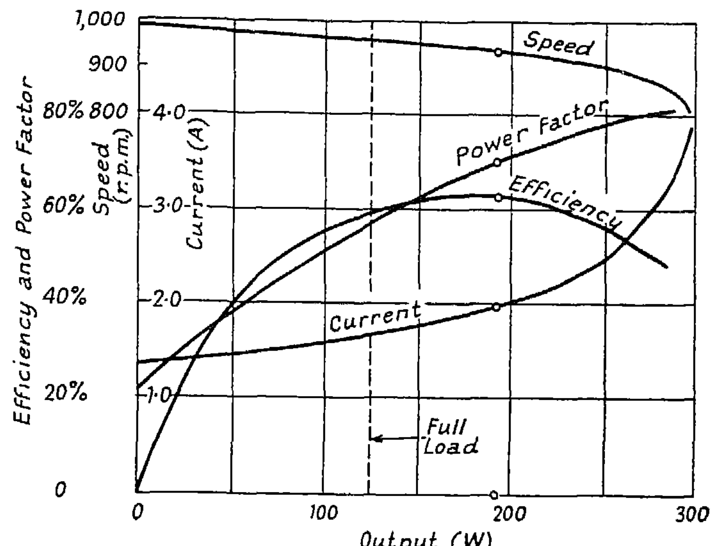


FIG. 17.7. RUNNING CHARACTERISTICS (EXAMPLE 17.1.)

Similar calculations can be made for other values of s giving the curves plotted in Fig. 17.7 to a base of output over the normal working range, and in Fig. 17.6 to a base of speed over the whole range from standstill to synchronism.

It can be seen from the above that calculations on the single-phase motor are laborious, and the procedure can be facilitated by the use of a desk computer or network analyser.

Cross-field Theory

In this theory the flux is resolved into two components acting along and at right-angles to the stator-winding axis as shown in

Fig. 17.8. The short-circuited rotor conductors are considered in two groups, one group being linked with each of the component fluxes; in Fig. 17.8 each group is represented by a single turn.

MOTOR STATIONARY. With the motor stationary and with a voltage V applied to the stator winding an m.m.f. will be set up in the horizontal or quadrature axis and this results in a quadrature flux Φ_q in this axis which is linked with the turn ab; an e.m.f. E_{taq} is induced in this turn which sets up a current I_{2q}. This current sets up an opposing m.m.f. along the quadrature axis so that the quadrature flux is set up by the resultant of these two m.m.f.'s. As they are along the same axis there is no torque produced by them. The coil cd has its axis at 90° to the quadrature flux and therefore has no e.m.f. induced in it and carries no current. No torque is thus

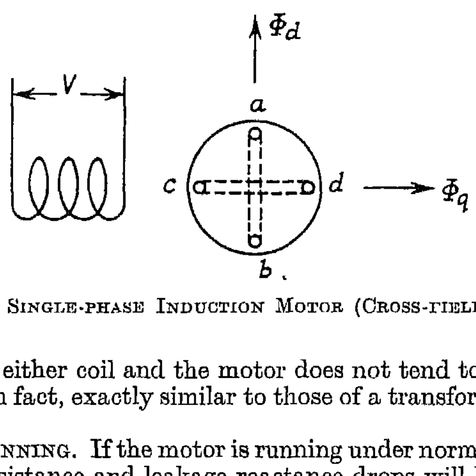


FIG. 17.8. SINGLE-PHASE INDUCTION MOTOR (CROSS-FIELD THEORY)

produced on either coil and the motor does not tend to start. Conditions are, in fact, exactly similar to those of a transformer on short circuit.

MOTOR RUNNING. If the motor is running under normal conditions the stator resistance and leakage reactance drops will be small and the back e.m.f. induced in the stator winding will be approximately equal to the applied voltage. With a constant applied voltage, therefore, this back e.m.f. will be approximately constant and the quadrature flux Φ_q which is inducing it will also be constant and independent of motor load or speed.

With the motor running the coil cd is moving in the quadrature flux and therefore has a rotational e.m.f. E_{raq} set up in it; since the coil is short-circuited a current I_{2d} will flow in it, and this sets up an m.m.f. and consequent flux Φ_d acting in the vertical (direct) axis.

Torque can now be produced as a result of the interaction of the flux Φ_d with the current I_{2q} in coil ab and also of the interaction of the flux Φ_q with the current I_{2d} in coil cd. The phase relations between these fluxes and currents and the magnitudes of the resulting torques can be seen by drawing the complexor diagram.

COMPLEXOR DIAGRAM. Since Φ_q is constant this forms a convenient starting point and it is drawn horizontally as shown in Fig. 17.9.

The coil *cd* has induced in it the rotational e.m.f. E_{raq} in phase with Φ_q. This e.m.f. sets up a current I_{2d} in the coil; as there is no other coil in the direct axis this current sets up the flux Φ_d.

Due to the high total reactance of the coil caused by this flux the current will lag by nearly 90° behind the e.m.f. producing it, and

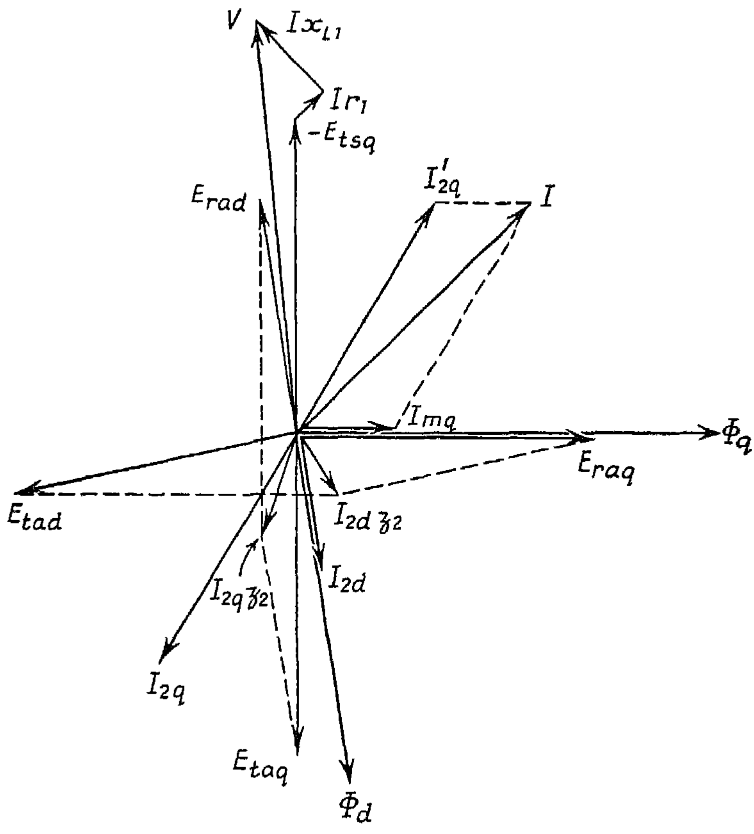


FIG. 17.9. COMPLEXOR DIAGRAM OF SINGLE-PHASE INDUCTION MOTOR (CROSS-FIELD THEORY)

the flux will, neglecting iron losses, be in phase with the current as shown. The total reactance e.m.f. of the coil *cd* is made up chiefly of a transformer e.m.f. E_{tad} lagging Φ_d by 90°, and the resultant of this and the rotational e.m.f. in the coil is equal to the resistance and leakage reactance drop of the coil $I_{2d}z_2$.

Since $E_{raq} = (\sqrt{2})\pi T f_r \Phi_q$ and $E_{tad} = (\sqrt{2})\pi T f_1 \Phi_d$ and these two are approximately equal

$$\Phi_q/\Phi_d = f_1/f_r$$

i.e. at speeds less than synchronism, which correspond to normal operation, the exciting flux is less than the quadrature flux in the ratio of running speed to synchronous speed. Since the running speed is only a little less than synchronous speed the two fluxes are always nearly equal.

Consider now the coil *ab*—due to the flux Φ_q a transformer e.m.f. E_{taq} will be induced lagging 90° behind Φ_q; due to Φ_d a rotational e.m.f. E_{rad} will be induced which must be approximately equal and opposite to E_{taq}. The rotational e.m.f. must, however, be in phase with or in phase opposition to the flux Φ_d so that, to satisfy the above, it must be drawn in phase opposition to Φ_d. Since $E_{taq} = (\sqrt{2})\pi T f_1 \Phi_q$ and $E_{rad} = (\sqrt{2})\pi T f_r \Phi_d$, $f_r < f_1$, and $\Phi_d < \Phi_q$, therefore $E_{rad} < E_{taq}$. The resultant of these two e.m.f.'s which is equal to the impedance drop $I_{2q}z_2$, is thus downwards as shown, and the current I_{2q} lags this drop by the impedance angle of the rotor.

In the stator winding an e.m.f. E_{tsq} is induced by the quadrature flux and the stator terminal voltage has to overcome this as well as the resistance and leakage reactance drops Ir_1 and Ix_{L1}. The stator current is made up of a component I_{2q}' which corresponds to the rotor current I_{2q} and the magnetizing current I_{mq} setting up the quadrature flux.

Torque. Torque is produced by the interaction of I_{2q} and Φ_d and of I_{2d} and Φ_q. It can be seen that I_{2d} and Φ_q are nearly at 90° so that they produce little torque, the main effect being produced by the current I_{2q} and the direct-axis flux.

The rotor powers corresponding to these torques are

$$E_{rad}I_{2q}\cos \widehat{E_{rad}I_{2q}} \quad \text{and} \quad E_{raq}I_{2d}\cos \widehat{E_{raq}I_{2d}}$$

It can be seen from the diagram that $\widehat{E_{rad}I_{2q}}$ is greater than 90° so that, since the power depends on the cosine of this angle (page 215), it is negative (motor torque) whereas the other torque is positive (generator torque). The torque of the motor is thus made up of a main-motor torque due to I_{2q} and Φ_d minus a small generator torque due to I_{2d} and Φ_q.

Analytical Treatment. The cross-field theory lends itself to analytical treatment similar to that used for the single-phase commutator motor based on eqs. (12.3) of pages 218 and 219. The cage rotor winding is represented by circuits in the direct and quadrature axes as shown in Fig. 17.8. The equations are thus—

Stator winding in quadrature axis

$$V_{1q} = r_{1q}I_{1q} + jx_{L1q}I_{1q} + jx_{mq}(I_{1q} + I_{2q}) \quad . \quad (17.2)$$

Rotor winding in direct axis

$$0 = r_{2d}I_{2d} + jx_{L2d}I_{2d} + jx_{md}I_{2d} + Sx_{mq}(I_{1q} + I_{2q}) \quad . \quad (17.3)$$

Rotor winding in quadrature axis

$$0 = r_{2q}I_{2q} + jx_{L2q}I_{2q} + jx_{mq}(I_{1q} + I_{2q}) - Sx_{md}I_{2d} \quad . \quad (17.4)$$

As an additional refinement the terms $Sx_{L2q}I_{2q}$ and $-Sx_{L2d}I_{2d}$ can be added to eqs. (17.3) and (17.4) respectively to represent the rotational e.m.f.'s induced in the rotor by the rotor leakage flux.

In a normal induction motor V_{1q} and I_{1q} are the supply voltage and current V_1 and I_1 respectively; also since the construction is symmetrical $x_{md} = x_{mq} = x_m$, the mutual reactance, $r_{2d} = r_{2q} = r_2$, the rotor resistance, and $x_{L2d} = x_{L2q} = x_{L2}$, the rotor leakage reactance. All quantities are referred to the primary winding. The above equations can thus be written, including the rotational e.m.f. due to leakage flux—

$$
\begin{aligned}
V_1 &= r_1I_1 + \mathrm{j}x_{L1}I_1 + \mathrm{j}x_m(I_1 + I_{2q})\\
0 &= r_2I_{2d} + \mathrm{j}x_{L2}I_{2d} + \mathrm{j}x_mI_{2d} + Sx_m(I_1 + I_{2q}) + Sx_{L2}I_{2q}\\
0 &= r_2I_{2q} + \mathrm{j}x_{L2}I_{2q} + \mathrm{j}x_m(I_1 + I_{2q}) - Sx_mI_{2d} - Sx_{L2}I_{2d}
\end{aligned}
$$

Solving these gives—

$$I_1 = V_1\frac{(1 - S^2)(x_m + x_{L2})^2 - r_2^2 - \mathrm{j}2r_2(x_m + x_{L2})}{U + \mathrm{j}W}$$

$$I_{2d} = V_1 \times Sx_mr_2/(U + \mathrm{j}W)$$

$$I_{2q} = -V_1\frac{x_m\{(1 - S^2)(x_m + x_{L2}) - \mathrm{j}r_2\}}{U + \mathrm{j}W}$$

where

$$
\begin{aligned}
U = 2r_2r_1(x_m + x_{L2}) &+ r_2x_m(x_m + 2x_{L2})\\
&+ (1 - S^2)r_1(x_m + x_{L2})^2 - r_1r_2^2
\end{aligned}
$$

and

$$
\begin{aligned}
W = (1 - S^2)\{x_{L1}(x_m + x_{L2}) &+ x_{L2}x_m\}(x_m + x_{L2})\\
&- r_2^2(x_{L1} + x_m) - 2r_1r_2(x_m + x_{L2})
\end{aligned}
$$

The fluxes are—

Direct-axis component: $\Phi_d = (x_m + x_{L2})I_{2d}/2\pi fT_1$

Quadrature-axis component: $\Phi_q = x_m(I_1 + I_{2q})/2\pi fT_1$

Torque is produced by the interaction of I_{2q} and Φ_d and of I_{2d} and Φ_q. Multiplying the flux by the component of current in phase with the flux in each case, adding and then multiplying by $2\pi fT_1$ gives the gross torque in synchronous watts. The net torque may be obtained by subtracting the iron, friction and windage loss, usually 4–8 per cent of the output.

It may be noted that at standstill ($S = 0$) I_{2d} is zero so that Φ_d is zero and there is thus no starting torque. Also at synchronous speed ($S = 1$) the two rotor currents become equal in magnitude and displaced by 90°.

General Conclusions Regarding Behaviour

Although, in general, the behaviour and characteristics of a single-phase induction motor are similar to those of a three-phase motor, there are certain important differences which can be observed by considering either of the foregoing theories.

STARTING TORQUE. According to the rotating-field theory, both components of the field are rotating relative to the conductors when the rotor is stationary; as the fields are moving in opposite directions the torques produced by each are equal and opposite so that the net torque when stationary, i.e. the starting torque, is zero. This is illustrated by the torque-speed curve of Fig. 17.6; this curve shows further that if the motor is given a start in either direction the forward torque for that direction of rotation will immediately exceed the backward torque and the motor will run up to speed in the direction of the initial impulse.

According to the cross-field theory the cross or direct-axis field upon which the main torque depends is produced as a result of rotational e.m.f., and therefore there can be no torque when the motor is stationary.

As mentioned previously, special starting arrangements have to be incorporated in any commercial motor and these are discussed in subsequent sections.

TORQUE PULSATION. The three-phase motor gives a steady torque but the single-phase motor produces a pulsating torque which may give rise to vibration and noise.

According to the rotating-field theory a part of the torque is produced by the interaction of the double-frequency rotor currents with the forward field component so that this part of the total torque pulsates at double the supply frequency.

Using the cross-field theory, both motor and generator torques are produced by the interaction of alternating currents and fluxes so that both are pulsating as shown in Fig. 12.4 (page 214).

FULL-LOAD SPEED. It can be seen from the curves of Fig. 17.6 that the torque drops to zero at slightly less than synchronous speed. In the three-phase motor this occurs at synchronous speed so that the slip of a single-phase motor will always be slightly higher than that of a corresponding three-phase motor.

IRON LOSS. The iron losses in a single-phase motor are greater than those in a corresponding three-phase motor on account, chiefly, of the losses occurring in the rotor core. According to the rotating-field theory the forward component of the field produces a negligible iron loss in the rotor as it is moving at nearly the same speed as the rotor as in the polyphase machine; the backward field, however, is moving relative to the rotor at nearly twice synchronous speed and thus causes a considerable iron loss.

In the cross-field theory there are two alternating components of field associated with the rotor and both set up rotor iron losses. The losses due to the quadrature field are supplied from the stator

winding in the usual way, but those due to the direct-axis field must be transferred inductively across the gap to the rotor as there is no stator winding in this axis; these losses are represented by the generator torque referred to on page 294.

TYPICAL EFFICIENCIES. Efficiency and power factor of simple 4-pole single-phase induction motors are as shown in Table 17.1.

TABLE 17.1

EFFICIENCY AND POWER FACTOR

Rating h.p.	Efficiency (%)	Power Factor
1/20	38	0·46
1/8	48	0·51
1/4	57	0·56
1/2	65	0·62
1·0	69	0·64

Methods of Starting

As already mentioned, the single-phase induction motor has no inherent starting torque and special means must be adopted to make it self-starting. Three general methods are in use—

1. Split-phase starting.
2. Shaded-pole starting.
3. Repulsion-motor starting.

SPLIT-PHASE STARTING. An additional winding, known as the starting or auxiliary winding, is wound on the stator in phase quadrature with the main running winding as shown in Fig. 17.10; this winding is supplied with current displaced in time from the current in the main winding by as nearly 90° as practicable. A rotating field is thus set up, and the motor starts as a somewhat imperfect two-phase motor but has sufficient starting torque for most purposes. The requisite phase displacement between the currents in the main and auxiliary windings is obtained by connecting a suitable impedance in series with one of them; if this impedance is a resistor the *resistor-split-phase* motor is obtained while if it is a capacitor the *capacitor-split-phase* motor is obtained. With these split-phase motors the auxiliary winding is cut out of circuit after the motor has run up to speed although with the capacitor type it may be left in, together with a part of the capacitor, to give an improved power factor. Such motors are usually referred to as *capacitor* motors, and they are more fully discussed in Chapter 20. In both cases the current in the auxiliary winding leads that in the main winding, and the rotor conductors move in a direction from the auxiliary winding to the main winding of the same polarity.

Occasionally a reactor may be added to the main winding to increase its angle of lag, giving a *reactor-start* motor.

SHADED-POLE STARTING. If a portion of the pole is shaded by a short-circuited winding the flux encircled by the winding will lag the remainder of the flux by an angle of 20° to 30°—two fluxes displaced in both space and time are thus obtained and a rotating field is again produced. The efficiency of this type is low and it is only suitable for powers of less than about 1/10 h.p. Gramophone motors and motors for small domestic fans are, however, typical applications, the former being only about 1/1,800 h.p. This type is more fully discussed in Chapter 19.

REPULSION-MOTOR STARTING. Where a high starting torque is required the motor may carry a commutator winding instead of the

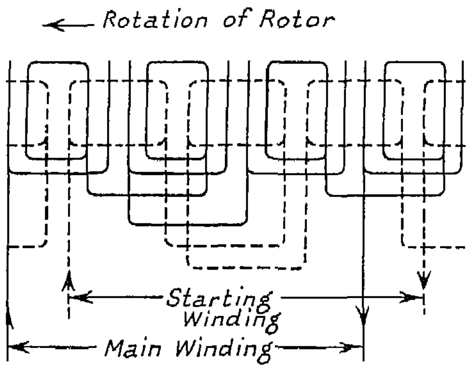


FIG. 17.10. MAIN AND STARTING WINDINGS OF SPLIT-PHASE MOTOR

ordinary cage winding and be started as a repulsion motor. In the *repulsion-start* motor the rotor winding is a commutator winding exactly similar to that of an ordinary repulsion motor. When switched on, the motor starts as a repulsion motor with a high starting torque—at a predetermined speed a centrifugal device short-circuits all the commutator segments thus making the winding equivalent to a cage winding and the motor continues to run as an induction motor.

In the *repulsion-induction* motor the rotor carries a commutator winding as well as a cage winding. At starting, the repulsion winding has a preponderating effect giving a good starting torque, while when running the cage winding gives the motor the constant-speed induction-motor characteristic. Either type of motor is built in sizes between 1/4 and 5 h.p., and they are more fully described in the next chapter.

Typical starting torques and the usual range of output for all types are given in Table 17.2.

Split-phase Starting

Various practicable arrangements are shown in Fig. 17.11 together with complexor diagrams showing the currents I_a in the auxiliary winding and I_g in the main winding.

The resistor split-phase motor of Fig. 17.11(*a*) involves only a simple resistor or, alternatively, the starting winding may be made

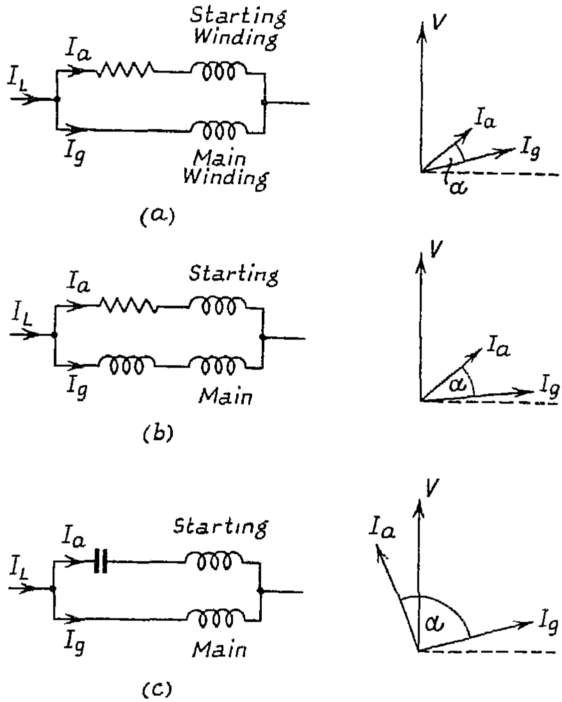


FIG. 17.11. TYPES OF SPLIT-PHASE MOTORS

(*a*) Resistor.
(*b*) Resistor and reactor.
(*c*) Capacitor.

to have a high inherent resistance by making it of small-section wire. The angle between the two currents cannot conveniently be greater than 30° to 40° so that although the arrangement is cheap and simple the starting torque available is rather low.

A slight improvement can be effected by putting a choke in series with the main winding as shown in Fig. 17.11(*b*) but the cost hardly warrants the small extra torque which is obtainable.

By using a capacitor as in Fig. 17.11(*c*) the current in the starting winding can be made to lead the voltage, and an angle of approximately 90° between the main and starting winding currents can be

obtained—the starting torque is thus much increased as indicated by the figures of Table 17.2.

TABLE 17.2

STARTING PERFORMANCE OF SPLIT-PHASE MOTORS

Type of Motor	Range of Output (h.p.)	Starting Current (Times F.L. Current)	Starting Torque (Times F.L. Torque)
Split-phase:			
Resistor	$\frac{1}{100}$–$\frac{1}{2}$	5–7	0·75–2·0
Capacitor	$\frac{1}{8}$–1	4–6	2–3·5
Repulsion-start	$\frac{1}{8}$–5	2–3	2–4
Repulsion-induction	$\frac{1}{2}$–5	3–4	2–4
Shaded-pole	$\frac{1}{1000}$–$\frac{1}{8}$	1–1·5	0·2–0·3
Capacitor-run:			
Single Capacitor	$\frac{1}{8}$–$\frac{1}{2}$	2–3	0·25–0·75
Two Capacitors	$\frac{1}{8}$–5	4–6	2–3·5

STARTING TORQUE. Suppose that, at starting, the currents in the main and starting windings are I_g and I_a respectively with an angle α between them. These set up fluxes Φ_g and Φ_a in the two axes which will also be displaced by the angle α in time and can be represented on a complexor diagram as shown in Fig. 17.12.

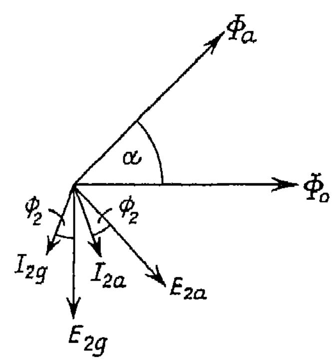


FIG. 17.12. COMPLEXOR DIAGRAM AT STARTING

The flux Φ_g sets up a transformer e.m.f. $E_{2g} = (\sqrt{2})\pi T_2' f \Phi_g$ volts in the main-axis rotor circuit and Φ_a similarly sets up $E_{2a} = (\sqrt{2})\pi T_2' f \Phi_a$ volts in the quadrature- or starting-axis circuit. These e.m.f.'s set up corresponding currents $I_{2g} = E_{2g}/\sqrt{(r_2^2 + x_{L2}^2)}$ and $I_{2a} = E_{2a}/\sqrt{(r_2^2 + x_{L2}^2)}$ each lagging their respective e.m.f.'s by the angle ϕ_2 given by $\cos \phi_2 = r_2/\sqrt{(r_2^2 + x_{L2}^2)}$.

The flux Φ_g interacts with the current I_{2a} and the flux Φ_a with I_{2g} to produce torque. Starting from the torque eq. (12.1) (page 215)

and putting $z = 2a\pi T_2'$, it can be seen that the torque set up is given by—

$$\text{Torque} = (1/\sqrt{2})T_2'p\Phi_g I_{2a} \cos(90 + \phi_2 - \alpha)$$
$$- (1/\sqrt{2})T_2'p\Phi_a I_{2g} \cos(90 + \phi_2 + \alpha) \text{ newtons} \quad . \quad (17.5)$$

Since $I_{2a}/I_{2g} = E_{2a}/E_{2g} = \Phi_a/\Phi_g$, $I_{2a} = I_{2g}\Phi_a/\Phi_g$

$$\text{Torque} = (1/\sqrt{2})T_2'p\Phi_a I_{2g}\{\cos(90 + \phi_2 - \alpha) - \cos(90 + \phi_2 + \alpha)\}$$
$$= (1/\sqrt{2})T_2'p\Phi_a I_{2g}(2\cos\phi_2 \sin\alpha)$$

But $$\Phi_a = \frac{E_{2a}}{(\sqrt{2})\pi f T_2'} = \frac{I_{2a}\sqrt{(r_2^2 + x_{L2}^2)}}{(\sqrt{2})\pi f T_2'}$$

so that, substituting for Φ_a and $\cos\phi_2$,

$$\text{Torque} = 2\frac{1}{\sqrt{2}}T_2'p\frac{I_{2a}\sqrt{(r_2^2 + x_{L2}^2)}}{(\sqrt{2})\pi T_2' f} \times I_{2g}\frac{r_2}{\sqrt{(r_2^2 + x_{L2}^2)}}\sin\alpha$$
$$= (1/2\pi n)2I_{2a}I_{2g}r_2 \sin\alpha \text{ newtons}$$

Referring all quantities to the main winding (by multiplying currents by (T_2'/T_1') and impedances by $(T_1'/T_2')^2$ and converting to synchronous watts—

$$\text{Torque} = 2I_a'I_g r_2' \sin\alpha \text{ synch. watts} \quad . \quad . \quad (17.6)$$

An alternative form of the expression can be obtained by expressing $\sin\alpha$ in terms of the motor impedances since, if ϕ_g and ϕ_a are the phase angles of the main- and starting-winding currents

$$\alpha = \phi_g - \phi_a$$

and $$\sin\alpha = \sin\phi_g \cos\phi_a - \cos\phi_g \sin\phi_a$$
$$= (x_g/z_g)(r_a/z_a) - (r_g/z_g)(x_a/z_a)$$

Hence

$$\text{torque} = 2I_a'I_g r_2'(x_g r_a - r_g x_a)/z_g z_a \text{ synch. watts}$$

In terms of V, the applied voltage, this becomes, since $I_g = V/z_g$ and $I_a = V/z_a$—

$$\text{Torque} = 2V^2(T_g/T_a)r_2'(x_g r_a - r_g x_a)/z_g^2 z_a^2 \text{ synch. watts}$$

Resistor Split-phase Motors

Of the split-phase motors mentioned on page 297 the resistor type is the cheapest but does not give such a good starting performance as the capacitor type. It is, however, usually quite adequate in sizes up to about 1/4 h.p., and its cheapness and simplicity can be accentuated by incorporating the necessary resistance of the starting winding in the winding itself, by making it of small section

wire, instead of by using an external resistor. In order to secure a reasonable running efficiency and to avoid overloading the starting winding a switch, either manual or automatic, must be included to cut the starting winding out of service as soon as the motor is up to speed.

The main winding is designed from the running characteristics required, particularly maximum torque; the problem is then to design a starting winding to operate in conjunction with this running winding to give the desired starting torque without excessive starting current. A convenient procedure is to assume a number of turns for

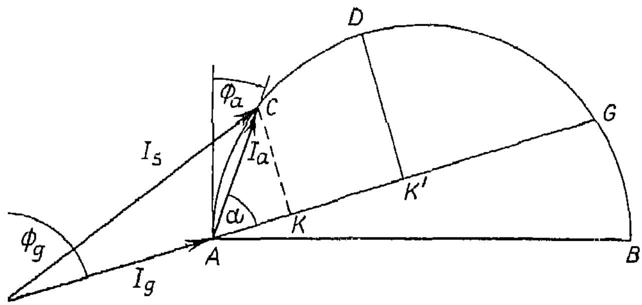


FIG. 17.13. LOCUS OF STARTING-WINDING CURRENT FOR RESISTOR SPLIT-PHASE MOTOR

the starting winding, calculate the best value of starting-winding resistance and then modify the number of turns to give the desired starting torque.

CHOICE OF STARTING-WINDING RESISTANCE. Referring to the complexor diagram of Fig. 17.13, I_g represents the starting or locked rotor current of the main winding. If the number of turns of the starting winding is assumed its reactance will be fixed at, say, x_a ohms so that if the starting-winding resistance is varied the locus of the starting-winding locked-rotor current I_a $(= AC)$ is a semicircle on AB as diameter, the length of AB being V/x_a amperes. The total starting current I_s is thus OC $(= \mathbf{I}_g + \mathbf{I}_a)$ which also follows the locus ACB.

Since I_g is constant, the starting torque is, from eq. (17.6), page 301, proportional to $I_a \sin \alpha$; from the geometry of Fig. 17.13 this is equal to CK, the angle CKA being 90°.

MAXIMUM STARTING TORQUE. It can be seen that the torque is a maximum when C is midway between A and G, i.e. at point D.

Under these maximum torque conditions, since $AD = DG$ and $AK' = GK'$, it can be shown that $\phi_a = \phi_g/2$.

Hence $\qquad \cot \phi_a = \cot (\phi_g/2)$

$$= (1 + \cos \phi_g)/\sin \phi_g$$

$$\therefore \qquad \frac{r_a}{x_a} = \frac{1 + r_g/z_g}{x_g/z_g}$$

$$= (r_g + z_g)/x_g$$

Hence the resistance to give maximum torque is

$$r_a = (x_a/x_g)(r_g + z_g) \quad . \quad . \quad . \quad (17.7)$$

Since reactance is proportional to the square of the number of turns, this may also be written

$$r_a = (T_a/T_g)^2(r_g + z_g)$$
$$= (1/k^2)(r_g + z_g) \quad . \quad . \quad . \quad (17.8)$$

where $k = T_g/T_a$.

The starting-winding impedance is $z_a = \sqrt{(r_a^2 + x_a^2)}$. Substituting for r_a from eq. (17.8) and $x_a = x_g/k^2$ gives, for the starting-winding current,

$$I_a = V/z_a = Vk^2/\sqrt{2z_g(r_g + z_g)} \quad . \quad . \quad (17.9)$$

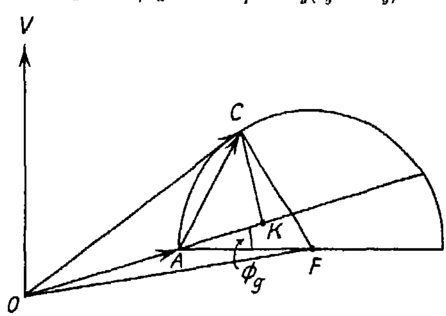


FIG. 17.14. CONDITIONS FOR MAXIMUM TORQUE PER AMPERE

For a given number of turns the resistance of the starting winding, the torque and the current can thus be calculated for a winding giving the maximum possible torque. By trying different numbers of turns an appropriate design can be selected. Curves have been published* which assist this procedure and which also enable magnetizing current to be taken into account.

An alternative criterion for starting behaviour is the *torque per ampere* of starting current. To make this a maximum, CK/OC (on Fig. 17.13) must be a maximum and this occurs when OC is a tangent to the circle as shown on Fig. 17.14. Point F is the centre

* T. C. Lloyd and J. H. Karr, "Design of Starting Windings for Split-phase Motors," *Trans. Amer. Instn. Elect. Engrs.*, **63**, p. 0, Jan. 1944.

of the semi-circle and triangle OCF is right-angled at C so that the line starting current I_s is given by—

$$I_s^2 = OC^2 = OF^2 - CF^2 = OF^2 - AF^2$$
$$= OA^2 + AF^2 + 2OA \cdot AF \sin\phi_g - AF^2$$
$$= I_g^2 + 2I_g(V/2x_a)(x_g/z_g)$$
$$= I_g^2(1 + x_g/x_a)$$

Hence $$x_a = x_g/\{(I_s/I_g)^2 - 1\} \quad . \quad . \quad . \quad (17.10)$$

Considering the main and starting windings in parallel

$$I_s = I_g(z_g + z_a)/z_a$$
$$(I_s^2/I_g^2) = (z_g + z_a)^2/z_a^2 = 1 + x_g/x_a$$

so that $$\{(r_g + r_a)^2 + (x_g + x_a)^2\}/(r_a^2 + x_a^2) = 1 + x_g/x_a$$

giving $$r_a = \frac{r_g x_a + z_g\sqrt{\{x_a(x_g + x_a)\}}}{x_g} \quad . \quad . \quad . \quad (17.11)$$

From x_a the number of turns in the starting winding can be found so that eqs. (17.10) and (17.11) above completely define the starting winding which will give the maximum starting torque per ampere for a given permissible total starting current I_s.

Capacitor Split-phase Motor

Improvements in capacitor design and construction are tending to make the capacitor split-phase or capacitor-start motor more effective than the resistor split-phase motor; although a little more expensive it gives a considerably higher starting torque and, due to the higher power factor, a lower starting current. In the capacitor-start motor the capacitor is switched out by a centrifugal switch when the motor is up to speed; there is, however, an alternative type of capacitor motor, described in Chapter 20, in which improved running characteristics are obtained by leaving capacitance permanently in the auxiliary-winding circuit.

CAPACITANCE FOR MAXIMUM TORQUE. The best value of capacitance for a given motor can be found by a method similar to that used in finding the resistance for the resistor split-phase motor. In the complexor diagram of Fig. 17.15 OA represents the locked-rotor main-winding current, I_g; assuming a given auxiliary-winding resistance, r_a, the locus of the locked-rotor auxiliary-winding current is the circle on AB as diameter, AB being V/r_a amperes. As with the resistor split-phase motor, the starting torque is a maximum when the perpendicular $I_a \sin\alpha = I_a \sin(\theta_a + \phi_g)$, from the circle on to OA, is a maximum, i.e. when I_a is at a point D such that DK passes through the centre of the circle. This condition is represented in Fig. 17.15.

From the geometry of the figure

$$\theta_a = (90 - \phi_g)/2$$

also

$$\begin{aligned}\tan \theta_a &= \sqrt{(1 - \cos 2\phi_a)/(1 + \cos 2\phi_a)} \\ &= \sqrt{(1 - \sin \phi_g)/(1 + \sin \phi_g)} \\ &= \sqrt{(z_g - x_g)/(z_g + x_g)} \\ &= r_g/(z_g + x_g)\end{aligned}$$

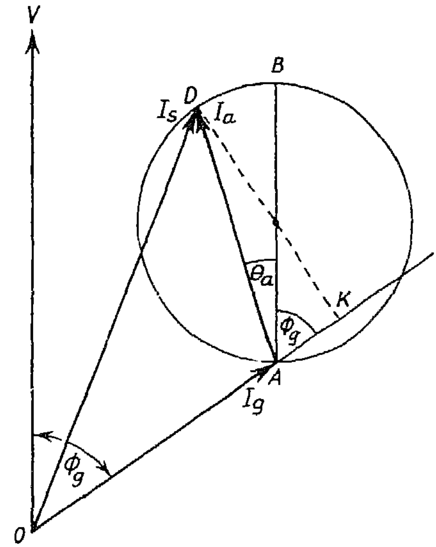


FIG. 17.15. COMPLEXOR DIAGRAMS FOR MAXIMUM STARTING TORQUE WITH CAPACITOR-START MOTOR

But

$$\tan \phi_a = X_a/r_a$$

where

$$X_a = x_a - x_c$$

and x_c is the reactance of the capacitor. Hence

$$x_a - x_c = r_a r_g/(z_g + x_g)$$

and

$$x_c = x_a - r_a r_g/(z_g + x_g)$$

This will always be negative indicating that the reactance will be capacitive.

CAPACITANCE FOR MAXIMUM TORQUE PER AMPERE. The maximum torque per ampere occurs when CK/OC in Fig. 17.16 is a maximum,

i.e. when OC, the line starting current, is a tangent to the circle as shown. From the figure—

$$I_s^2 = OC^2 = OF^2 - CF^2 = OF^2 - AF^2$$
$$= OA^2 + AF^2 + 2OA \,.\, AF \cos\phi_g - AF^2$$
$$= OA^2 + 2OA \,.\, AF \cos\phi_g$$
$$= I_g^2 + 2I_g(V/2r_a)(r_g/z_g)$$
$$= I_g^2(1 + r_g/r_a)$$

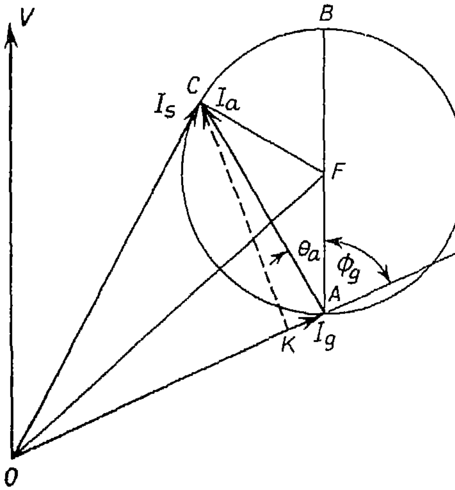


FIG. 17.16. COMPLEXOR DIAGRAM FOR MAXIMUM STARTING TORQUE PER AMPERE WITH CAPACITOR-START MOTOR

Hence the required resistance of the auxiliary winding can be found from

$$r_a = \frac{r_g}{(I_s^2/I_g^2) - 1}$$

Considering the main and starting-winding circuits in parallel—

$$I_s = I_g \,.\, \sqrt{\{r_g + r_a\}^2 + \{x_g + (x_a - x_c)\}^2}/z_a$$

Substituting for I_s/I_g in the above gives

$$x_a - x_c = \{x_g r_a - z_g\sqrt{r_a(r_g + r_a)}\}/r_g$$

from which x_c can be found.

The size of capacitor normally required varies from 20 to 30 μF for a 1/10 h.p. motor up to 60 to 100 μF for a 1/2 or 3/4 h.p. motor. A mica or paper capacitor of this value would be large and expensive

so that electrolytic capacitors are generally employed. Such capacitors are suitable for 20 to 30 starts per hour but not for continuous operation.

Reactor Split-phase Motor

A similar analysis can be made for the reactor split-sphase motor although this is rarely used. The reactance added to the main winding to give maximum starting torque is—

$$x = r_g r_a/(z_a + x_a) - x_g$$

and to give maximum starting torque per ampere—

$$x = \{x_a r_a + z_a\sqrt{r_a(r_g + r_a)}\}/r_g - x_g$$

Design Features

Many of the design features follow those of polyphase induction motors and single-phase commutator motors. In the fractional-horse-power size, however, cheapness of manufacture, e.g. by the use of mass-production methods, may have to outweigh considerations of good performance.

Specific Loadings. Flux densities must be kept low to minimize the magnetizing current and values between 0·35 to 0·55 Wb/m^2 are usual. The electric loading under running conditions is also low on account of the space occupied by the starting winding and usually lies between 5,000 and 15,000 A-conductors/m. Motors with repulsion starting, which have only a single stator winding, can have higher values, e.g. 10,000 to 20,000 A-conductors/m.

Full-load efficiency and power factor are between about 70 per cent and 0·65 respectively for a 1-h.p. motor and 50 per cent and 0·55 for a 1/10-h.p. motor.

The output coefficient can be found from—

$$G = (\text{h.p.} \times 746)/(D^2Ln) = (\pi^2/\sqrt{2})\hat{B}ac\eta \cos\phi \text{ volt-ampere units}$$

typical values being shown in Table 17.1.

Table 17.1

Values of Output Coefficient

Watts/r.p.m.	0·06	0·12	0·2	0·3
G volt-ampere units	9,400	12,000	15,600	18,000

Core length is generally made about equal to the pole-pitch but the precise dimensions are governed by manufacturing considerations.

Air Gap. The gap length is made as small as possible consistent with mechanical clearance between stator and rotor. It is thus

dependent on the core diameter and the peripheral speed. The following empirical formula gives satisfactory values—

$$\text{Gap length} = (0{\cdot}007 \times \text{rotor diameter})/\sqrt{(2p)}$$

Thus a 4-pole motor with a rotor of 6 cm diameter will have a gap length of $(0{\cdot}007 \times 6)/2 = 0{\cdot}021$ cm (0·35 per cent of the diameter).

Stator Winding. Single-layer concentric windings with enamelled wire and a 0·3 to 0·4 mm slot liner are generally employed. current densities are usually 3 and 4 A/mm^2; slots per pole are generally 9 or 12.

The arrangement of the winding is governed largely by the necessity of minimizing harmonic fluxes which may otherwise give rise to noise and an uneven accelerating torque; such harmonics arise from the non-sinusoidal shape of the m.m.f. wave (page 2) and from the presence of the slots. M.m.f.-wave harmonics can be reduced by using different numbers of conductors in the slots in order to give a more nearly sinusoidal m.m.f. wave as described in Chapter 1 (page 3). Tooth harmonics are not appreciably affected by the winding arrangement but can be minimized by proper choice of the number of stator and rotor slots and also by skewing the rotor slots through one stator slot pitch.

Rotor Winding. The cage rotor winding may be either of copper bars and end rings or of cast aluminium; technical advantages lie with copper but manufacture may be cheaper with aluminium and the joints between the bars and end rings are eliminated.

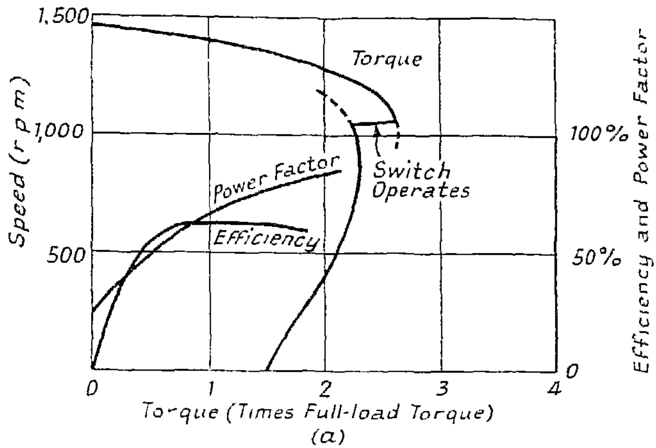
Semi-closed slots are generally preferred to tunnel slots since with the latter the leakage permeance is somewhat indeterminate and also surface losses are higher.

Many empirical rules have been suggested for choosing the best ratio of stator to rotor slots in order to give freedom from harmonic effects. One useful rule states that the number of rotor slots should equal the number of stator slots plus twice the number of poles; a 36 slot stator for a 4-pole machine would thus have 44 rotor slots and such a machine would give silent and satisfactory operation. Other combinations may, however, prove equally satisfactory, although the rotor rarely has more than 1·5 times as many slots as the stator.

Starting Winding. The number of slots available for the starting winding is usually about half that for the running winding, and the number of turns required is between 50 and 100 per cent of the running-winding turns. With a resistor split-phase motor the required resistance is usually obtained by using small-section wire, e.g. about 0·25 of that of the running winding. The current density at starting may thus be nearly 100 A/mm^2, but this is permissible as the winding is only in service for 1 or 2 sec. Similar considerations apply to the starting winding for a capacitor split-phase motor although the resistance should not be so high.

Typical Performance Characteristics

Characteristics for 1/4-h.p. resistor and capacitor split-phase motors are given in Fig. 17.17.



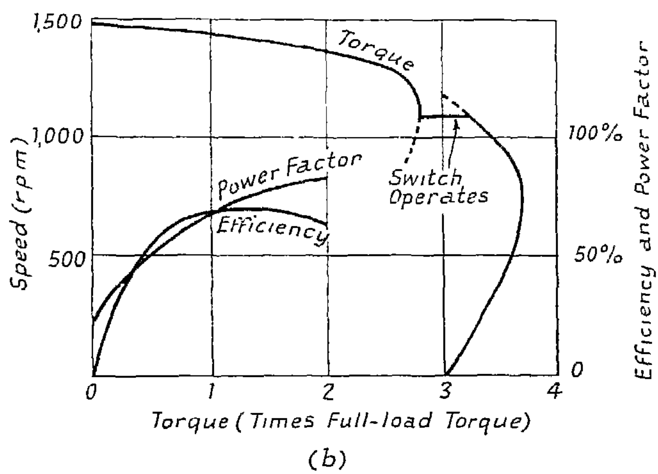


FIG. 17.17. TYPICAL CHARACTERISTICS OF SPLIT-PHASE MOTORS

(a) 1/4 h.p. resistor split-phase motor.
(b) 1/4 h.p. capacitor split-phase motor.

EXERCISES 17

1. A 2-pole, 50-c/s, 1-ph. induction motor has an effective rotor resistance and leakage reactance of 0·5 Ω and 5·0 Ω respectively. If the motor is running at 2,600 r.p.m. determine the frequencies of the rotor current components and the relative magnitude of the forward and backward fluxes. Neglect magnetizing currents and stator impedance.

2. Plot, using the rotating-field theory for a 50-c/s, 4-pole, 1-ph. induction motor having the constants given below, the torque-slip curves for the forward, backward and total torques and curves showing the component fluxes (in terms of e.m.f.) at various slips.

Applied voltage = 230 V
$r_1 = 2{\cdot}8\ \Omega$ $\quad x_1 = 3{\cdot}5\ \Omega$
$r_2' = 4{\cdot}6\ \Omega$ $\quad x_2' = 2{\cdot}4\ \Omega$
$x_m = 35{\cdot}5\ \Omega$

3. A 220-V, 1-ph. induction motor gave the following test results—

No-load: $V = 220$ V, $I = 6{\cdot}15$ A, $W = 348$ W
Locked rotor: $V = 126$ V, $I = 15{\cdot}0$ A, $W = 577$ W
Estimate the output and efficiency when running with a slip of 0·05.

4. A 1-ph., 4-pole, 50-c/s induction motor is running at 1,300 r.p.m. and the quadrature-axis flux is 0·1 Wb. The effective rotor turns are 20 and the effective stator turns are 100. The total effective impedance of the rotor is $4/\underline{80}\ \Omega$ under the above conditions and the leakage impedance is $0{\cdot}2/\underline{30}\ \Omega$. The stator resistance is 0·2 Ω and the stator leakage reactance is 1·0 Ω. Draw to scale the complexor diagram using the cross-field theory.

5. The data refer to a $\frac{1}{3}$-h.p., 110-V, 4-pole, 50-c/s, 1-ph. induction motor. Total standstill impedances (all values referred to main winding)—

Main winding: $r = 1{\cdot}47\ \Omega$ $\quad x = 3{\cdot}72\ \Omega$
Starting winding: $r = 4{\cdot}83\ \Omega$ $\quad x = 5{\cdot}14\ \Omega$
Rotor resistance = 0·5 Ω.
Calculate the resistance to be added to the starting winding in order to give (*a*) maximum starting torque and (*b*) maximum starting torque per ampere. Find, in each case, the starting torque and the starting torque per ampere.

6. A 230-V, 4-pole, 50-c/s split-phase induction motor has the following impedances at standstill.

Main winding: $r = 1{\cdot}5\ \Omega$ $\quad x = 4{\cdot}0\ \Omega$
Starting winding: $r = 2{\cdot}0\ \Omega$ $\quad x = 5{\cdot}0\ \Omega$

Draw a curve showing the relative starting torques for various values of (*a*) resistance and (*b*) capacitance in series with the starting winding.

CHAPTER 18

REPULSION-MOTOR STARTING

THE repulsion motor has, as explained in Chapter 15, a high starting torque for a given current, e.g. three times full-load torque with twice full-load current. Attempts have therefore been made to devise motors incorporating this feature together with the desirable constant-speed characteristic of the single-phase induction motor. As a result two types of motor have been found commercially practicable, i.e. the *repulsion-start* and the *repulsion-induction* motors.

Repulsion-start Motor

The motor is similar to an ordinary repulsion motor except that a centrifugal device short-circuits all the commutator segments

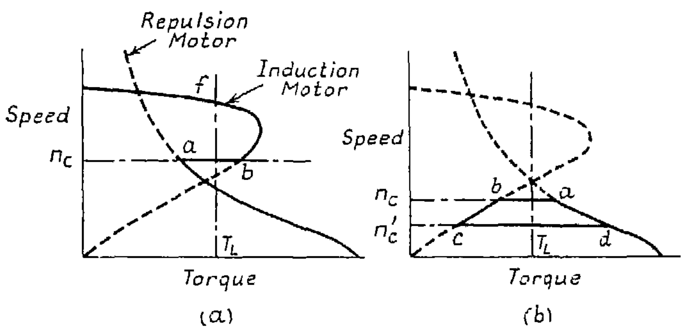


FIG. 18.1. ACTION OF REPULSION-START MOTOR
(*a*) Normal acceleration.
(*b*) Hunting.

when running above a predetermined speed, usually about two-thirds of synchronous speed; this short-circuits each coil of the winding and virtually converts the commutator winding into a cage winding, so that the motor starts as a repulsion motor with its high starting torque and runs as an induction motor with its constant-speed characteristic. Motors of this type can conveniently be built in sizes between about 1/8 and 5 h.p.

The speed-torque characteristics with and without the commutator segments short-circuited are shown in Fig. 18.1(*a*). At starting the motor runs up the repulsion-motor characteristic to the speed n_c at which the centrifugal device operates and changes conditions from point *a* on the repulsion-motor characteristic to point *b* on the

induction-motor characteristic; acceleration continues through the maximum-torque point to the running position at f. Unless the speed at which the centrifugal device operates happens to coincide with the speed at which the two curves cross there is inevitably a transient current and torque at the change-over.

If the load torque and the setting of the centrifugal device are such that after the change-over the motor torque is less than the load torque as shown in Fig. 18.1(*b*) a hunting phenomenon will occur. Since, at speed n_c, the load torque is greater than the motor torque, the speed will drop after the change-over and at a speed n_c'

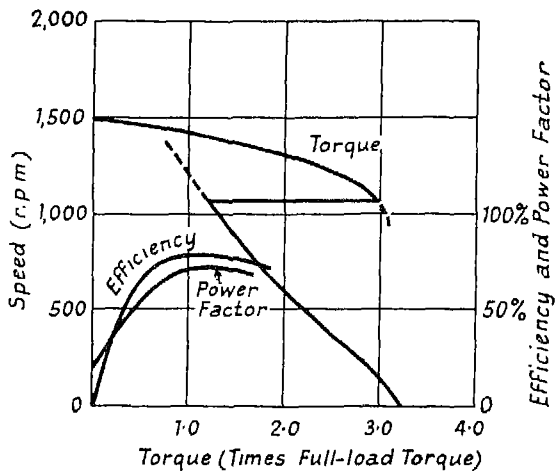


FIG. 18.2. TYPICAL CHARACTERISTICS OF 3-H.P., 4-POLE REPULSION-START MOTOR

the centrifugal device will operate to open the short-circuit and return conditions to repulsion-motor operation at point d. This cycle will continue and the motor will not run up to speed.

The design and performance of this motor are calculated as for the repulsion motor when considering starting conditions and as for the induction motor when considering running conditions. The resistance of the coil type of rotor winding is greater than that of the cage type of winding normally employed for single-phase induction motors so that the efficiency is slightly lower and the slip higher than usual for such motors. Brush friction may also add to the loss and introduce noise but this is usually eliminated by arranging for the centrifugal device to raise the brushes from the commutator simultaneously with the short-circuiting of the commutator segments. A typical set of characteristic curves is shown in Fig. 18.2.

Repulsion-induction Motor

The discontinuity in acceleration and the possibility of hunting that obtain with the repulsion-start motor can be avoided by fitting the rotor with both a commutator winding *and* a cage winding, the latter being in deep slots as shown in Fig. 18.3.

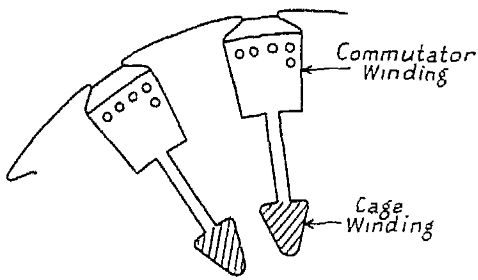


FIG. 18.3. ARRANGEMENT OF ROTOR SLOTS FOR REPULSION-INDUCTION MOTOR

On account of its relatively high permeance, the cage winding offers a high impedance at starting when the currents are at supply frequency; most current then flows in the commutator winding giving a high starting torque by repulsion-motor action. The

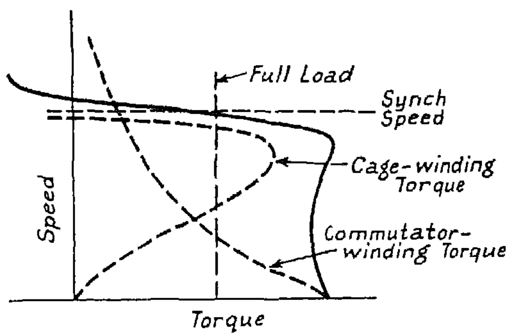


FIG. 18.4. SPEED-TORQUE CHARACTERISTIC OF REPULSION-INDUCTION MOTOR

resistance of the cage winding is, however, considerably lower than that of the commutator winding, so that, when running near synchronous speed, the current flows chiefly in the cage winding giving induction-motor running characteristics. The speed-torque curve is thus as shown in Fig. 18.4. At full load each winding contributes to the total torque, at synchronous speed (between half and full load) the cage winding contributes no torque, and at lower loads the cage winding has a braking effect and limits the speed to

2–3 per cent above synchronous speed. The acceleration of the motor is quite smooth and no centrifugal device is required; the brushes are, however, not raised from the commutator when running so that brush friction lowers the efficiency and may result in noise. Commutation is good as the cage winding acts somewhat as a discharge winding; the power factor is also improved by the action of the cage winding.

ANALYTICAL TREATMENT. The cross-field theory can conveniently be used to investigate the behaviour. Using the symbols of Fig. 18.5

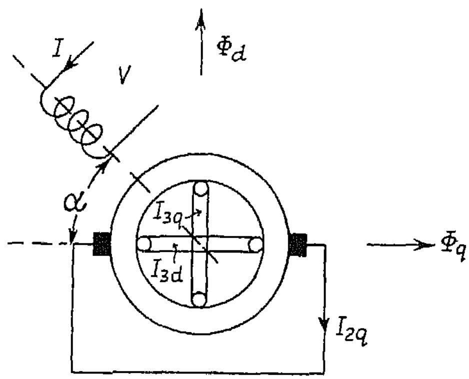


FIG. 18.5. SCHEMATIC DIAGRAM OF REPULSION-INDUCTION MOTOR

the equations of the repulsion motor and the induction motor can be combined to give the following expressions. For simplicity the coils short-circuited by the brushes and the rotational e.m.f.'s due to the leakage flux are omitted.

Stator winding—

$$V = r_1 I_1 + jx_{L1} I_1 + jx_m \sin\alpha(I_1 \sin\alpha + I_{3d}) + jx_m \cos\alpha(I_1 \cos\alpha + I_{2q} + I_{3q})$$

Commutator winding in quadrature axis—

$$0 = r_2 I_{2q} + jx_{L2}(I_{2q} + I_{3d}) + jx_m(I_1 \cos\alpha + I_{2q} + I_{3q}) - Sx_m(I_1 \sin\alpha + I_{3d})$$

Cage winding in direct axis—

$$0 = r_3 I_{3d} + jx_{L3} I_{3d} + jx_m(I_1 \sin\alpha + I_{3d}) + Sx_m(I_1 \cos\alpha + I_{2q} + I_{3q})$$

Cage winding in quadrature axis—

$$0 = r_3 I_{3q} + jx_{L3} I_{3q} + jx_m(I_1 \cos\alpha + I_{2q} + I_{3q}) - Sx_m(I_1 \sin\alpha + I_{3d})$$

The solution of these equations is laborious but has been fully done by West.*

DESIGN FEATURES. The shape of the characteristics is sensitive to variations in the parameters, particularly those of the cage winding. Raising the cage-winding leakage reactance increases the starting torque per ampere, reduces the maximum torque and reduces the efficiency; raising the cage-winding resistance reduces efficiency, may cause a high no-load speed and reduces the re-entrancy of the speed-torque curve.

The speed tends to rise steeply near the zero-torque ordinate as shown in Fig. 18.4; increasing the brush shift away from the stator-

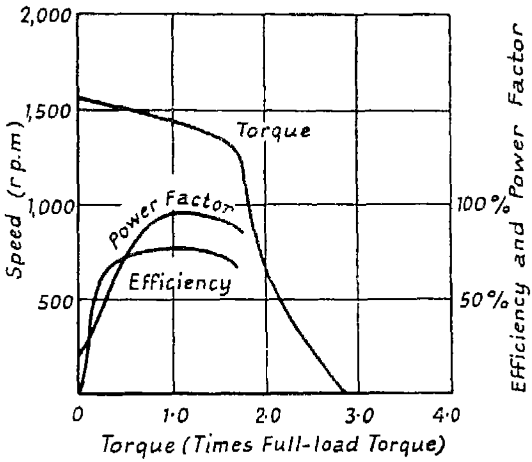


FIG. 18.6. TYPICAL CHARACTERISTICS OF 3-H.P., 4-POLE REPULSION-INDUCTION MOTOR

winding axis moves the speed-torque curve to the left, thus putting the rise in speed well beyond the no-load point and preventing racing at no-load. For this reason the brush shift α is usually greater than in a plain repulsion motor.

Typical characteristics for the repulsion motor are shown in Fig. 18.6.

* H. R. West, "Theory and Calculation of the Squirrel-cage Repulsion Motor," *Trans. Amer. Inst. Elect. Engrs.*, **43** (1924).

CHAPTER 19

THE SHADED-POLE MOTOR

Principle

It is well known that if part of a pole carrying an alternating flux is encircled by a short-circuited coil as shown in Fig. 19.1, the flux, Φ_s, within the coil lags the remaining flux, Φ_d, by a considerable

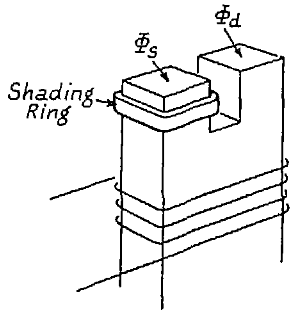


FIG. 19.1. SHADED POLE

angle. The short-circuited coil is commonly known as a *shading ring* and usually consists of a single turn of relatively heavy copper wire or strip.

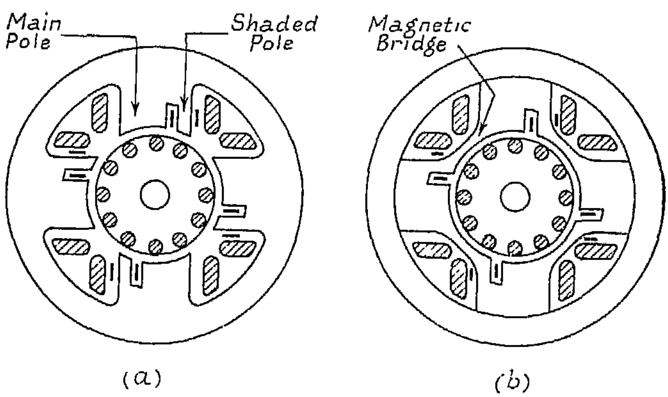


FIG. 19.2. TYPICAL FOUR-POLE ARRANGEMENTS

If a single-phase induction motor is built on this principle as shown in Fig. 19.2(a), each complete pole will set up two fluxes, Φ_d and Φ_s, displaced in time by an angle α which may be 30°–40°

and displaced in space by an angle which is usually nearly 90°. These flux conditions, being similar to those of other types of split-phase motor, make the shaded-pole motor self-starting, the rotor conductors tending to move from the unshaded towards the shaded part of the pole. The starting torque is usually not more than 30–50 per cent of the full-load torque.

It is not practicable to provide facilities for open-circuiting the shading ring when the motor is up to speed, so that during normal operation it will continuously carry currents, resulting in a low efficiency (20–50 per cent); this type of motor is therefore suitable only for outputs of less than about 1/10 h.p., and is not commonly used above 1/100 h.p. In such sizes it is suitable for driving domestic fans, gramophone turntables and other small appliances such as valves and regulators. It has the merits of cheapness, simplicity and freedom from radio interference.

Constructional Details

In Fig. 19.2 are shown typical arrangements of the stator stampings. With arrangement (*a*) the main winding is of the usual former-wound type slipped over the pole-piece. Improved characteristics can be obtained if the pole leakage flux is increased by magnetic wedges between the pole tips or by the construction shown at (*b*) in which the stampings are made in two parts. The rotor is of normal cage construction with copper or cast aluminium bars with considerable skew in order to minimize the effect of space harmonics due to the concentrated stator winding.

Analytical Treatment

By making certain simplifying assumptions, equations can be written down according to the cross-field theory. The assumptions are: sinusoidal flux distributions, no magnetic bridge between pole tips and an angle of 90° in both space and time between the two flux components. Conditions are thus as shown in Fig. 19.3; it can be seen that the main primary coil is linked with both components of flux so that the circuits can be represented as in Fig. 19.3(*b*). Summing the e.m.f.'s in the four circuits gives the following equations in which $r_1 + jx_{L1}$, $r_s + jx_{Ls}$, and $r_2 + jx_{L2}$ are the resistances and leakage reactances of the main primary, shading ring and secondary windings respectively, all referred to the primary winding.

Main primary winding—

$$V = (r_1 + jx_{L1})I_1 + jx_{md}(I_1 + I_{2d}) + jx_{mq}(I_1 + I_s + I_{2s}) \quad (19.1)$$

Shading ring—

$$0 = (r_s + jx_{Ls})I_s + jx_{mq}(I_1 + I_s + I_{2s}) \quad . \quad (19.2)$$

Rotor winding in main (direct) axis—

$$0 = (r_2 + jx_{L2})I_{2d} + jx_{md}(I_1 + I_{2d}) + Sx_{mq}(I_1 + I_s + I_{2s}) \quad (19.3)$$

Rotor winding in shading (quadrature) axis—

$$0 = (r_2 + jx_{L2})I_{2s} + jx_{mq}(I_1 + I_s + I_{2s}) - Sx_{md}(I_1 + I_{2d}) \quad (19.4)$$

The complete solution of these equations is laborious but for *starting conditions*, which are commonly the most important, the behaviour can be found by putting $S = 0$.

From eq. (19.3)

$$I_{2d} = -I_1 \frac{jx_{md}}{r_2 + j(x_{L2} + x_{md})} \quad . \quad . \quad (19.5)$$

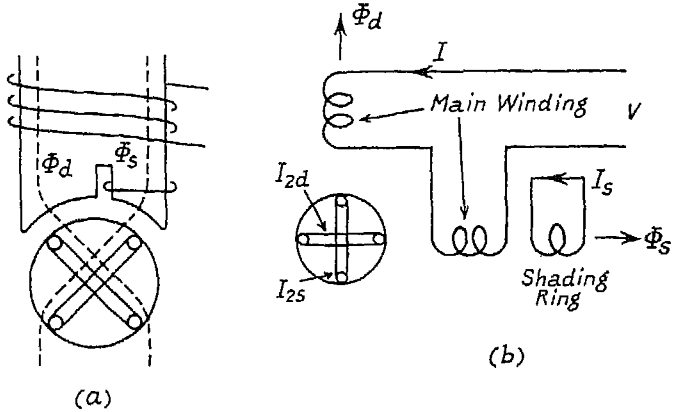


FIG. 19.3. CIRCUIT DIAGRAM

From eq. (19.4)

$$I_{2s} = -(I_1 + I_s) \frac{jx_{mq}}{r_2 + j(x_{L2} + x_{mq})} \quad . \quad . \quad (19.6)$$

From eq. (19.2)

$$0 = (r_s + jx_{Ls})I_s + jx_{mq}(I_1 + I_s) \left\{1 - \frac{jx_{mq}}{r_2 + j(x_{L2} + x_{mq})}\right\}$$

$$\therefore \quad I_s = -I_1 \frac{jx_{mq}\left\{1 - \dfrac{jx_{mq}}{r_2 + j(x_{L2} + x_{mq})}\right\}}{r_s + jx_{Ls} + jx_{mq}\left\{1 - \dfrac{jx_{mq}}{r_2 + j(x_{L2} + x_{mq})}\right\}} \quad . \quad (19.7)$$

All currents can now be calculated in terms of I and substituted in eq. (19.1); inserting the value of applied voltage V then enables I to be calculated.

The starting torque can be found from eq. (17.5) on page 301 which may be written—

$$\text{Torque} = (1/\sqrt{2})T_1 p\{\Phi_d \operatorname{conj} I_{2s} - \Phi_q \operatorname{conj} I_{2d}\}_{\text{real terms}}$$
$$\text{newton-metres} \quad . \quad (19.8)$$

Since $\Phi_d = \frac{(\sqrt{2})x_{md}}{\omega T_1}(I_1 + I_{2d})$ and $\Phi_q = \frac{(\sqrt{2})x_{mq}}{\omega T_1}(I_1 + I_s - I_{2s})$ this becomes

$$TM = (p/2\pi f)\{x_{md}(I_1 + I_{2d}) \operatorname{conj} I_{2s} - x_{mq}(I_1 + I_s + I_{2s}) \operatorname{conj} I_{2d}\}_{\text{real terms}} \quad . \quad (19.8b)$$

A more detailed analysis, based on the above, has been given by Trickey* and an analysis using matrix methods has been given by Kron.†

Example 19.1. A 1/100-h.p., 4-pole, shaded-pole motor has the following constants—

$r_1 = 47{\cdot}4\ \Omega$	$x_{L1} = \mathrm{j}20{\cdot}6\ \Omega$	$x_{md} = \mathrm{j}277\ \Omega$
$r_s = 90{\cdot}5\ \Omega$	$x_{Ls} = \mathrm{j}7{\cdot}23\ \Omega$	$x_{mq} = \mathrm{j}139\ \Omega$
$r_2 = 39{\cdot}0\ \Omega$	$x_{L2} = \mathrm{j}52{\cdot}9\ \Omega$	Primary turns = 3,000

Estimate the currents, fluxes and torque at starting, and draw the complexor diagram.

From eq. (19.5)

$$I_{2d} = -I_1\,\mathrm{j}277/\{39 + \mathrm{j}(52{\cdot}9 + 277)\} = -I_1 \times 0{\cdot}835\underline{/6{\cdot}7^\circ}$$

From eq. (19.6)

$$I_{2s} = -(I_1 + I_s)\mathrm{j}139/\{39 + \mathrm{j}(52{\cdot}9 + 139)\}$$
$$= -(I_1 + I_s)0{\cdot}714\underline{/11{\cdot}5^\circ}$$

From eq. (19.7)

$$I_s = -I_1 \frac{\mathrm{j}139\left\{1 - \dfrac{\mathrm{j}139}{39 + \mathrm{j}(52{\cdot}9 + 139)}\right\}}{90{\cdot}5 + \mathrm{j}7{\cdot}23 + \mathrm{j}139\left\{1 - \dfrac{\mathrm{j}139}{39 + \mathrm{j}(52{\cdot}9 + 139)}\right\}}$$
$$= -I_1 \times 0{\cdot}385\underline{/40{\cdot}8^\circ}$$

Hence

$$I_1 + I_{2d} = I_1(1 - 0{\cdot}835\underline{/6{\cdot}7^\circ}) = I_1 \times 0{\cdot}196\underline{/-29{\cdot}7^\circ}$$

and

$$I_1 + I_s + I_{2s} = (I_1 + I_s)(1 - 0{\cdot}714\underline{/11{\cdot}5^\circ}) = I_1 \times 0{\cdot}251\underline{/-42{\cdot}8^\circ}$$

Substituting in eq. (19.1),

$$230 = (47{\cdot}4 + \mathrm{j}20{\cdot}6)I_1 + \mathrm{j}277 \times 0{\cdot}196\underline{/-29{\cdot}7^\circ}\,I_1 + \mathrm{j}139 \times 0{\cdot}251\underline{/-42{\cdot}8^\circ}\,I_1$$
$$= I_1 \times 134\underline{/43{\cdot}5^\circ}$$

* P. H. Trickey, "An Analysis of the Shaded-pole Motor," *Trans. Amer. Instn. Elect. Engrs.*, **55**, p. 1007 (1935).
P. H. Trickey, "Performance Calculations for the Shaded-pole Motor," *Trans. Amer. Instn. Elect. Engrs.*, **66**, p. 1431 (1947).
† G. Kron, "Equivalent Circuits of the Shaded-pole Motor with Space Harmonics," *Trans. Amer. Instn. Elect. Engrs.*, **69** (1950).

$$\therefore \quad I_1 = 230/(134\underline{/43{\cdot}5^\circ}) = 1{\cdot}72\underline{/-43{\cdot}5} \text{ amperes}$$

$$I_{2d} = -1{\cdot}72\underline{/-43{\cdot}5^\circ} \times 0{\cdot}835\underline{/6{\cdot}7^\circ} = -1{\cdot}43\underline{/-36{\cdot}8^\circ}$$

$$I_s = -1{\cdot}72\underline{/-43{\cdot}5^\circ} \times 0{\cdot}385\underline{/40{\cdot}8^\circ} = -0{\cdot}665\underline{/-2{\cdot}7^\circ}$$

$$I_{2s} = -(I_1 - 0{\cdot}385\underline{/40{\cdot}8^\circ}\, I_1)0{\cdot}714\underline{/11{\cdot}5^\circ}$$

$$= -0{\cdot}93\underline{/-51{\cdot}4^\circ}$$

$$I_1 + I_{2d} = 1{\cdot}72\underline{/-43{\cdot}5^\circ} \times 0{\cdot}196\underline{/-29{\cdot}7^\circ} = 0{\cdot}338\underline{/-73{\cdot}2^\circ}$$

$$I_1 + I_s + I_{2s} = 1{\cdot}72\underline{/-43{\cdot}5^\circ} \times 0{\cdot}251\underline{/-42{\cdot}8^\circ} = 0{\cdot}431\underline{/-86{\cdot}3^\circ}$$

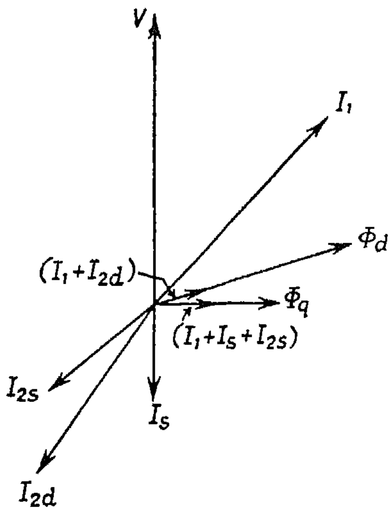


FIG. 19.4. COMPLEXOR DIAGRAM (EXAMPLE 19.1)

The two component fluxes are thus

$$\Phi_d = (\sqrt{2})277/(2\pi \times 50 \times 3{,}000) \times 0{\cdot}338\underline{/-73{\cdot}2^\circ}$$

$$= 1{\cdot}41 \times 10^{-5}\underline{/73{\cdot}2^\circ} \text{ Wb}$$

$$\Phi_s = (\sqrt{2})139/(2\pi \times 50 \times 3{,}000) \times 0{\cdot}431\underline{/-86{\cdot}3^\circ}$$

$$= 0{\cdot}9 \times 10^{-5}\underline{/-86{\cdot}3^\circ} \text{ Wb}$$

The complexor diagram of these quantities is thus as shown in Fig. 19.4.

From eq. (19.8) the starting torque is

$$TM = 3{,}000\sqrt{2}\{1{\cdot}41 \times 10^{-5}\underline{/-73{\cdot}2^\circ} \times (-0{\cdot}93\underline{/51{\cdot}4^\circ})$$

$$- 0{\cdot}9 \times 10^{-5}\underline{/-86{\cdot}3^\circ} \times (-1{\cdot}43\underline{/36{\cdot}8})\}_{\text{real}}$$

$$= 0{\cdot}0136 \text{ N-m } (140 \text{ g-cm})$$

This is only about 0·3 of the full-load torque.

Effect of Magnetic Bridge

The purpose of the magnetic bridge is to increase the phase angle between the two fluxes Φ_d and Φ_s in order to improve the starting torque.

With no bridge as in Fig. 19.5(*a*) the flux entering the rotor opposite the shaded pole includes the leakage flux Φ_{Ls}. This flux, although small, is in phase with the shading-ring current and lags Φ_d by about 90°, thereby tending to increase the lag of the total shaded flux. If a bridge is fitted as in Fig. 19.5(*b*), it affords an easy

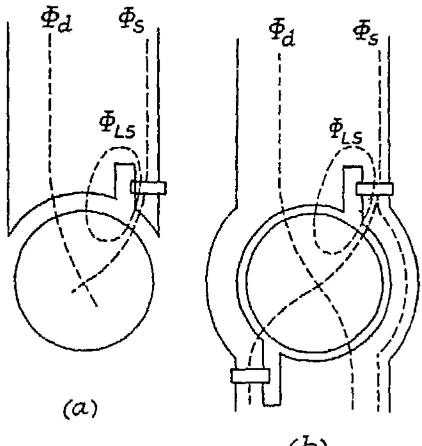


FIG. 19.5. FLUX DISTRIBUTION WITH MAGNETIC BRIDGE
(*a*) No bridge.
(*b*) With bridge.

path for the flux through the shading ring and therefore increases its total magnitude; the shading-ring current is correspondingly increased so that the majority of the flux crossing the gap is leakage flux, and this becomes the dominating factor in controlling the angle α and enables it to approach 90°.

Design Features

On account of the complication of any accurate analytical treatment design must largely be done by trial and error. The effect of making various changes on the starting characteristics is illustrated in Fig. 19.6. At (*a*) is shown the effect of rotor resistance; it is desirable to work near the maximum point of this curve since, even so, the starting torque is less than full-load torque. A rotor designed with proportions similar to those of an ordinary fractional-horsepower motor usually fulfils this requirement. The effect of shading-ring resistance is shown in (*b*)—a low resistance results in excessive losses and poor running characteristics so that it is generally

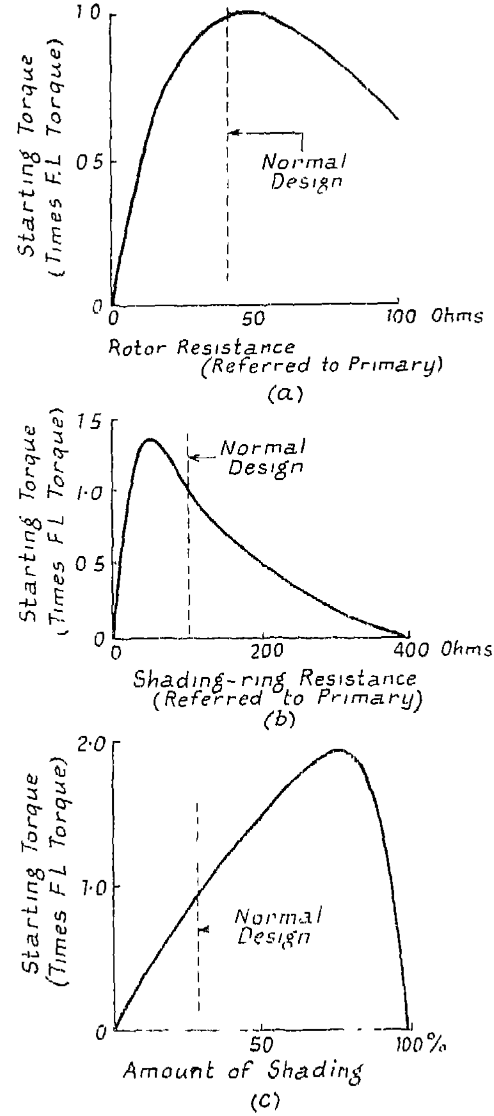


FIG. 10.6. EFFECT OF VARYING PARAMETERS ON STARTING TORQUE

(a) Effect of rotor resistance.
(b) Effect of shading-ring resistance.
(c) Effect of amount of shading.

desirable to use a higher resistance than that corresponding to maximum starting torque. The amount of shading is also a compromise between good starting and good running characteristics, and about 1/3 of the pole is usually shaded as shown in (c).

The length of the air gap is generally between 0·01 and 0·02 in.: with lower values there is a tendency for considerable variations of starting torque at different rotor positions due to the effect of the rotor slots.

Typical Characteristics

A set of typical characteristics is given in Fig. 19.7 from which it can be seen that the efficiency is low: with the small sizes in which

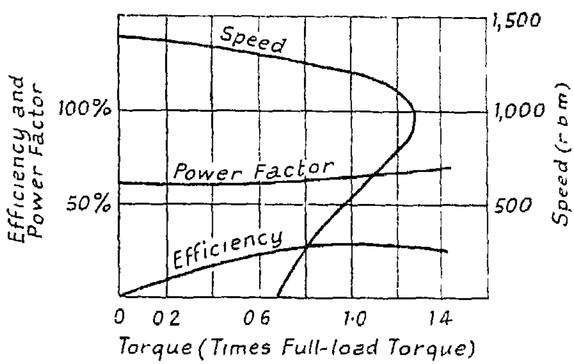


Fig. 19.7. Typical Characteristics of 1/100 h.p., 4-pole Motor

this type of motor is built this is of little importance compared with the starting torque and commercial considerations affecting production costs. The smaller motors used for gramophones have efficiencies between 4 and 6 per cent.

Starting torque is usually between 0·4 and 0·9 of full-load torque and the maximum torque between 1·1 and 1·3 times full-load torque.

CHAPTER 20

THE CAPACITOR MOTOR

THE capacitor-start motor in which a capacitor is used to give a split-phase effect for starting has been described in Chapter 17 (page 304). Here is discussed the motor in which a capacitor is left in circuit continuously so that the motor operates as a modified form of two-phase motor; motors of this type are sometimes referred to as *capacitor-start-and-run* motors and have the advantage over

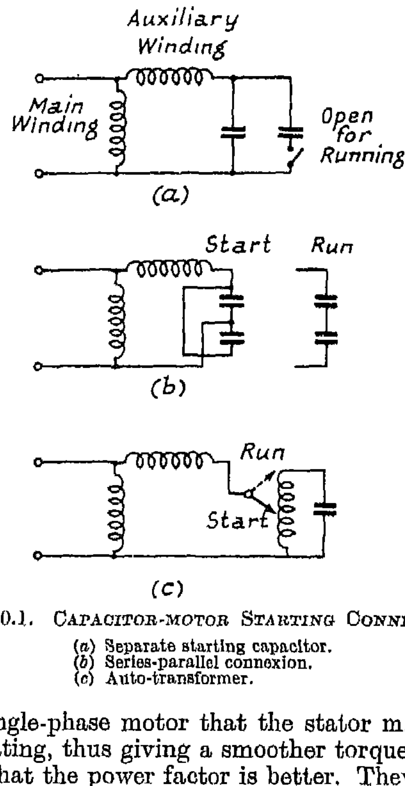


FIG. 20.1. CAPACITOR-MOTOR STARTING CONNEXIONS

(a) Separate starting capacitor.
(b) Series-parallel connexion.
(c) Auto-transformer.

the ordinary single-phase motor that the stator m.m.f. is rotating instead of pulsating, thus giving a smoother torque and less vibration, and also that the power factor is better. They are commonly built in sizes between 1/4 h.p. and 5 h.p.

Capacitor Connexions

The size of capacitor required to give satisfactory running characteristics is about one-quarter to one-third of that required to give the best starting torque; also a mica or paper capacitor is desirable

for continuous operation, whereas an electrolytic capacitor can be used for the intermittent starting duty.

Some inexpensive fractional-horse-power motors designed for a low starting torque are built with a single capacitor serving both purposes but, in general, two separate capacitance values are required with a centrifugally-operated switch for changing the connexions when the motor is up to speed.

Typical methods of effecting this change in capacitance are shown in Fig. 20.1. In the simple arrangement of (*a*) the starting capacitor is switched out by the centrifugal device; at (*b*) the capacitors are connected in parallel at starting and in series for running, giving a

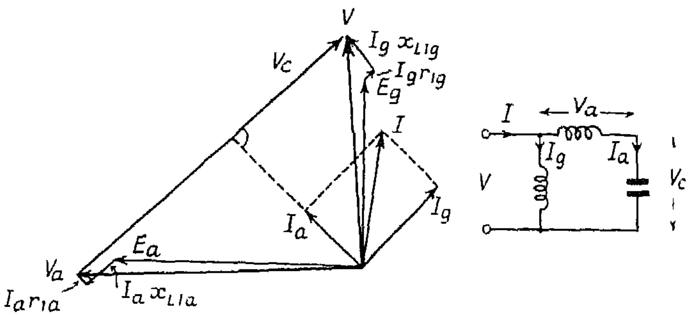


FIG. 20.2. COMPLEXOR DIAGRAM AT FULL-LOAD

4:1 capacitance ratio; at (*c*) a step-up auto-transformer is used to increase the effective capacitance at starting; the capacitor can withstand the higher voltage for the short starting period so that a smaller capacitor can be used and also switching in the capacitor circuit is avoided.

With motors above about 1 h.p. it is often necessary to reduce the starting current by a resistor in the stator circuit, by additional starting turns on the main winding or by using a slip-ring rotor with rotor-resistance starting.

As explained later, it is usual to design capacitor motors so that the back e.m.f.'s, E_g and E_a, of the main and auxiliary windings are equal and 90° displaced in phase so that the complexor diagram is of the form shown in Fig. 20.2. It can be seen that the voltage, V_c, across the capacitor is about $\sqrt{2}$ times the line voltage although higher values of E_a and V_c, with a correspondingly smaller capacitor, may result in a cheaper arrangement.

Analytical Treatment

As in the plain induction motor either the rotating-field or the cross-field theory can be used; with the same initial assumptions both give, of course, similar results and calculations are equally laborious in both cases.

ROTATING-FIELD THEORY. The component fluxes produced by the main and auxiliary windings may be considered to act separately and each may be resolved into forward and backward components as for the plain single-phase induction motor described on page 283. In Fig. 20.3 are drawn equivalent circuits for the two windings

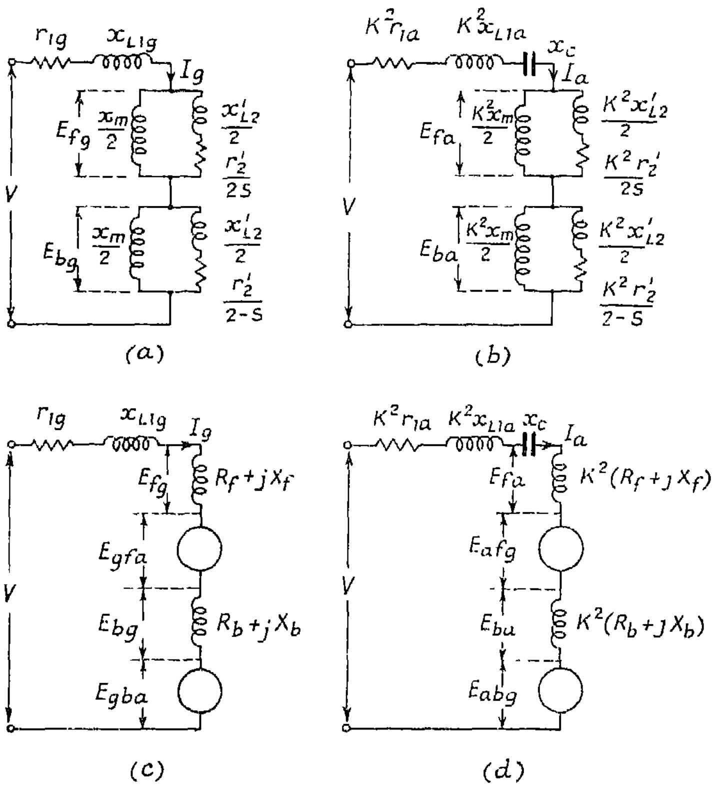


FIG. 20.3. EQUIVALENT CIRCUITS FOR ROTATING-FIELD THEORY

(a) Main winding.
(b) Auxiliary winding.
(c) Main winding.
(d) Auxiliary winding.

similar to that of Fig. 17.4. At (a) and (b) it is assumed that each winding acts quite independently of the other; rotor quantities are all referred to their respective stator windings and with the auxiliary-winding parameters the turns ratio K has been introduced to refer them to the main stator winding, K being the ratio (auxiliary-winding effective turns)/(main-winding effective turns) and usually having a value between 1 and 1·5. The auxiliary-winding circuit also includes the capacitor reactance x_c.

When the windings are considered together, in addition to the e.m.f.'s shown at (*a*) and (*b*), there will be further e.m.f.'s induced in one winding by the forward and backward flux components due to the other. The e.m.f. induced in the main winding by the forward component of the auxiliary-winding flux is E_{gfa} and lags by 90° the e.m.f. E_{fa} produced by the same flux component in the auxiliary winding so that

$$E_{gfa} = -\mathrm{j}E_{fa}/K$$

For simplicity, the two parallel branches of the circuits at (*a*) and (*b*) can be replaced by equivalent impedances $(R_f + \mathrm{j}X_f)$ and $(R_b + \mathrm{j}X_b)$ so that

$$E_{gfa} = -\mathrm{j}KI_a(R_f + \mathrm{j}X_f)$$

Similarly the e.m.f. induced in the main winding by the backward component of the auxiliary flux is

$$E_{gba} = \mathrm{j}E_{ba}/K = \mathrm{j}KI_a(R_b + \mathrm{j}X_b)$$

For the e.m.f.'s induced in the auxiliary winding by the main-winding flux—

$$E_{afg} = \mathrm{j}KE_{fg} = \mathrm{j}KI_g(R_f + \mathrm{j}X_f)$$
$$E_{abg} = -\mathrm{j}KE_{bg} = -\mathrm{j}KI_g(R_b + \mathrm{j}X_b)$$

The equivalent circuits thus become as shown in Fig. 20.3(*c*) and (*d*), and the following equations can be written down—

$$\begin{aligned} V &= I_g(r_{1g} + \mathrm{j}x_{L1g}) + E_{fg} + E_{gfa} + E_{bg} + E_{gba} \\ &= I_g(r_{1g} + \mathrm{j}x_{L1g} + R_f + \mathrm{j}X_f + R_b + \mathrm{j}X_b) \\ &\qquad + \mathrm{j}KI_a(R_b + \mathrm{j}X_b - R_f - \mathrm{j}X_f) \end{aligned}$$

$$\begin{aligned} V &= I_a\{K^2(r_{1a} + \mathrm{j}x_{L1a}) - \mathrm{j}x_c\} + E_{fa} + E_{afg} + E_{ba} + E_{abg} \\ &= I_a\{K^2(r_{1a} + \mathrm{j}x_{L1a}) - \mathrm{j}x_c + K^2(R_f + \mathrm{j}X_f + R_b + \mathrm{j}X_b)\} \\ &\qquad + \mathrm{j}KI_g(R_f + \mathrm{j}X_f - R_b - \mathrm{j}X_b) \end{aligned}$$

Solving these two equations gives—

$$I_g = V\frac{K^2(r_{1a} + R_f + R_b) + \mathrm{j}\{K^2(x_{L1a} + X_f + X_b) - x_c\} + \mathrm{j}K\{(R_f - R_b) + \mathrm{j}(X_f - X_b)\}}{Q}$$

$$I_a = V\frac{r_{1g} + R_f + R_b + \mathrm{j}(x_{1g} + X_f + X_b) - \mathrm{j}K\{(R_f - R_b) + \mathrm{j}(X_f - X_b)\}}{Q}$$

where

$$\begin{aligned} Q = {} & [K^2(r_{1a} + R_f + R_b) + \mathrm{j}\{K^2(x_{L1a} + X_f + X_b) - x_c\}] \\ & \times [(r_{1g} + R_f + R_b) + \mathrm{j}(x_{L1g} + X_f + X_b) \\ & \qquad - K^2\{(R_f - R_b) + \mathrm{j}(X_f - X_b)\}] \end{aligned}$$

If these two currents are written $I_g = A + jB$ and $I_a = C + jD$ the average torque can be shown to be

$$TM = (I_g^2 + K^2 I_a^2)(R_f - R_b) + 2K(AD - BC)(R_f + R_b)$$

synch. watts

Iron loss has not been included in the above but may be allowed for by adding the appropriate resistor to the magnetizing branch in the initial equivalent circuit or by subtracting the loss from the gross output of the motor; the former method adds considerably to the labour.

CROSS-FIELD THEORY. Using the circuit diagram of Fig. 20.4 the following equations can be written down, as on page 217, for each of the four circuits, all quantities being referred to the main stator winding.

Stator winding in direct axis (auxiliary winding)—

$$V_{1d} = I_{1d}(r_{1d} + jx_{L1d} - jx_c) + jx_{md}(I_{1d} + I_{2d})$$

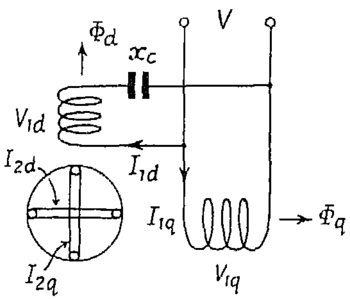


FIG. 20.4. CIRCUIT DIAGRAM FOR CROSS-FIELD THEORY

Stator winding in quadrature axis—

$$V_{1q} = I_{1q}(r_{1q} + jx_{L1q}) + jx_{mq}(I_{1q} + I_{2q})$$

Rotor winding in direct axis—

$$0 = I_{2d}(r_{2d} + jx_{L2d}) + jx_{md}(I_{1d} + I_{2d}) + Sx_{mq}(I_{1q} + I_{2q}) + Sx_{L2q}I_{2q}$$

Rotor winding in quadrature axis—

$$0 = I_{2q}(r_{2q} + jx_{L2q}) + jx_{mq}(I_{1q} + I_{2q}) - Sx_{md}(I_{1d} + I_{2d}) - Sx_{L2d}I_{2d}$$

For a normal motor some simplifications of the symbols can be made since $V_{1d} = V_{1q} = V$, the applied voltage; $x_{md} = x_{mq} = x_m$, the mutual reactance; $r_{2d} = r_{2q} = r_2$, the rotor resistance; $x_{L2d} = x_{L2q} = x_{L2}$, the rotor leakage reactance; suffix g is used as before

for the main axis and suffix a for the auxiliary axis. The equations thus become—

$$V = I_a(r_{1a} + jx_{L1a} - jx_c) + jx_m(I_a + I_{2a})$$
$$V = I_g(r_{1g} + jx_{L1g}) + jx_m(I_g + I_{2g})$$
$$0 = I_{2a}(r_2 + jx_{L2}) + jx_m(I_a + I_{2a}) + Sx_m(I_g + I_{2g}) + Sx_{L2}I_{2g}$$
$$0 = I_{2g}(r_2 + jx_{L2}) + jx_m(I_g + I_{2g}) - Sx_m(I_a + I_{2a}) - Sx_{L2}I_{2a}$$

Solving these gives

$$I_g = V(DF - CG)/(AD - BC)$$

and

$$I_a = V(AG - BF)/(AD - BC)$$

where

$$A = \{r_{1g} + r_2 + (r_{1g}x_{L2} + r_2x_{L1g})/x_m\} + j\{x_{L1g} + x_{L2} + (x_{L1g}x_{L2} - r_{1g}r_2)/x_m\}$$
$$B = S\{x_{L2} + x_{L1g}(1 + x_{L2}/x_m)\} - jSr_{1g}(1 + x_{L2}/x_m)$$
$$C = -S\{x_{L2} + (x_{L1a} - x_c)(1 + x_{L2}/x_m)\} + jSr_{1a}(1 + x_{L2}/x_m)$$
$$D = r_2\{1 + (x_{L2} - x_c)/x_m\} + r_{1a}(1 + x_{L2}/x_m) + j\{x_{L2} + (x_{L1a} - x_c)(1 + x_{L2}/x_m) - r_2r_{1a}/x_m\}$$
$$F = (1 + x_{L2}/x_m) + j\{S(1 + x_{L2}/x_m) - r_2/x_m\}$$
$$G = (1 + x_{L2}/x_m) - j\{r_2/x_m + S(1 + x_{L2}/x_m)\}$$

For the rotor currents—

$$I_{2g} = -I_g(1 + x_{L1g}/x_m) + j(V - I_gr_{1g})/x_m$$
$$I_{2a} = -I_a\{1 + (x_{L1a} - x_c)/x_m\} + j(V - I_ar_{1a})/x_m$$

The torque is given by

$$E_gI_a \sin (E_g \times I_a) + E_aI_g \sin (E_a \times I_g) \text{ synch. watts}$$

where E_g and E_a are the e.m.f.'s (referred to the main stator winding) of the main and auxiliary stator windings and are given by—

$$E_g = V - I_g(r_{1g} + jx_{L1g})$$
$$E_a = V - I_a(r_{1a} + jx_{L1a} - jx_c)$$

Design Features

In designing a capacitor motor the aim is to ensure balanced operation, i.e. no transfer of power from one winding to the other and with each winding producing equal flux components so that the rotor operates with balanced currents; these conditions give operation similar to that of a two-phase motor although, of course, they can only obtain at a single load, usually about full load. Any

departure from the above conditions may result in a cheaper motor but impairs the performance due to circulating currents and the generation of harmonics.

A convenient design procedure is thus to design a two-phase motor to give the desired running characteristics at full load and then modify the auxiliary winding to give improved starting conditions or to reduce the cost of the capacitor.

It can be seen from the complexor diagram of Fig. 20.2 that at full-load the capacitor voltage for a balanced motor will be $\sqrt{2}$

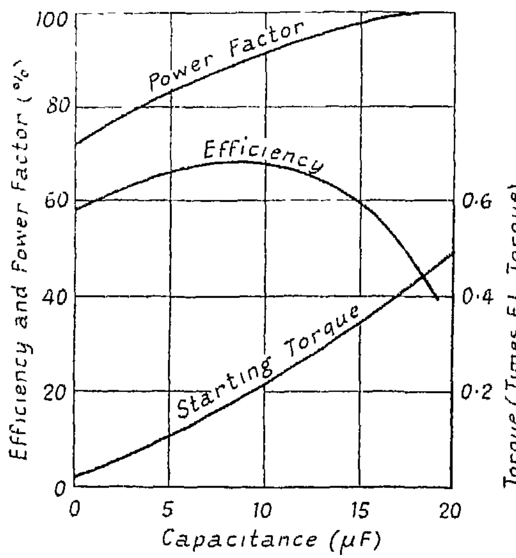


FIG. 20.5. EFFECT OF VARYING CAPACITANCE WITH 1/2-H.P. MOTOR OPERATING AT FULL-LOAD

times the winding or line voltage; the currents will also be the same so that the power factor will be about unity.

Reducing the turns on the auxiliary winding reduces both the auxiliary-winding voltage and the capacitor voltage so that a larger capacitor will be required; this would give a higher starting torque but this is rarely necessary except for small motors using a single capacitor for both starting and running. Increasing the auxiliary-winding turns results in higher voltages and therefore in a smaller and cheaper capacitor; the power factor is somewhat reduced and the total current increased, resulting in a lower efficiency. As a suitable compromise a value of K between about 1·2 and 1·5 is common. Calculation of the best ratio is usually done by trial and error, but investigations into the effect of varying the ratio have

been carried out by Trickey* and Macfarland.† The effect of varying the capacitance with a given motor is given in Fig. 20.5; about 10μF would be most satisfactory for running conditions but

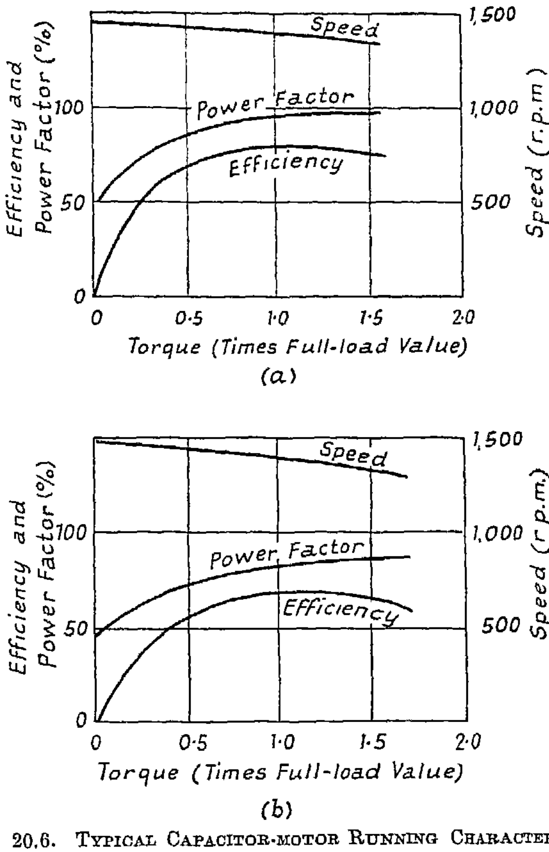


FIG. 20.6. TYPICAL CAPACITOR-MOTOR RUNNING CHARACTERISTICS

(a) 5-h.p. 4-pole motor.
(b) 1/4-h.p. 4-pole motor.

about 40 μF would be required to give an adequate starting torque (about 1·5 times full-load torque).

Typical Characteristics and Applications

Operating characteristics for a 1/4-h.p. and a 5-h.p. motor are given in Fig. 20.6. It is seen that in both cases the power factor is well

* P. H. Trickey, "The Equal-volt-ampere Method of Designing Capacitor Motors," *Trans. Amer. Instn. Elect. Engrs.*, **60**, p. 990 (1941).

† T. C. Macfarland, "Turn Ratio of the Capacitor Motor," *Trans. Amer. Instn. Elect. Engrs.*, **62**, p. 892 (1943).

above that of a plain single-phase induction motor. If a separate starting capacitor, or its equivalent, is used starting torques up to 1 or 2 times full-load torque can be obtained with 3 to 4 times full-load current.

The most common application of the capacitor motor is for driving refrigerators, but they are also widely used for any drive requiring a constant speed, silent operation and continuous running.

APPENDIX 1

HARMONIC ANALYSIS

TIME variations of voltage or current, and space variations of air-gap m.m.f. or flux density, are commonly periodic in character: such periodic functions can be resolved into a number of sine and cosine waves having periods equal to that of the original and sub-multiples thereof. Waves having periods equal to that of the original function are said to be of fundamental frequency and are known as *fundamentals*; the waves having frequencies of n times the fundamental are known as *harmonics*.

A periodic function, of period 2π, can thus be written as—

$$f(\theta) = a_0 + a_1 \cos\theta + a_2 \cos 2\theta + a_3 \cos 3\theta + \ldots + a_n \cos n\theta + b_1 \sin\theta + b_2 \sin 2\theta + b_3 \sin 3\theta + \ldots + b_n \sin n\theta \quad \text{(A.1.1)}$$

where $a_0, a_1, a_2, \ldots b_1, b_2, b_3, \ldots$ are constant coefficients. It may also be expressed in the form—

$$f(\theta) = a_0 + c_1 \sin(\theta + \psi_1) + c_2 \sin(2\theta + \psi_2) + \ldots + c_n \sin(n\theta + \psi_n) \quad \text{(A.1.2)}$$

where $c_n = \sqrt{(a_n^2 + b_n^2)}$ and $\psi_n = \tan^{-1}(a_n/b_n)$.

Mathematical text-books on Fourier analysis show that

$$a_0 = (1/2\pi)\int_0^{2\pi} f(\theta)\,d\theta = \text{average height of curve over a period} \quad \text{(A.1.3)}$$

$$a_n = (1/\pi)\int_0^{2\pi} f(\theta)\cos n\theta\,d\theta \quad \text{(A.1.4)}$$

$$b_n = (1/\pi)\int_0^{2\pi} f(\theta)\sin n\theta\,d\theta \quad \text{(A.1.5)}$$

If succeeding half waves are identical but reversed, as is usual for waves associated with rotating machinery, expressions (A.1.4) and (A.1.5) may be written—

$$a_n = (2/\pi)\int_0^{\pi} f(\theta)\cos n\theta\,d\theta \quad \text{(A.1.6)}$$

$$b_n = (2/\pi)\int_0^{\pi} f(\theta)\sin n\theta\,d\theta \quad \text{(A.1.7)}$$

The process of harmonic analysis is the determination of these constants, thus enabling any periodic wave, such as the m.m.f.

waves mentioned in Chapter 1, to be resolved into their fundamentals and harmonics. The behaviour of machines is governed chiefly by the fundamentals of the voltage, current and m.m.f., and only these are usually taken into account in discussing behaviour as they can easily be represented by complexor or space vector diagrams. The harmonics have, however, important and usually disadvantageous secondary effects and frequently have to be considered when discussing noise, losses and other such items.

Various graphical, arithmetical and analytical methods of harmonic analysis are available, a typical example of each being given below.

Graphical Method

As mentioned above the value of constant a_0 is the average height of the wave over a period; this can easily be determined by

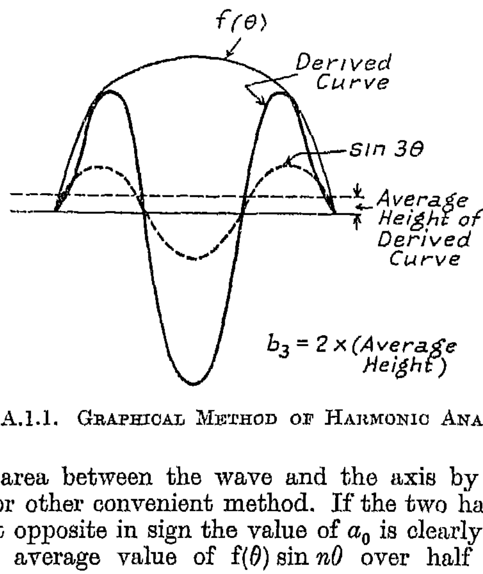


FIG. A.1.1. GRAPHICAL METHOD OF HARMONIC ANALYSIS

finding the area between the wave and the axis by means of a planimeter or other convenient method. If the two half waves are identical but opposite in sign the value of a_0 is clearly zero.

Since the average value of $f(\theta)\sin n\theta$ over half a period is $(1/\pi)\int_0^{\pi} f(\theta)\sin n\theta\, d\theta$ the coefficient b_n is, from expression (A.1.7), equal to *twice* this average value; similarly, coefficient a_n is twice the average value of $f(\theta)\cos n\theta$ over half a period.

To find a coefficient, say b_3, of the wave $f(\theta)$ in Fig. A.1.1 the wave must first be drawn to a suitable scale. On the same base $\sin 3\theta$ is then drawn as shown and corresponding ordinates of the two curves multiplied together to give a third curve, $f(\theta) . \sin 3\theta$, known as the *derived* curve. The average height of this derived curve can be found by a planimeter (or otherwise) and twice this average height

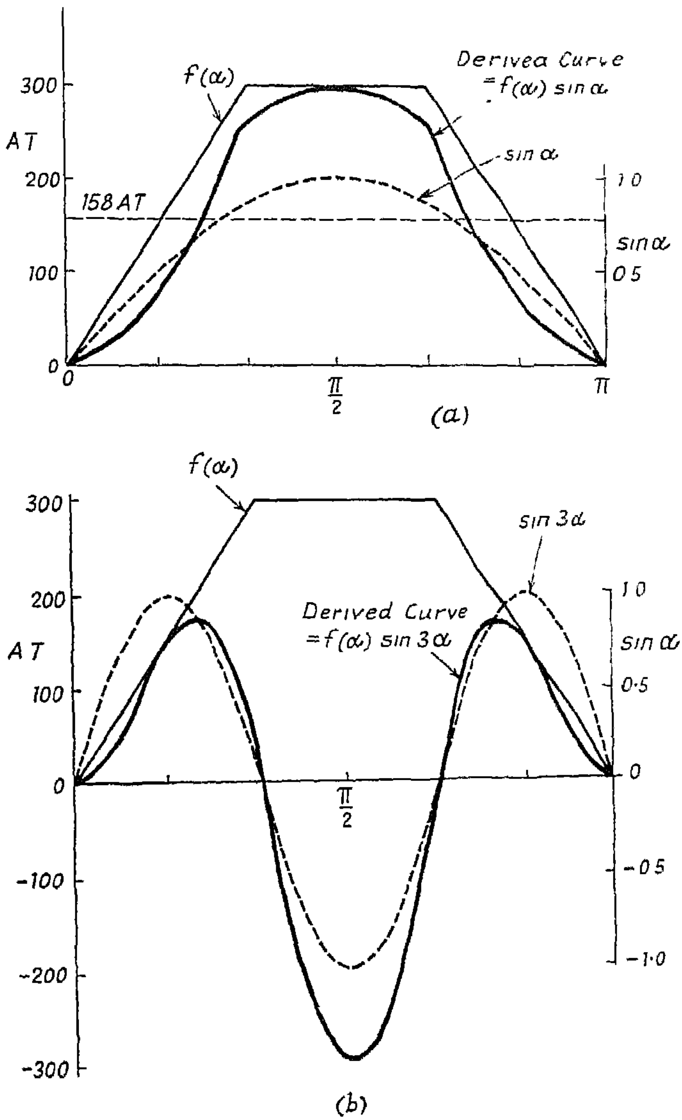


FIG. A.I.2. GRAPHICAL METHOD OF HARMONIC ANALYSIS

(a) Determination of coefficient b_1.
(b) Determination of coefficient b_3.

gives the required coefficient b_3. A similar procedure can be adopted to determine the fundamental and other harmonics as shown in Example A.1.1.

Example A.1.1. Determine, by the graphical method, the fundamental and third harmonic of the trapezoidal m.m.f. wave shown in Fig. 1.3, Chapter 1.

(i) FUNDAMENTAL. The m.m.f. wave and $\sin \alpha$ are plotted in Fig. A.1.2(*a*), the general angle, θ, being replaced in this case by the angle, α, the angular distance around the periphery; multiplying these gives the derived curve, $f(\alpha) \sin \alpha$, as shown by the thick curve. The average height of this is 158 AT. The coefficient b_1 is thus $2 \times 158 = 316$ AT.

(ii) THIRD HARMONIC. In Fig. A.1.2(*b*) the m.m.f. wave to be analysed is replotted together with $\sin 3\alpha$: the derived curve obtained by multiplying ordinates of these two curves is now $f(\alpha) \sin 3\alpha$ and its average height is zero. Coefficient b_3 is thus zero.

A similar procedure will show that $b_5 = -12{\cdot}7$ AT and $b_7 = 6{\cdot}5$ AT and also that all the *a* coefficients are zero. The latter conclusion could, however, have been reached by inspection as described later.

The expression for the m.m.f. wave is thus

$$F = 316 \sin \alpha - 12{\cdot}7 \sin 5\alpha + 6{\cdot}5 \sin 7\alpha + \ldots$$

Arithmetical Methods

In arithmetical or tabular methods the drawing of the derived curve is avoided by estimating its area from a limited number of ordinates. To secure reasonable accuracy the number of ordinates in a half period should not be less than 12 or $2n$, whichever is the greater.

The procedure is best set out in tabular form using the following columns—

1. Angle θ.
2. Ordinate at θ.
3. $n\theta$.
4. $\sin n\theta$.
5. Col. (2) $\times$ col. (4) $= f(\theta) \sin n\theta$.

Adding the fifth column and dividing by the number of ordinates gives the average value of $f(\theta) \sin n\theta$. Twice this is the value of the desired coefficient. The procedure is illustrated in Example A.1.2.

Example A.1.2. Repeat Example A.1.1 using the arithmetic method.

The results are set out in the table on page 337.

It can be seen that the values of b_1 and b_3 agree with the values obtained by the graphical method but that, due to the limited number of ordinates, there is a slight error in finding b_5.

		Fundamental		3rd Harmonic			5th Harmonic		
1	2	3	4	5	6	7	8	9	10
$\alpha°$	Ord. at $\alpha°$	sin α	(2) × (3)	3α	sin 3α	(2) × (6)	5α	sin 5α	(2) × (9)
0	0	0	0	0	0	0	0	0	0
15	75	0·259	19·4	45	0·707	53	75	0·966	72·5
30	150	0·5	75	90	1·0	150	150	0·5	75
45	225	0·707	159	135	0·707	159	225	− 0·707	− 159
60	300	0·866	260	180	0	0	300	− 0·866	− 260
75	300	0·966	290	225	− 0·707	− 212	375	0·259	77·7
90	300	1·0	300	270	− 1·0	− 300	450	1·0	300
105	300	0·966	290	315	− 0·707	− 212	525	0·259	77·7
120	300	0·866	260	360	0	0	600	− 0·866	− 260
135	225	0·707	159	405	0·707	159	675	− 0·707	− 159
150	150	0·5	75	450	1·0	150	750	0·5	75
165	75	0·259	19·4	495	0 707	53	825	0·966	72·5
Sum		1,906·8		0			− 87·6		
Divided by 12		158·9		0			− 7·3		
Multiplied by 2		$b_1 = 317·8$		$b_3 = 0$			$b_5 = - 14·6$		

Analytical Method

When the curve is of such a simple shape that it can be represented by a simple mathematical expression the coefficients can be found by direct integration. The procedure is illustrated in Example A.1.3. Use is made of the following standard integrals—

$$\int \theta \sin n\theta \, d\theta = \frac{\sin n\theta}{n^2} - \frac{\theta \cos n\theta}{n}$$

$$\int \theta \cos n\theta \, d\theta = \frac{\cos n\theta}{n^2} - \frac{\theta \sin n\theta}{n}$$

Example A.1.3. Repeat Example A.1.1 by direct integration.

The curve is made up of three parts—

(i) from 0 to δ (60°) $\quad f(\alpha) = F_m \,.\, \alpha/\delta$

(ii) from δ to $(\pi - \delta)$ (60° to 120°) $\quad f(\alpha) = F_m$

(iii) from $(\pi - \delta)$ to π (120° to 180°) $\quad f(\alpha) = F_m(\pi - \alpha)/\delta$

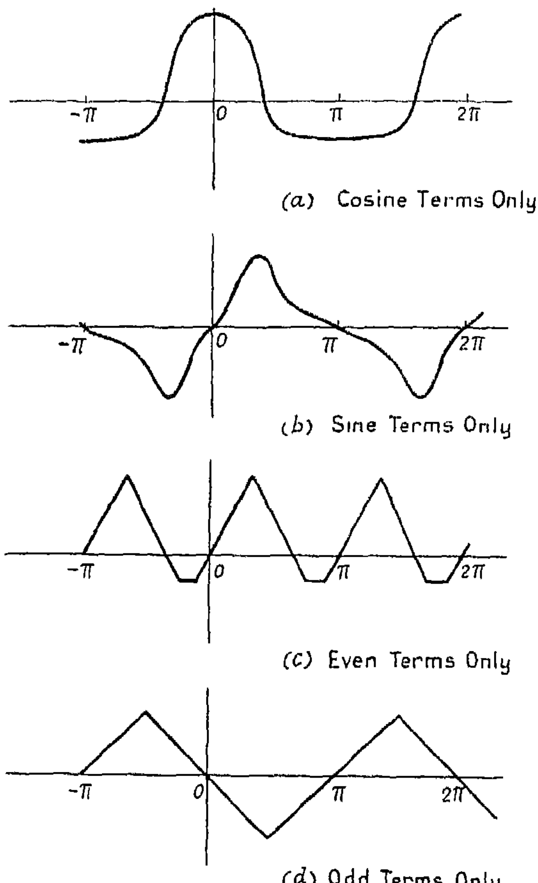


FIG. A.1.3. CONDITIONS FOR ELIMINATING CERTAIN GROUPS OF HARMONICS

(a) Wave symmetrical about ordinate $\theta = 0$: cosine terms only.
(b) Wave symmetrical about ordinate $\theta = 0$ but inverted: sine terms only (and $a_0 = 0$).
(c) Second half-wave repeats first half-wave: even terms only.
(d) Second half-wave repeats first half-wave but inverted: odd terms only (and $a_0 = 0$).

Thus

$$b_n = (2/\pi)\left\{\int_0^\delta f(\alpha)\sin n\alpha\, d\alpha + \int_\delta^{\pi-\delta} f(\alpha)\sin n\alpha\, d\alpha + \int_{\pi-\delta}^{\pi} f(\alpha)\sin n\alpha\, d\alpha\right\}$$

$$= (2F_m/\pi)\left\{\int_0^\delta (1/\delta)\alpha\sin n\alpha\, d\alpha + \int_\delta^{\pi-\delta}\sin n\alpha\, d\alpha + \int_{\pi-\delta}^{\pi}(1/\delta)(\pi-\alpha)\sin n\delta\, d\delta\right\}$$

$$= (2F_m/\pi)\left\{2/\delta\int_0^\delta \alpha\sin n\alpha\, d\alpha + \int_\delta^{\pi-\delta}\sin n\alpha\, d\alpha\right\}$$

$$= (F_m/\pi)(4/\delta)(\sin n\delta)/n^2$$

Hence putting $n = 1$

$$b_1 = \frac{300}{\pi}\times\frac{4}{\pi/3}\times\frac{\sin(\pi/3)}{1} = 366\times\sqrt{3}/2 = 316 \text{ AT}$$

$$b_3 = \frac{300}{\pi}\times\frac{4}{\pi/3}\times\frac{\sin\pi}{9} = 0$$

$$b_5 = \frac{300}{\pi}\times\frac{4}{\pi/3}\times\frac{\sin(5\pi/3)}{25} = -\frac{366}{25}\times\sqrt{3}/2 = -12{\cdot}7 \text{ AT}$$

$$b_7 = \frac{300}{\pi}\times\frac{4}{\pi/3}\times\frac{\sin(7\pi/3)}{49} = \frac{366}{49}\times\sqrt{3}/2 = 6{\cdot}5 \text{ AT}$$

These results agree with those of the graphical method. Similar analysis will show that all the a coefficients are zero.

Preliminary Inspection of Wave

It is often possible, by inspection of the wave, to see that certain harmonics are zero, thereby saving labour in calculation. The conditions are set out in Fig. A.1.3.

APPENDIX 2

CURRENT LOCUS IN CIRCUIT CONTAINING RESISTANCE AND REACTANCE

A CIRCUIT with variable resistance and reactance is shown in Fig. A.2.1(a). It is shown below that if the ratio of X_2 to R_2 is constant the locus of the end of the current complexor is a circle.

Let R and X be the total resistance and reactance of the circuit and let G and B be the corresponding conductance and susceptance. The circuit of Fig. A.2.1(a) can then be represented by that of

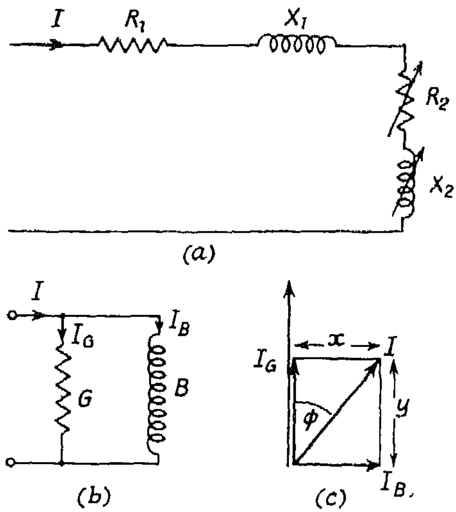


FIG. A.2.1. CIRCUIT WITH VARIABLE RESISTANCE AND REACTANCE

(a) Circuit with variable resistance and reactance.
(b) Equivalent circuit of (a).
(c) Complexor diagram

Fig. A.2.1(b), and the currents in it by the complexor diagram of Fig. A.2.1(c) in which

$$x = I \sin \phi = V \,.\, B$$

and

$$y = I \cos \phi = V \,.\, G$$

For the circuits to be equivalent

$$R = R_1 + R_2 = \frac{G}{G^2 + B^2} = \frac{y/V}{y^2/V^2 + x^2/V^2} = \frac{Vy}{x^2 + y^2}$$

$$\therefore \qquad Vy - R_1(x^2 + y^2) = R_2(x^2 + y^2)$$

Also

$$X = X_1 + X_2 = \frac{B}{G^2 + B^2} = \frac{x/V}{y^2/V^2 + x^2/V^2} = \frac{Vx}{x^2 + y^2}$$

$$\therefore \qquad Vx - X_1(x^2 + y^2) = X_2(x^2 + y^2)$$

Dividing,

$$\frac{Vx - X_1(x^2 + y^2)}{Vy - R_1(x^2 + y^2)} = \frac{X_2}{R_2} = K$$

which reduces to

$$x^2 + y^2 - \frac{Vx}{X_1 - KR_1} + \frac{KVy}{X_1 - KR_1} = 0$$

This is the equation to a circle of the form—

$$x^2 + y^2 + 2gx + 2fy - C = 0$$

so that the co-ordinates of its centre are

$$x' = -g = \frac{V}{2(X_1 - KR_1)} = \frac{V}{2X_1(1 - K\,R_1/X_1)}$$

$$y' = -f = -\frac{KV}{2(X_1 - KR_1)} = Kx'$$

ANSWERS TO EXERCISES

Exercises 1, page 16

1. 270 AT; 90 AT, 33·3 per cent; 54 AT, 20 per cent.

2. (*a*) 860 AT. (*b*) 645 AT. (*c*) 1,290 AT, 9·6 turns.

3. $\hat{F} = 382$ AT; Conductors per slot, 10, 10, 9, 7, 5, 0, 0.

4. $\hat{F} = 690$ AT; $F_1 = 630$ AT.

5. (*a*) $F_1 = 3{,}650$ AT. (*b*) $F_1 = 1{,}740$ AT. (*c*) 5,050 AT.

6. F_R at 54·8° to 700-AT winding = 860 AT.

Exercises 2, page 26

1. (*a*) 169 V; 50 c/s. (*b*) 56·3 V; 50 c/s. (*c*) 150 V; 50 c/s. $e = 0{\cdot}4 \sin 314t \,.\, \sin 126t$.

2.

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
Brushes	240 V; 0 c/s	158 V; 3·33 c/s	181 V; 3·33 c/s	208 V; 0 c/s 104 V;	137 V; 3·33 c/s
Adj. rings	85·2 V; 50 c/s	70 V; 46·7 c/s	91 V; 53·3 c/s	85·2 V; 50 c/s	70 V; 46·7 c/s
Diam. rings	170·4 V; 50 c/s	158 V; 46·7 c/s	181 V; 53·3 c/s	170·4 V; 50 c/s	158 V; 46·7 c/s

3. 28·7°; 141 V; 50 c/s.

4. 5·77 A; 27·9 V; 18·5 V.

5. 80 V; 40 V; 0·00118 Wb.

6. 27·7 V; 87·5 V; 43·7 V.

Exercises 3, page 42

1. 0·75 V.

2. (*a*) 100 A; 1·96 V. (*b*) 141 A; 1·96 V. (ci) 141 A; 0·49 V. (cii) 141 A; 0·33 V. At 120°, 1·96 and 1·31 V.

3. 3·33 V. 2·22 V. 1,500 r.p.m.

4. (i) 0·95 V. (ii) 3·8 V.

Exercises 4, page 56

1.

	s	$\cos \phi_2$	I_w
(i)	$\pm$ 0·0202	0·993	993 A
(ii)	$\pm$ 0·0423	0·993	993 A
(iii)	$\pm$ 0·0215	0·929	929 A
(iv)	$\pm$ 0·0212	0·946	946 A
(v)	0·23 or 0·179	0·6	600 A
(vi)	$-$ 0·23 or $-$ 0·179	0·6	600 A
(vii)	$\pm$ 0·0174	0·914	914 A
(viii)	$\pm$ 0·0224	0·895	895 A
(ix)	$\pm$ 0·0123	0·913	913 A

2. 4·7 kVA.

3. (*a*) 300 kW; 700 kW; 700 kW; 14,200 Lb-ft; 700 kW.
   (*b*) − 300 kW; 1,300 kW; 1,300 kW; 14,200 Lb-ft; 1,300 kW.
   (*a*) 300 kW; 700 kW; 1,000 kW; 20,000 Lb-ft; 1,000 kW.
   (*b*) − 300 kW; 1,300 kW; 1,000 kW; 10,800 Lb-ft; 1,000 kW.

4. 0·0274 Ω/ph. (equiv. star); 13 kVA.

5. Slip power: (*a*) (i) $1{,}000s/(1-s)$ (*b*) (i) $1{,}000s$
   (ii) $1{,}000s$ (ii) $1{,}000s(1-s)$
   (iii) $1{,}000s(1-s)$ (iii) $1{,}000s(1-s)^2$
   (iv) $1{,}000s(1-s)^2$ (iv) $1{,}000s(1-s)^3$

6. Slip power $= 10{,}000(s - 2s^2 + s^3)$.

7. 283 kW; 246 kW; 123 kW; 49 kW; 102 kW; 8·6 kW;
   712 kW; 643 kW; 772 kW; −313 kW; 244 kW; 8·6 kW.

Exercises 5, page 75

1. Speed, 100 r.p.m. with field: $E = 6$ V; $\phi = 33{\cdot}5°$.
   Speed, 100 r.p.m. against field: $E = 37$ V; $\phi = 82{\cdot}2°$.

2. 10,450 μF.

3. 545 r.p.m.

4. 14,200 μF; 9·0 A.

5. (i) 36·2 − j13·3 A; 37·7 − j28·2 A; 0·8 lag.
   (ii) 19° or 87°; $s = 0{\cdot}00727$ or 0·0221; 21 A at 0·897 lead or 46·2 A at 0·998 lead.

6. 0·77 lead; 29·5 kW.

7. $s = \sqrt{[(a')^2 \sin^2\delta + q^2]/[r_1{}^2 + (x_{L1} + x_{L2}')^2]}$

Exercises 6, page 95

1. (*a*) Voltage ratio = 0·58; frequency ratio = 1/3.
   (*b*) Voltage ratio = 0·41 (max.); frequency ratio = 0/50.

2. (*a*) $116\angle{-40°}$ A; 0·766 lag; 622 h.p.
   (*b*) $136\angle{22°}$ A; 0·927 lead; 920 h.p.; 36 kW.
   (*c*) $89\angle{27°}$ A; 0·89 lag; 620 h.p.

3. At 225 r.p.m.: 104·5 V/ph.; 278 A.
   At 350 r.p.m.: 82 V/ph.; 277 A.

Exercises 7, page 104

1. 50 kW; 33·3 kW; 16·67 kW; 16·67 kW.

2. − 50 V; 500 r.p.m.; 25·8°.

3. 0·4 Ω (capacitive).

Exercises 8, page 125

1. (*a*) 386 AT; 17 V.
   (*b*) 772 AT; 34 V.

2. (3 brushes)/(6 brushes) = 4/3.

3. (*a*) Added res. = 0·1 Ω: TM = 374 K; TM per ampere = 0·87 K.

(*b*) E.m.f. = 40 V· TM = 525 K; TM per ampere = 0·275 K

4. 400–1,600 r.p.m.

5.

	Load	Speed (r.p.m.)	cos ϕ
$E_j = -100$ V	F.L.	430	0·93 lag
	10%	650	0·054 lead
$E_j = 0$	F.L.	936	0·923 lag
	10%	1,161	0·054 lead
$E_j = 100$ V	F.L.	1,436	0·93 lag
	10%	1,668	0·054 lead

6. Stator current = 6·3 A; total input current = 4·35 A; cos ϕ = 0·67; torque = 13·5 Lb-ft; 1·14 h.p.; 1·47 kW.

Exercises 9, page 167

1. 28·7° (mech.); 141 V; 50 c/s.

2. 800 r.p.m.

3. 126 turns.

4. 16·2 A; 14·5 V.

5. 1,630 AT.

6. x_{L2} = 0·069 Ω; 19·5 A at cos ϕ = 0·67 (lag); 165 r.p.m.

7. $s = -0·4$, $\rho = -5°$.

Circuit	(*a*)	(*b*)	(*c*)	(*d*)
Input current (A)		$96·5\underline{/14·7°}$	$86·7\underline{/31·4°}$	$83\underline{/12·7°}$
Input cos ϕ . .		0·97 lead	0·853 lead	0·975 lead
Input power (kW) .	as (*b*)	65	51·5	50
Output power (kW) .		54·3	41·8	38·5
Torque (Lb-ft) . .		360	280	260
Secondary current (A/ph.)		59·5	64·5	66

Exercises 10, page 181

1. Primary current = 20 A. (*a*) cos ϕ = 0·76 lag; (*b*) cos ϕ = 0·92 lead.

2. $41·8\underline{/35°}$ A/ph.; cos ϕ = 0·82 lead; 50·5 h.p.; 1,531 r.p.m.

3. (*a*) 4·42 V; 0·925 V. (*b*) 2·21 V; 0·463 V. (*c*) 3·84 V; 0·454 V (*d*) 7·3 V; 0·86 V.

4. 16 per cent.

5. 0·36 Wb.

Exercises 11, page 209

1. Speed = 250 r.p.m.; $\cos\phi = 0{\cdot}8$.

2.

S	(*a*)			(*b*)			(*c*)		
	V_1	V_2	$\cos\phi$	V_1	V_2	$\cos\phi$	V_1	V_2	$\cos\phi$
1·0	690	690	0	620	310	0	310	620	0
0·5	620	310	0·73	520	155	0·28	690	690	0·74
0	420	0	0·95	420	0	0·44	420	0	0·87 lead
−1·0	210	210	1·0	300	150	0·64	150	300	0·59 lead

3. 0·62.

Exercises 13, page 233

1. (*a*) 47 V. (*b*) 0·00028 Wb. (*c*) 0·21 h.p. (*d*) 9,300 r.p.m.

2. 1,710 r.p.m.; 0·87.

4. 0·62 A; 0·7 p.f.; 0·071 h.p.

5. 43 turns; 0·955 Wb/m^2.

6. $D = 1\frac{3}{8}$ in. $L = 1\frac{1}{2}$ in. Field turns per pole = 200. Armature turns = 1,700.

Exercises 14, page 247

1. $E_{tsd} = 103$ V; $E_{taq} = 109$ V; $E_{rad} = 400$ V.

2. $V = 265$ V; 0·906; 1·67 V.

3. $I = 500$ A. 770 r.p.m.; 0·92 p.f.; 100 per cent F.L

4. 5 turns; 750 AT.

5. 0·0135 Wb; $E_{rad} = 158$ V; 26; 10 poles.

Exercises 15, page 262

1. 2·2 A; 0·76 p.f.; 1·3 Lb-ft.

2. 12 A; 0·68 p.f.

3. $I = z_{1q} + \dfrac{\mathrm{j}(x_{mq}z_{1d} + x_{mq}z_{2q} - x_{md}x_{mq}S) - x_{md}x_{mq}}{z_{1d} + z_{2q} - Sx_{md} + \mathrm{j}(x_{md} + x_{mq})}$

4. (*a*) 1,200–1,750 r.p.m. (*b*) 770–2,000 r.p.m.

Exercises 17, page 309

1. 6·67 c/s; 93·3 c/s. $\Phi_f/\Phi_b = 1{\cdot}37$.

2. $s = 0{\cdot}5$. $TM_f = 2{,}100$ synch. W; $TM_b = 700$ synch. W. Nett. $TM = 1{,}400$ synch. W. $E_f \propto \Phi_f = 102$ V. $E_b \propto \Phi_b = 42$ V.

3. 1,630 W; 69·5 per cent.

4. $I = 15$ A; $V = 236$ V.

5. (*a*) 2·72 Ω; 0·87 Lb-ft; 0·0225 Lb-ft.
(*b*) 4·67 Ω; 0·865 Lb-ft; 0·0241 Lb-ft.

6. (*a*) Added resistance for maximum torque = 5·75 Ω; torque = 100 per cent.
(*b*) Capacitance for maximum torque = 600 μF; torque = 850 per cent.

INDEX